This booklet contains 24 printed pages **No.:**

PAPER - 1 : MATHEMATICS, CHEMISTRY & PHYSICS

Test Booklet Code

R

Do not open this Test Booklet unit you are asked to do so.
Read carefully the Instructions on the Back Cover of this Test Booklet

Important Instructions:

1. Immediately fill in the particulars on this page of the Test Booklet with Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet out the Answer sheet and fill in the particulars carefully.
3. The test of **3 hours** duration
4. The Test Booklet consists of 90 questions. The maximum marks are 360
5. There are **three** parts in the questions paper A, B, C, consisting of **Mathmatics, Chemistry** and **Physics** having 30 question in each part of equal weightage. Each question is allotted **4(four)** marks for each correct response.
6. Candidates will be awarded marks as stated above in instruction No. 5 correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
7. There is only correct response for each question. filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.
8. Use **Blue/Black Ball Point Pen only** for writing particulars/marking reponeses on side-1 and **side-2** of the Answer Sheet. **Use of pencil is strictly prohibited.**
9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phon, any electronic device etc. except the Admit Card inside the examination hall/room.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in pages (pages 21-23) at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. **However, the candidates are allowed to take aways this Test Booklet with them.**
12. The CODE for this Booklet is **R.** Make sure that the CODE printed on Side-2 of the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
13. **Do not fold or make any stray marks on the Answer Sheet.**

SEAL

Name of the Candidate (in Capital letters) : ____________________

Roll Number :in figures | | | | | | | | |

:in words ____________________

Examination Center Number : | | | | | | |

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Candidate's Signature : ____________ Invigilator;s Signature: ____________

PART A - MATHEMATICS

1. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval:

(1) $\left(\frac{11}{12}, 1\right]$

(2) $\left(\frac{1}{2}, \frac{3}{4}\right]$

(3) $\left(\frac{3}{4}, \frac{11}{12}\right]$

(4) $\left[0, \frac{1}{2}\right]$

Key.4
Soln: 1

$$1 - p^5 \geq \frac{31}{32}$$

$$\Rightarrow p^5 \leq \frac{1}{2^5} \Rightarrow p \leq \frac{1}{2}$$

$$\Rightarrow p \in [0, 1/2]$$

2. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is :

(1) 132

(2) 144

(3) -132

(4) -144

Key.4
Soln: 2

Coefficient of x^7 in $\{(1-x) - x^2(1-x)\}^6$

$$\{(1-x)^6 (1-x^2)^6\}$$

$$\Rightarrow 6C_1.6C_3 - 6C_3.6C_2 + 6C_5.6C_1$$

$= -144$

3. $\lim_{x \to 2} \left(\frac{\sqrt{1 - \cos\{2(x-2)\}}}{x-2} \right)$

(1) equals $\frac{1}{\sqrt{2}}$

(2) does not exist

(3) equal $\sqrt{2}$

(4) equal $-\sqrt{2}$

Key.2
Soln: 3

$$\lim_{x \to 2} \frac{\sqrt{1 - \cos 2(x-2)}}{x-2} = \lim_{x \to 2} \frac{\sqrt{2}\,|\sin(x-2)|}{x-2}$$

R.H.L. at x = 2 $\Rightarrow \lim_{h \to 0} f(2+h) = \sqrt{2} \lim_{h \to 0} \frac{\sin h}{h} = \sqrt{2}$

L.H.L. at x = 2 $\Rightarrow \lim_{h \to 0} f(2-h) = -\sqrt{2} \lim_{h \to 0} \frac{\sin h}{h} = -\sqrt{2}$

As R.H.L. $\neq$ L.H.L. i.e. $\lim_{x \to 2} f(x)$ does not exist.

4. Let R be the set of real numbers.

Statement - 1 :

$A = \{(x, y) \in R \times R : y - x \text{ is an integer}\}$ is an equivalence relation on R

Statement - 2 :

$B = \{(x, y) \in R \times R : x = xy\}$ for some relation number an equivalence relation on R

(1) Statement-1 is false, Statement -2 is true

(2) Statement-1 is ture, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(4) Statement-1 is true, Statement - 2 is false.

Key.4
Soln: 4

Statement -1 is reflexive, symmetric, Transitive and hence equivalance.
Statement -2 is non-symentric and hence not equivalence.

5. Let α, β be real and z be a complex number.

If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re z = 1, then it is necessary that:

(1) $\beta \in (1, \infty)$

(2) $\beta \in (0, 1)$

(3) $\beta \in (-1, 0)$

(4) $|\beta| = 1$

Key.1

Soln: 5 $z = 1 + iy$, $y \leftarrow R$, β = (1+iy) (1-iy)

$$\Rightarrow \beta = 1 + y^2 \Rightarrow \beta \in (1, \infty)$$

6. $\frac{d^2 x}{d y^2}$ equals:

(1) $-\left(\frac{d^2y}{dy^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

(2) $-\left(\frac{d^2y}{dy^2}\right)^{-1}$

(3) $-\left(\frac{d^2y}{dy^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

(4) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

Key.1

Soln: 6

$$\frac{d^2n}{d\,y^2} = \frac{d}{dy}\left\{\frac{d\,x}{d\,y}\right\} = \frac{d}{dn}\left\{\left(\frac{d\,y}{d\,x}\right)^{-1}\right\} \cdot \frac{d\,x}{d\,y}$$

$$= -\left(\frac{dy}{dx}\right)^{-3} \cdot \frac{d^2y}{dx^2}$$

7. The number of values of k for which the linear equations

4x + ky + 2z = 0

kx + 4y + z = 0

2x + 2y + z = 0

possess a non-zero solution is :

(1) Zero

(2) 3

(3) 2

(4) 1

Key.3

Soln: 7

$$\Rightarrow 4\,(4\text{-}2) - K\,(K\text{-}2) + 2(2k - \theta) = 0$$

$$\Rightarrow \theta - K^2 + 2K + 4K - 16 = 0$$

$$\Rightarrow K^2 - 6K + \theta = 0 \Rightarrow \boxed{K = 2,\ 4}$$

8. **Statement-1 :**

The point A(1, 0, 7) is the mirror image of the point B(1, 6, 3) in the line:

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

Statement-2 :

The line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ bisects the line segment joining A(1, 0, 7) and B(1, 6, 3)

(1) Statement-1 is false, Statement-2 is true.

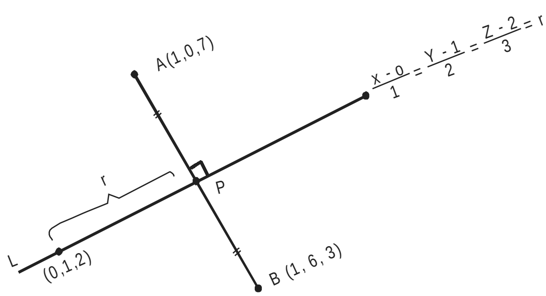
(2) Statement-1 is true, Statement-2 is true Statement-2 is a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is true Statement-2 is not a correct explanation for Statement-1

(4) Statement-1 is true, Statement-1 is false.

Key.3

Soln: 8



Dr's of A B $\Rightarrow$ 0, 6, -4

Dr's of L $\Rightarrow$ 1, 2, 3

$\Rightarrow a_1 a_2 + b_1 b_2 + c_1 c_2 = 0+12-12=0$

i. e. $L \perp AB$

P: (1, 3, 5) = (r, 1+2r, 2+3r)

= r =1, 1+2r = 3, 2+3r=5

A is Image of B $\Rightarrow$ (i) A B should be $\perp$ to L

(ii) Mid-point of A B. I.e. P should lie on L.

9. Consider the following statement

P : Suman is brilliant

Q : Suman is rich

R : Suman is honest

The negation of the Statement ``Suman is brilliant and dishonest if and only if Suman is rich'' can be expressed as :

(1) $\sim(P \wedge \sim R) \leftrightarrow Q$

(2) $\sim P \wedge (Q \leftrightarrow \sim R)$

(3) $\sim(Q \leftrightarrow (P \wedge \sim R))$

(4) $\sim Q \leftrightarrow \sim P \wedge R$

Key.3

Soln: 9 $\sim (Q \leftrightarrow (P \wedge \sim R))$

10. The lines L_1 : y - x = 0 and L_2 : 2x + y =0 intersect the line L_3 : y + 2 = 0 at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement-1

The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$.

Statement-2

In any triangle, bisector of an angle divides the traingle into two similar trangles.

(1) Statement-1 is false, Statement-2 is true

(2) Statement-1 is true, Statement-2 is true is a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is true Statement-2 is **not** a correct explanation for Statement-1

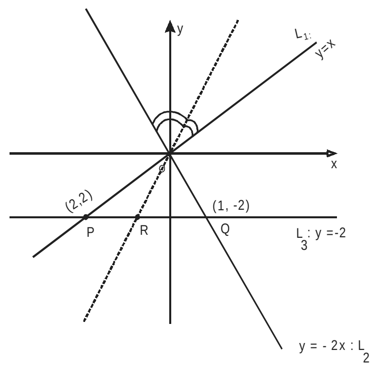
(4) Statement-1 is true, Statement-2 is false.

Key.4

Soln: 10

$PR : RQ = OP : OQ$

$= \sqrt[2]{2} : \sqrt{5}$.



11. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after:

(1) 21 months
(2) 18 months
(3) 19 months
(4) 20 months

Key.1

Soln: 11

$$\underbrace{200 + 200 + 200 + 250 + 200 + \ldots\ldots\ldots}_{n\text{- months}}$$

$$400 + \frac{n}{2}.\{400 + (n-1).\,40\} = 11040$$

$\Rightarrow$ n = 19 That means total months = 19 + 2

$\boxed{= 21.}$

12. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (-3, 1) and has eccentricity $\sqrt{2/5}$ is :

(1) $5x^2 + 3y^2 - 32 = 0$

(2) $3x^2 + 5y^2 - 32 = 0$

3) $5x^2 + 3y^2 - 48 = 0$

(4) $3x^2 + 5y^2 - 15 = 0$

Key.2
Soln: 12

Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{9}{a^2} + \frac{1}{b^2} = 1 \quad \text{------ } 1$$

$$e^2 = 1 - \frac{b^2}{a^2} \Rightarrow$$

$$\frac{b^2}{a^2} = 3/5 =, \frac{5}{a^2} = \frac{3}{b^2} \quad \text{------- } 2$$

Using 1 and 2

$$\Rightarrow \frac{1}{a^2} = \frac{3}{32}, \frac{1}{b^2} = \frac{5}{32}$$

$$\Rightarrow \boxed{3x^2 + 5y^2 - 32 = 0}$$

13 If A = $\sin^2 x + \cos^4 x$, then for all real x :

(1) $\frac{3}{4} \le A \le \frac{13}{16}$

(2) $\frac{3}{4} \le A \le 1$

(3) $\frac{13}{16} \le A \le 1$

(4) $1 \le A \le 2$

Key.2
Soln: 13

$$A = \sin^2 x + \cos^y x$$

$$= 1 - \cos^2 x + \cos^y x + \frac{1}{4} - \frac{1}{4}$$

$$= \left(\cos^2 X - \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$\boxed{\frac{3}{4} \le A \le 1}$$

14 The value of $\int_0^1 \frac{8 \log(1+x)}{1+x^2} dx$ is :

(1) log 2

(2) π log 2

(3) $\frac{\pi}{8}$ log 2

(4) $\frac{\pi}{2}$ log 2

Key.2
Soln: 14

$$8\int_0^1 \frac{\log(1+x)}{1+x^2} dx$$

Let $x = \tan\theta \Rightarrow dn = See^2 \theta$ do

$$\Rightarrow I = 8\int_0^{\lambda/4} \log(1+\tan\theta)\, do \text{-------- } 1$$

$$I = 8\int_0^{\lambda/4} \log\left\{1+\tan\left(\frac{\lambda}{4} - \theta\right)\right\} d\theta$$

$$I = 8\int_0^{\lambda/4} \log\left(1+\frac{1-\tan\theta}{1+\tan\theta}\right) d\theta$$

$$I = 8\int_0^{\lambda/4} \log\left(\frac{2}{1+\tan\theta}\right) d\theta$$

$$I = 8\int_0^{\lambda/4} \log 2\, d\theta$$

$$\boxed{I = \lambda \log 2}$$

15. If the angle between the lien

$x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane

$x + 2y + 3z = 4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, λ

equals:

(1) $5/3$

(2) $2/3$

(3) $3/2$

(4) $2/5$

Key.2
Soln: 15

$$L : \frac{x-0}{1} = \frac{y-1}{2} = \frac{2-3}{\lambda}$$

P : x + 2y + 32 = 4

$$\text{Sin}\theta = \frac{1}{\sqrt{14}} = \frac{1+4+3\lambda}{\sqrt{5+\lambda^2}\sqrt{14}}$$

$$\Rightarrow 9(5+\lambda^2) = 25 + 9\lambda^2 + 30\lambda$$

$$20 = 30\lambda \quad \Rightarrow \boxed{\lambda = \frac{2}{3}}$$

16. For $x \in \left(0, \frac{5\pi}{2}\right)$, define $f(x) = \int_0^x \sqrt{t}$ sint dt

Then f has:

(1) local maximum at π and local minimum at 2π.
(2) local maximum at π and 2π.
(3) local minimum at π and 2π.
(4) local minimum at π and local maximum at 2π.

Key.1

Soln: 16 $f^1(x) = \sqrt{x}$. smx

+ - +
$\overline{\wedge}$ 2$\overline{\wedge}$

$X = \overline{\wedge}$ is point of local maxima.
$X = 2\overline{\wedge}$ is a point of local minima.

17 The domain of the function

$f(x) = \dfrac{1}{\sqrt{1 \text{ x } 1 \text{ - x}}}$ *is*

(1) $(-\infty, \infty)$ - {0)
(2) $(-\infty, \infty)$
(3) $(-0, \infty)$
(4) $(-\infty, 0)$

Key.4

Soln: 17 $f(x) = \dfrac{1}{\sqrt{1\ X\ 1-x}}$

1 X 1-X > 0 $\Rightarrow$ < 0 = 1 x $\in$ (-∞, 0)

18 If the mean deviation about the median of the numbers a, 2a, ..., 50 a is 50, then $|a|$ equals:

(1) 5
(2) 2
(3) 3
(4) 4

Key.4

Soln: 18 $M = \left(\frac{25+26}{2}\right)$ a = 25.5 a

$$50 = \frac{\sum l \text{ x i - ml}}{N} \Rightarrow \boxed{lal = 4}$$

19. If $\vec{a} = \frac{1}{\sqrt{10}}\left(3\hat{i} + \hat{k}\right)$ and

$\vec{b} = \frac{1}{7}\left(2\hat{i} + 3\hat{i} - 6\hat{k}\right)$, then the value

of $\left(2\vec{a} - \vec{b} - 6\hat{k}\right) \cdot \left[(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})\right]$ is

(1) 3
(2) -5
(3) -3
(4) 5

Key.2

Soln: 19 $|\vec{a}| = 1$, $|\vec{b}| = 1$, $\vec{a} . \vec{b} = 0$

$\Rightarrow \left(\vec{b} - 2\vec{a}\right) . \left\{\left(\vec{a}+2\vec{6}\right) \text{x} \left(\vec{a}+\vec{b}\right)\right\}$

$\Rightarrow \left(\vec{b} - 2\vec{a}\right) . \left\{\left(\vec{a}.\vec{b}\right)\vec{a} - a^2\ b^2 + 2\ b^2\vec{a} - 2\left(\vec{a}.\vec{b}\right)\vec{b}\right\}$

$\Rightarrow \left(\vec{b} - 2\vec{a}\right) . \left\{2\vec{a} - \vec{b}\right\} = 2\ \vec{a}.\vec{b} - 1 - 4 + 2\vec{a}.\vec{b} = -5$

20. The values of p and q for which the function

$$f(x) = \begin{cases} \dfrac{\sin (p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \dfrac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}}, & x > 0 \end{cases}$$

is continuous for all x in R, are :

(1) $p = \frac{1}{2}, q = \frac{3}{2}$

(2) $p = \frac{1}{2}, q = -\frac{3}{2}$

(3) $p = \frac{2}{5}, q = \frac{1}{2}$

(4) $p = -\frac{3}{2}, q = \frac{1}{2}$

Key.4

Soln: 20 t (x) is continuous at x = 0

$\Rightarrow f\omega = \lim_{x \to 0^+} t(x) = \lim_{x \to 0^-} t(x)$

$\Rightarrow {}^{\circ}V = \frac{1}{2} = \rho + 2$

$\boxed{\Rightarrow \rho = -3/2\ ,\ {}^{\circ}V = 1/2}$

21. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ (c > 0) touch each other if :

(1) $|a| = 2c$

(2) $2|a| = c$

(3) $|a| = c$

(4) $a = 2c$

Key.3

Soln: 21

$S_1 : (x - \frac{a}{2})^2 + y^2 = \frac{a^2}{4}$

$S_2 : x^2 + y^2 = C^2$

$C_1 C_2 = r_1 \pm r_2 \Rightarrow$

$\boxed{|a| = C}$

22 Let I be the purchase value of an equipment and V (t) be the value after it has been used for t years. The value V (t) depreciates at a rate given by differential equation

$\frac{dv(t)}{dt} = -k\ (T - t)$, where K > 0 is a constant and T is the total life in years of the equipment. Then the scrap value V (T) of the equipment is :

(1) e^{-kT}

(2) e^{-kT}

(3) $T^2 - \frac{I}{k}$

(4) $I - \frac{kT^2}{2}$

Key.3

Soln: 22 $\int_I^{V(T)} dv(L) = \int_{t=0}^{T} -k(T-t)dt$

$\Rightarrow V(T) = I - \frac{KT^2}{2}$

23 If C and D are two event such that $C \subset D$ and P (D) $\neq$ O, then the correct statement among the following is :

(1) $P(C|D) = \frac{P(D)}{P(C)}$

(2) $P(C|D) = P(C)$

(3) $P(C|D) \geq P(C)$

(4) $P(C|D) < P(C)$

Key.3

Soln: 23 $C \cap D = C$

$P(C/D) \cdot P(D) = P(C \cap D)$

$= P(C)$

$P\left(\frac{C}{D}\right) = \frac{P(C)}{P(D)}$, 9.5 $0 < P(D) \le 1$

$\boxed{P\left(\frac{C}{D}\right) \ge P(c)}$

24 Let A and B be two symmetric matrices of order 3

Statement - 1

A(BA) and (AB) A are symmetric matrices.

Statement - 2

AB is symmetric matrix if matrix multiplication of A with B is commutative.

(1) Statement-1 is fales, Statement-2 is true,

(2) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1

(3) Statement-2 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1

(4) Statement-1 is true , Statement-2 is false

Key.3

Soln: 24 $A^1 = A^1$, $B^1 = B$

$(A(BA))^1 = (BA)^1 A^1 = A^1 B^1 A^1 = ABA$

25 If $\omega (\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A + B\omega$ Then (A, B) equals :

(1) (-1, 1)

(2) (0, 1)

(3) (1,1)

(4) (1,0)

Key.3

Soln: 25 $(1+w)^7 = (-w^2)^7 = -w^{14}$

$= -w^{3(4)+2} = -w^2 = 1 + w = A + Bw$

$\Rightarrow \boxed{A = 1, B = 1}$

26 **Statement - 1**

The number of ways of distributing 10 identical balls in distinct boxes such that no box is empty ${}^9C_3$.

Statement - 2

The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$.

(1) Statement-1 is false, Statement-2 is true

(2) Statement-1 is true, Statement-2 is true is a correct explanation for Statement-2

(3) Statement-1 is true, Statement-2 is true Statement-2 is not a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is false.

Key.4

Soln: 26 ${}^{n-1}C_{r-1} = {}^9C_3$

27. The shortest distance between line y-x=1 and curve $x = y^2$ is:

(1) $\frac{4}{\sqrt{3}}$

(2) $\frac{\sqrt{3}}{4}$

(3) $\frac{3\sqrt{2}}{8}$

(4) $\frac{8}{\sqrt[3]{2}}$

Key.3

Soln:27 Short distance is along the normal

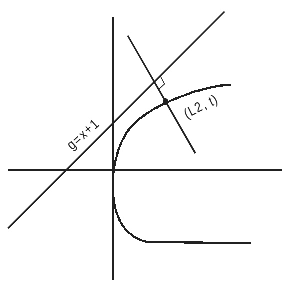
$y^2 = x$

$2y.y^1 = 1$

$y^1 = \frac{1}{2l} = +1$

$t = +1/2$

$\left(\frac{1}{4}, \frac{1}{2}\right)$ *is* point



$\Rightarrow \perp$ distance of the point on the line

$\Rightarrow \left|\frac{\frac{1}{4} - \frac{1}{2} + 1}{\sqrt{2}}\right| = \frac{3}{4\sqrt{3}} = \boxed{\frac{3\sqrt{2}}{8}}$

28. The area of the region enclosed by the curves y = x, y = $1/x$ and the positive x-axis is :

(1) $5/2$ square units

(2) $1/2$ square units

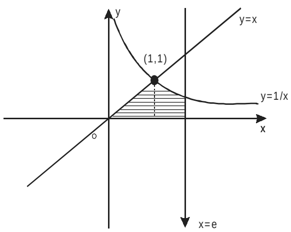
(3) 1 square units

(4) $3/2$ square units

Key.4
Soln: 28

$$= \frac{1^{Area}}{2} (1), (1)+\int_I^e \frac{1}{x}\, dx$$

$$= \frac{1}{2} + 1 = \boxed{\frac{2}{3}}$$



29. If $\frac{dy}{dx}$ = y + 3 > 0 and y(0) = 2 then y(In 2) is equal to:

(1) -2

(2) 7

(3) 5

(4) 13

Key.2
Soln: 29

$$\left|\frac{dy}{y+3} = \right| dx + c$$

In (y+3) = x+c

y+3 = e^{x+c}

$y+3 =$ k.e^x

$y(o) =$ 2 $\Rightarrow$ 5 = k

y+3 = 5 e^x = y = 5 e^x -3

$y(\ln_2) = 5.\ e^{\ln 2}$ - 3 = 10-3 =7

30. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicaular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying : $\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$ and $\vec{a} \cdot \vec{d}$ = 0 Then the vector $\vec{d}$ is equal to:

(1) $\vec{c} - \left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$

(2) $\vec{b} - \left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$

(3) $\vec{c} + \left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$

(4) $\vec{b} + \left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$

Key.1
Soln: 30

$\vec{a} \cdot \vec{b} = 0$

given : $\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$

$= \vec{a} \times (\vec{b} \times \vec{c}) = \vec{a} \times (\vec{b} \times \vec{d})$

$(\vec{a} \cdot \vec{c})\,\vec{b} - (\vec{a}.\vec{b})\,\vec{c} = (\vec{a}.\vec{d})\,\vec{b} - (\vec{a}.\vec{b})\,\vec{d}$

$$\Rightarrow \boxed{\vec{d} = \vec{c}\ \frac{(\vec{a}.\vec{c})}{(\vec{a}.\vec{b})}\ \vec{b}}$$

PART B - CHEMISTRY

31. In context of the lanthanoids, which of the following statements is **not** correct ?

(1) Availability of 4f electrons results in the formation of componds in +4 state for all the members of the series.

(2) There is a gradual decrease in the radii of the members with increasing atomic number in the series.

(3) All the members exhibit +3 oxidation state.

(4) Because of similar properties the separation of lanthanoidsis not easy.

Key. 1

Soln. 31 lanthanoids exhibt +3 oxidation state without are exception mont of the compounds fromed by lanthanoids are in this oxidation state

32. In a face centred cubic lattice, atom A occupies the corner positions and atom B occupies the face centre positions. If one atom of B is missing from one of the face centred points, the formula of the compound is :

(1) A_2B_5

(2) A_2B

(3) AB_2

(4) A_2B_3

Key. 1

Soln.32 No of aloms per unit all at the corners

(Z) $A = \frac{1}{8} x\ 8 = 1.$

No of aloms per unit all at the corners

(Z) $B = 5 \times \frac{1}{2} = \frac{5}{2} = 2.5$

Have formula for the coumpounds will be $A_1\ B_{2.5}$ or A_2B_5.

33. The magnetic moment (spin only) of $[NiCl_4]^{2-}$ is :

(1) 1.41 BM

(2) 1.82 BM

(3) 5.46 BM

(4) 2.82 BM

Key. 4

Soln.33 (Ni) At No. = 28 configuration [Ar] $4s^2\ 3d^8$

Ni^{+2} = 26 configuration [Ar] $4s^2\ 3d^8$

Since Cl^- it is a weak field ligand and therefore $d^8\ Ni^{+2}$as will not undergo pairing and hence will have two unpaired electrons. $\therefore$ n = 2

Magnetic Moment = $\sqrt{n(n+2)}$ BM

$= \sqrt{4x2)} = \sqrt{8} = 2.82$ BM.

34. Which of the following facts about the complex $[Cr(NH_3)_6]Cl_3$ is wrong ?

(1) The complex gives white precipitate with silver nitrate solution.

(2) The complex involves d^2sp^3 hybridisation and is octaehedral in shape.

(3) The complex is paramagnetic.

(4) The complex is an outer orbital complex

Key.4

Soln.34 Z (Cr) At No.= 25 [Ar] $4s^1\ 3d^5$

Cr^{+3} = 22 electronic [Ar] $4s^2\ 3d^8$

Since NH_3 is a strong field ligand so it fouces inner orbital complex of hybridisation $d^2\ sp^3$ which is octahedral in shape inner d-orbital is involved in hybridization so statement (4) is wrong.

35. The rate of a chemical reation doubles for every 10°C rise of temperature. If the temperature is raised by 50°C, the rate of the reaction increases by about :

(1) 64 times

(2) 10 times

(3) 24 times

(4) 32 times

Key.4

Ans.35 temperature coefficient

$(\theta) = \frac{rate \text{ at } (t+10)^{\circ}C}{rate \text{ at } t\ ^{\circ}C} = 2$ for this reaction

so increase in the rate of reaction =
where n is the no. of times by which temperature is raised at a multiple of 10^0c.

$\theta = \frac{50^{\circ}C}{10^{\circ}C} = 5$ Hence rate of reaction=2^5

=32 times answer.

36. 'a' and 'b' are van der Waals' constants for gases. Chlorine is more easily liquefied than ethane because

(1) a for Cl_2 > a for C_2H_6 but b for Cl_2 < b for C_2H_6
(2) a and b for Cl_2 > a and b for C_2H_6
(3) a and b for Cl_2 < a and b for C_2H_6
(4) a for Cl_2 < a for C_2H_6 but b for Cl_2 > b for C_2H_6

Key.1

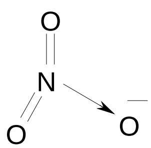
Ans.36 Compressible gases have greater force of attraction so the value of ``a`` should be greater in the case of $Cl_2(g)$ which is easily liquified and also it must have reduced volume so ``b`` should be lesser.

37. The hybridisation of orbitals of N atom in NH_3^-, NO_2^+ and NH_4^+ are respectively :

(1) sp^2, sp^3, sp
(2) sp, sp^2, sp^3
(3) sp^2, sp, sp^3
(4) sp, sp^3, sp^2

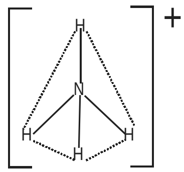
Key. 3

Ans.36



$\rightarrow$ 3 σ bonds have sp^2 hybridized

$[O = \overset{+}{N} = O] \rightarrow 2\sigma$ bonds hance sp hybridized



$\rightarrow 4\sigma$ bonds have sp hybridized

38. Ethylene glycol is used as an antifreeze in a cold climate. Mass of ethylene glygol which should be added to 4kg of water to prevent it from freezing at -6°C will be :
(K_f for water = 1.86 K kg mol^{-1},and molar mass of ethylene glycol = 62 g mol^{-1})

(1) 304.60 g
(2) 804.32 g
(3) 204.30 g
(4) 400.00 g

Key. 2

Soln.36 $\Delta T_F = K_F \cdot m$

$$\Delta T_F = K_F \times \frac{wt(solute)}{mol.wt} \times \frac{1000}{wt.\text{ of solvent}}$$

$$O - (-6^{\circ}C) = \frac{1.86 \times wt(solute) \times 1000}{62 \times 4000}$$

$$6 = \frac{1.86 \times wt(solute) \times 1000}{62 \times 4000}$$

so wt(solute) = 804. 32g on solving

39. The outer electron configuration of Gd (Atomic No. : 64) is :

(1) $4f^7 5d^1 6s^2$
(2) $4f^3 5d^5 6s^2$
(3) $4f^8 5d^0 6s^2$
(4) $4f^4 5d^4 6s^2$

Key. 1

Soln.39 The electronic configuration of Gd (Atomic No. 64) $= [Xe]\ 4f^7\ 5d^1\ 6s^2$

It must be half filled configuration which is stable in nature.

40. The structure of IF_7 is :

(1) pentagonal bipyramid
(2) square pyramid
(3) trigonal bipyramid
(4) octahedral

Key . 1

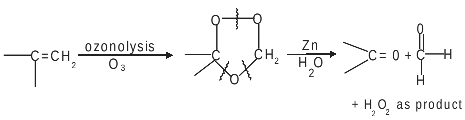
Soln.40 Pentagonal bipyramid shape $sp^3 d^3$ Hybridization.

41. Ozonolysis of an organic compound gives foemaldehyde as one of the products. This confirms the presence of :

(1) an acetylenic triple bond
(2) two ethylenic double bonds
(3) a vinyl group
(4) an isopropyl group

Key. 3

Soln.41 The organic compound must have at least one vinyl in order to give formaldehyde as one of the products on ozonolysis as per the rxn:



42. The degree of dissociation (α) of a weak electrolyte, A_xB_y is related to van't Hoff factor (i) by the expression :

(1) $\alpha = \frac{x+y+1}{i-1}$

(2) $\alpha = \frac{i-1}{(x+y-1)}$

(3) $\alpha = \frac{i-1}{x+y-1}$

(4) $\alpha = \frac{x+y-1}{i-1}$

Key.2

Ans.42

$$Van't\ Hoff\ factor(i) = \frac{\textit{observed}\ \text{colligative properhy}}{\textit{Normal}\ \text{colligative properhy}}$$

$$A_XB_Y \rightarrow xA^{Y+} + yB^{X-}$$

t=0	1	0	0
t=rxn	$1-x$	$x\alpha$	$y\alpha$

Hence $i = \frac{1-x+x\alpha+y\alpha}{1}$ get the answer.

43. A gas absorbs a photon of 355 nm and emits at two wavelengths. If one of the emissions is at 680 nm, the other is at :

(1) 518 nm
(2) 1035 nm
(3) 325 nm
(4) 743 nm

Key. 4

Soln.43

Energy of absorbed photon = Sum of energies of emitted photons. since $E = \frac{hc}{\lambda}$

Hence, $\frac{hc}{355\times10^{-9}} = \frac{hc}{680\times10^{-9}} + \frac{hc}{x(\text{find})}$ solve x

44. Identify the compound that exhibits tautomerism.

(1) Phenol
(2) 2-Butene
(3) Lactic acid
(4) 2-Pentanone

Key. 4

Soln.44

The prescence of alpha-H atom is necessary condition for the exihibiting keto-enol tautomerism.

45. The entropy change involved in the isothermal reversible expansion of 2 moles of an ideal gas from a volume of 10 dm^3 to a volume of 100

dm^3 at 27°C is :
(1) 42.3 $J\,mol^{-1}K^{-1}$
(2) 38.3 $J\,mol^{-1}K^{-1}$
(3) 35.8 $J\,mol^{-1}K^{-1}$
(4) 32.3 $J\,mol^{-1}K^{-1}$

Key. 2

Soln.45 As per the formula

$$\Delta S = 2.303 nR \log(\frac{V_{final}}{V_{initial}})$$

Hence

$$\Delta S = 2.303 \times 2 \times 8.314 \log(\frac{100}{10})$$

On solving we get answer 38.3 J mol^{-1} K^{-1}.

46. Silver Mirror test is given by which one of the following compounds ?
(1) Benxophenone
(2) Acetaldehyde
(3) Acetone
(4) Formaldehyde

Key.2 & 4

Soln.46

$$R-CHO + Ag(NH_3)^+ + OH^- \rightarrow RCOO^- + Ag + NH_4^+$$

Silver Mirror

47. Trichloroacetaldehyde was subjected to Cannizzaro's reaction by using NaOH. The mixture of the products contains sodium trichloroacetate and another compound. The other compound is :
(1) Chloroform
(2) 2,2,2 - Trichloroethanol
(3) Trichloromethanol
(4) 2,2,2-Trichloropropanol

Key.2

Soln.47

Asper the cannizaro's reaction:

$$CCl_3CHO \xrightarrow[Cannizaro's]{NaOH} CCl_3COO^-Na^+ + CCl_3CH_2OH$$

2,2,2 trichloroethanol

48. The reduction potential of hydrogen half-cell will be negative if :
(1) $p(H_2)$ = 2 atm and $[H^+]$ = 2.0 M
(2) $p(H_2)$ = 1 atm and $[H^+]$ = 2.0 M
(3) $p(H_2)$ = 1 atm and $[H^+]$ = 1.0 M
(4) $p(H_2)$ = 2 atm and $[H^+]$ = 1.0 M

Key. 4

Soln. 48

Reduction half cell reaction: $H^+ + e^- \rightarrow 1/2H_2$

Apply nernst equation:

$$E = 0 - \frac{0.0591}{1}\log(\frac{p_{H_2}^{1/2}}{[H^+]})$$ **since p_{H2}>$[H^+]$**

so the emf will be negative.

49. Phenol is heated with a solution of mixture of KBr and KBrO3. The major product obtained in the above reaction is :
(1) 2, 4, 6 - Tribromophenol
(2) 2 - Bromophenol
(3) 3 - Bromophenol
(4) 4 - Bromophenol

Key. 1

Soln.49 This is due to generation of Br_2 by reaction:

$$5KBr + KBrO_3 + 3H_2O \rightarrow 3Br_2 + 6KOH$$

which gives trisubstitution on the phenol molecule.

50. Among the following the maximum covalent character is shown by the compound :
(1) $MgCl_2$
(2) $FeCl_2$
(3) $SnCl_2$
(4) $AlCl_3$

Key. 4

Soln.50

Higher is the value of charge(oxidation state) of cat ion so higher is the polarisation & covalent charac ter as per the FAJAN's rule.

51. Boron cannot form which one of the following anions ?
(1) BO_2^-
(2) BF_6^{3-}
(3) BH_4^-
(4) $B(OH)_4^-$

Key. 2
Soln.51
Due to the abscence of d-orbitals in Boron(B) sp^3d^2 hybridization is not possible,consequently compound BF_6^{3-} is not possible.

52. Sodium ethoxide has reacted with ethanoyl chloride. The compound that is produced in the above reaction is :
(1) Ethyl ethanoate
(2) Diethyl ether
(3) 2-Butanone
(4) Ethyl chloride

Key. 1
Soln. 52

$$C_2H_5O^-Na^+ + CH_3COCl \rightarrow CH_3COOC_2H_5 + NaCl$$

53. Which of the following reagents may be used to distinguish between phenol and benzoic acid ?
(1) Neutral $FeCl_3$
(2) Aqueous NaOH
(3) Tollen's reagent
(4) Molisch reagent

Key.1
Soln. 52
Phenol gives voilet coloration with neutral $FeCl_3$ while benzoic acid does not.

54. A vessel at 1000K contains CO_2 with a pressure of 0.5 atm. Some of the CO_2 is converted into CO on the addition of graphite. If the total pressure at equilibrium is 0.8 atm, the value of K is :
(1) 0.18 atm
(2) 1.8 atm
(3) 3 atm
(4) 0.3 atm

Key.2
Soln.54

As per the Equilibrium reaction:

$$CO_{2(g)} + C_{(s)} \underset{\text{equlibrium}}{\rightleftharpoons} 2CO_{(g)}$$

t=0	0.5	-	0
t=eq	0.5-x	-	2x

Since 0.5-x + 2x = 0.8 atm so solve for x=0.3atm
Then

55. The strongest acid amongst the following compounds is :
(1) $ClCH_2CH_2CH_2COOH$
(2) CH_3COOH
(3) HCOOH
(4) $CH_3CH_2CH(Cl)CO_2H$

Key.4
Soln.55
Due to -I effect in the fourth compound which is the basic reason for the increase in acidity as compared to other compounds.

56. Which one of the following orders presents the correct sequence of the increasing basic nature of the given oxides ?
(1) $K_2O < Na_2O < Al_2O_3 < MgO$
(2) $Al_2O_3 < MgO < Na_2O < K_2O$
(3) $MgO < K_2O < Al_2O_3 < Na_2O$
(4) $Na_2O < K_2O < MgO < Al_2O_3$

Key.2
Soln.56
Metallic property in case of basic oxides increases moving down the group and decreases accross the period.

57. A 5.2 molal aqueous solution of methyl alcohol, CH3OH, is supplied. What is the mole fraction of methyl alchohol in the solution ?
(1) 0.050
(2) 0.100
(3) 0.190
(4) 0.086

Key. 4
Soln.57

5.2 molal CH_3OH means 1000g of solution contains 5.2 moles of CH_3OH.
So moles of water will be=(1000/18)=55.55
The mole Fraction of CH_3OH:--

$$= \frac{n_{CH_3OH}}{n_{CH_3OH} + n_{H_2O}} = \frac{5.2}{5.2 + 55.55} = 0.086$$

58. The presence or absence of hydroxy group on which carbon atom of sugar differentiates RNA and DNA ?
(1) 4^{th}
(2) 1^{st}
(3) 2^{nd}
(4) 3^{rd}

Key.3
Soln.58 As per the structures of DNA & RNA the 2nd carbon in DNA does not have -OH functional group attached.

59**.** Which of the following statement is **wrong** ?
(1) N_2O_4 has two resonance structures.
(2) The stability of hydrides increases from NH_3 to BiH_3 in group 15 of the periodic table.
(3) Nitrogen cannot from $d\pi - p\pi$ bond.
(4) Single N - N bond is weaker than the single P - P bond.

Key.2
Soln.59
As we move down the group, the tendency to form covalent bond with small H atom decreases hence M-H bond enthalpy decreases.

60**.** Which of the following statements regarding sulphur is **incorrect** ?
(1) The oxidation state of sulphur is never less than +4 in its compounds.
(2) S_2 molecule paragagnetic
(3) The vapour at 200°C consists mostly of S_8 rings.
(4) At 600°C the gas mainly consists of S_2 molecules.

Key.1
Soln. 60
The normal oxidation state range for Sulphur is between -2 to +6.

PART C - PHYSICS

61. A Carnot engine operating between temperatures T_1 and T_2 has efficiency has efficiency $\frac{1}{6}$. When T_2 is lowered by 62K, its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are respectively :

(1) 310 K and 248 K

(2) 372 K and 310 K

(3) 372 K and 330 K

(4) 330 K and 268 K

Key.3
soln.1
Efficiency of a carnot engine

$$\eta = 1 - \left(\frac{T_2}{T_1}\right)$$

given $1 - \left(\frac{T_2}{T_1}\right) = \frac{1}{6} \Rightarrow \frac{T_2}{T_1} = \frac{5}{6}$

and $1 - \left(\frac{T_2 - 62}{T_1}\right) = \frac{1}{3} \Rightarrow \frac{T_2 - 62}{T_1} = \frac{2}{3}$

solving which T_1 = 372K , T_2 = 330K

62. A pulley of radius 2m is rotated about its axis by a force F = (20t - 5t^2) newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m2, the number of rotations made by the pulley before its direction of motion if reversed, is :

(1) more than 9

(2) less than 3

(3) more than 3 but less than 6

(4) more than 6 but less than 9

Key.3

Soln.62

Given F = (20t - 5t^2), R = 2m

$\Rightarrow$ torque $\tau = FR = (40t - 10t^2)$

Now, angular impulse in a time interval

$\tau = 0$ to $\tau =$ t

$$J = \int_0^f \tau dt = \int (40t - 10t^2)dt \; = (20t^2 - \frac{10}{3}t^3)$$

at an instant when the "direction" of angular motion is "about' to reverse, the angular momentum should be

$$\Rightarrow \ell = 20t^2 - \frac{10t^3}{3} = 0$$

$$\Rightarrow 10t^2\left(2 - \frac{1}{3}t\right) = 0$$

$$\Rightarrow t = 6 \text{ sec}$$

Now, angular displacement as a function of time can be delivered from the relations angular momentum

$$\Rightarrow \ell = I\omega$$

$$\Rightarrow \left(20t^2 - \frac{10}{3}t^3\right) = 10\omega$$

$$\Rightarrow \omega = 2t^2 - \frac{t^3}{3}$$

$$\Rightarrow \int_0^\theta d\theta = \int_0^6 \left(2t^2 - \frac{t^3}{3}\right)dt$$

$$\Rightarrow \theta = \frac{2}{3}t^3 - \frac{t^4}{(3\times4)}\Bigg|_0^6$$

$$\Rightarrow \theta = 36 \text{ radians}$$

$\therefore$ no of revolutions n = $\frac{36}{2\pi}$ n = 5.73

63. Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is :

(1) $\frac{n_1^2T_1^2 + n_2^2T_2^2 + n_3^2T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$

(2) $\frac{(T_1 + T_2 + T_3)}{3}$

(3) $\frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$

(4) $\frac{n_1T_1^2 + n_2T_2^2 + n_3T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$

Key.3

Soln: 63

Assuming the final temperature to be T, the total initial energy

$U_f = n_1C_vT + n_2C_vT + n_3C_vT$

Euqality this with the initial value,

$U_f = n_1C_vT_1 + n_2C_vT_2 + n_3C_vT_3$

$$T = \frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$$

64. A boat is moving due east in a region where the earth's magnetic field is 0.5 x 10^{-5} NA^{-1} m-1 due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is 1.50 ms-1, the magnitude of the induced emf in the wire of aerial is :

(1) 0.15 mV
(2) 1 mV
(3) 0.75 mV
(4) 0.50 mV

Key.1

Soln.64

B = 5.0 × 10^{-5} N/A-m (due north)

$v =$ 1.5 m/s (due east)

length of the aerial is $\ell = 2$m (placed along vertical) since the direction of magnetic field and velocity and the length of the rod are mutually perpendicular, the induced voltage,

$\varepsilon = B\ell v$

$\Rightarrow \varepsilon = (5\times10^{-5})\times(2)\times(1.5)$

$\Rightarrow \varepsilon = 18\times10^{-5}$

$\Rightarrow \varepsilon = 0.15$ mv

65. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc :

(1) first increases and then decreases

(2) remains unchanged

(3) continuously decreases

(4) continuously increases

Key.1
Soln:65

As the insect moves from the along the diameter, as if approches the centre, the system's moment of inertia decreases while once it crosses the centre and moves outwards, the moment of inertia again increases . Since angular momentum $L = I\omega$ is 'conserved' during this process the angular velocity will first increase, then decrease.

66. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance d(d<<l) apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between them,

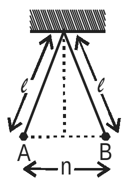
(1) $v \alpha x$

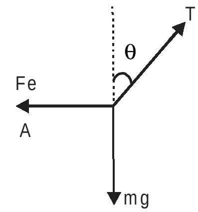
(2) $v \alpha x^{-1/2}$

(3) $v \alpha x^{-1}$

(4) $v \alpha x^{1/2}$

Key.2
Soln .66





$T\cos\theta = mg$

$T\sin\theta = Fe$

$$\Rightarrow \tan\theta = \frac{Fe}{mg}$$

$$\Rightarrow \frac{x}{2\ell} = \frac{\left(\frac{1}{4\pi f_0}\frac{\theta^2}{n^2}\right)}{mg}$$

$$\Rightarrow n^3 \alpha\ \theta^2$$

$$\Rightarrow \frac{d}{dt}(n^3)\ \alpha\ \frac{d}{dt}(\theta^2)$$

$$\Rightarrow 3n^2\left(\frac{dn}{dt}\right)\alpha\ 2\theta\left(\frac{dq}{dt}\right)$$

$$\Rightarrow \left(\frac{dx}{dt}\right) = v\ \alpha\ \frac{\theta}{n^2}\left(\frac{dq}{dt}\right)$$

given $\left(\frac{dq}{dt}\right)$ = constant

$$v\ \alpha\ \frac{1}{\sqrt{n^2}}$$

67. 100g of water is heated from 30°C to 50°C Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/kg/K) :

(1) 2.1 kJ

(2) 4.2 kJ

(3) 8.4 kJ

(4) 84 kJ

Key. 3
Soln.67

First law : $\theta = \Delta u + \omega$

where $\omega = \int Pdv$, now dv $\simeq 0$

$\therefore \theta = \Delta u$

therfore $\Delta u = \theta = ms\Delta T$

$= (0.1) \times (4184) \times 20$

$= 8363\ J \simeq 8.4\ kJ$

68. The half life of radioactive substance is 20 minutes. The approximate time interval (t_2 - t_1) between the time t_2 when $\frac{2}{3}$ of it has decayed and time t_1 when $\frac{1}{3}$ of it had decayed is :

(1) 28 min
(2) 7 min
(3) 14 min
(4) 20 min

Key.4
Soln.68

Given at time t = t_1, N = $\frac{2}{3}$No

and at time t = t_2 ; N = $\frac{1}{3}$No

Therefore in the time interval (t_2 - t_1), the number of samples have "halved", Therefore by defination

(t_2 - t_1) = $T_{1/2}$ = 20min

69. Energy requiredd for the electron excitation in Li^{++} from the first to the third Bohr orbit is :

(1) 122.4 eV

(2) 12.1 eV

(3) 36.3 eV

(4) 108.8 eV

Key. 4

Soln. 69 $\Delta E = -13.6 \times 7^2 \left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right] eV$

$= -13.6 \times 3^2 \left[\frac{1}{3^2} - \frac{1}{1^2}\right]$ = 108.8 eV

70. The electrostatic potential inside a charged spherical ball is given by $\phi = a r^2 + b$ where r is the distance from the centre ; a, b are constant s. Then the charge density inside the ball is :

(1) $-6\ a\varepsilon_0$

(2) $-24\pi\ a\varepsilon_0 r$

(3) $-6\ a\varepsilon_0 r$

(4) $-24\pi\ a\varepsilon_0$

Key. 1
Soln.70

Given electrostatic potential $\phi = a r^2 + b$, therfore

electric field $E = -\left(\frac{d\phi}{dr}\right) = -2ar$

Now, from Gauss' Law $\oint \vec{E} \cdot \vec{ds} = \frac{\phi_{enC}}{f_0}$

$\Rightarrow E \times 4\pi r^2 = \frac{\int \rho dv}{f_0}$ where ρ = change density

and dv = $4\pi r^2 dr$

$\Rightarrow \int \rho dv = 4\pi\varepsilon_o r^2 \times (-2ar) = -8\pi\varepsilon_0 ar^3$

$$\Rightarrow \frac{d(-8\pi\varepsilon_0 ar^3)}{dV} = \frac{-24\pi\varepsilon_0 ar^2 dr}{4\pi r^2 dr}$$

$\Rightarrow \rho \ = -6\varepsilon_0 a$

71. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (Surface tension of soap solution = 0.03 Nm^{-1})
(1) 0.4π mJ
(2) 4π mJ
(3) 0.2π mJ
(4) 2π mJ

Key. 1
Soln. 71
workdone = change in surface potential energy

$= T\times[8\pi R_2^2 - 8\pi R_1^2]$

$= 0.03\times 8\pi[5^2 - 3^2]\times 10^{-4}$

$\simeq 0.4\pi$ mJ

72. A resistor 'R' and 2μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that light up 5s after the switch has been closed. $(\log_{10} 2.5 = 0.4)$
(1) $3.3 \times 10^7\ \Omega$
(2) $1.3 \times 10^4\ \Omega$
(3) $1.7 \times 10^5\ \Omega$
(4) $2.7 \times 10^6\ \Omega$

Key. 4
Soln. 72
The transient charge on the capacitor

$q = Q[1 - e^{t/e}]$ where

$Q = C\varepsilon = (2\mu F \times 200V)$

$\ell = RC = (R \times 2\mu F)$

Therefore the voltage across the capacitor (also the near bulb)
$\therefore$ since at time t = 5 sec, v = 120v

$\Rightarrow 120 = 200[1 - e^{-5/e}]$

$\Rightarrow e^{-5/e} = (2/5)$

$\Rightarrow -\frac{5}{e} = \log(0.4) = 2.303[\log(0.4)]$

$\Rightarrow -\frac{5}{e} = \log\left(\frac{10}{4}\right)$

$\Rightarrow e = R \times 2\mu F = \frac{5}{(2.303 \times 0.4)}$ $(\log 2 = 0.3010)$

$\Rightarrow R = 2.7 \times 10^6 \Omega$

73. A current I flows in an infinitely long wire with cross section in the form of semicircular ring of radius R. The magnitude of the magnetic induction along its axis is :

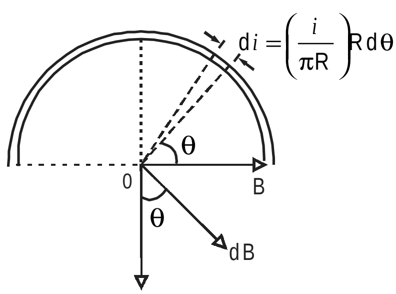
(1) $\frac{\mu_0 I}{4\pi R}$

(2) $\frac{\mu_0 I}{\pi^2 R}$

(3) $\frac{\mu_0 I}{2\pi^2 R}$

(4) $\frac{\mu_0 I}{2\pi R}$

Key. 2
Soln. 73



Take a current element "di" as shown in the figure, it produces a field

$dB = \frac{\mu_0 di}{2\pi R}$ at the point 0 dBcosθ : cancelling component

$$B = \int dB \sin\theta$$

$$B = \int \frac{\mu_0 di}{2\pi R} \sin\theta$$

$$B = \int_{\theta=0}^{\theta=\pi} \frac{\mu_0 i}{2\pi^2 R} \sin\theta d\theta$$

$$B = \frac{\mu_0 i}{\pi^2 R}$$

74. An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by :

$$\frac{dv}{dt} = -2.5\sqrt{v}$$

where v is the instantaneous speed. The time taken by the object, to come to rest would be:

(1) 8 s
(2) 1 s
(3) 2 s
(4) 4 s

Key. 3
Soln. 74

$$\frac{dv}{dt} = -2.5\sqrt{v}$$

$$\Rightarrow \int_{v=6.25}^{v=0} \frac{dv}{\sqrt{v}} = \int_{f=0}^{f=t} -2.5dt$$

$$\Rightarrow 2\sqrt{v}\Big|_{6.25}^{0} = -2.5t$$

$$\Rightarrow t = \frac{2\times\sqrt{6.25}}{2.5} = \frac{2\times 2.5}{2.5} = 2\,\text{sec}$$

75. Direction :

*The question has a paragraph followed by two statements, **Statement - 1** and **Statement - 2,** of the given four alternatives after the statements, choose the one that describes the statements.*

A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives and interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

Statement -1 :
When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of π .
Statement -2 :
The centre of the interference pattern is dark.

(1) Statement -1 is false, Statement -2 is true
(2) Statement -1 is true, Statement - 2 is false.
(3) Statement -1 is true, Statement -2 is true and Statement - 2 is the correct explana tion of Statement - 1.
(4) Statement -1 is true, Statement - 2 is true and Statement -2 is **not** the correct expla nation of Statement -1.

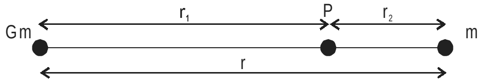
Key. 4
Soln. 75

Both statements are true but statement 2 does not imply statement - 1 as the central bridge is "dark" because of a phase inversion of "π" suffered by the reflected wave.

76. Two bodies of masses m and 4 m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is :

(1) $-\frac{9Gm}{r}$
(2) zero
(3) $-\frac{4Gm}{r}$
(4) $-\frac{6Gm}{r}$

Key. 1
Soln. 76



At the point 'P' , the field is zero

$$\Rightarrow \frac{Gm}{r_1^2} = \frac{m}{r_2^2} \Rightarrow r_1 : r_2 = 2 : 1$$

$$\Rightarrow r_1 = \frac{2}{3}r, \quad r_2 = \frac{1}{3}r$$

Therefore the Gravitational Potential

$$V = \frac{G(4m)}{r_1} - \frac{G(m)}{r_2} = -\frac{6Gm}{r} - \frac{3Gm}{r}$$

$$= -\frac{9Gm}{r}$$

77. This question has Statement -1 and Statement -2. of the four choices given after the statements, choose the one that best describes the two statements.

Statement - 1 :
Sky wave signals are in general, less stable than ground wave signals.

Statement - 2 :
The state of ionosphere varies from hour to hour, day to day and season to season.

(1) Statement -1 is false, Statement - 2 is true.
(2) Statement - 1 is true, Statement - 2 is false.
(3) Statement - 1 is true, Statement - 2 is true, Statement - 2 is the correct expla nation of Statement -1.
(4) Statement - 1 is true, Statement -2 is true, Statement -2 is **not** the correct explanation of Statement - 1.

Key. 3
Soln. 77

The Statement -2 is the correct explanation for the fact what sky wave signals are less stable as compound to ground wave signals.

78. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at t = 0. The time at which the energy is stored equally between the electric and the magnetic fields is :

(1) $\sqrt{LC}$
(2) $\pi\sqrt{LC}$
(3) $\frac{\pi}{4}\sqrt{LC}$
(4) $2\pi\sqrt{LC}$

Key. 3
Soln. 78

For the given L-C circuit (oscillation) the charge on the capacitor (transient) is given by

$q = q_0 \sin(\omega t + \pi/2)$

where $\omega = (\sqrt{LC})^{-1}$

Therefore at an instant when energy stored in the electric and magnetic fields are identical, the Potential Energy of the capacitor becomes half the initial value

$$\frac{1}{2}\frac{q^2}{C} = \frac{1}{2} \times \frac{1}{2}\frac{q_0^2}{C}$$

$$\Rightarrow q = \frac{q_0}{\sqrt{2}} \Rightarrow q_0 \sin\{(\omega t) + \pi/2\} = \frac{q_0}{\sqrt{2}}$$

$$\Rightarrow \omega t = \frac{\pi}{4} \Rightarrow t = \frac{\pi}{4}\sqrt{LC}$$

79. This question has Statement -1 and Statement -2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement -1 :
A metallic surface is irradiated by a monochromatic light of frequency $v > v_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.

Statement - 2 :

The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.

(1) Statement - 1 is false, Statement -2 is true.

(2) Statement -1 is true, Statement -2 is false

(3) Statement -1 is true, Statement-2 is true, Statement -2 is the correct explanation of Statement-1.

(4) Statement -1 is true, Statement -2 is true, Statement-2 is **not** the correct explanation of Statement - 1.

Key. 1

Soln. 79

$K_{max} = hf = Q$ where workfunction $Q = eV_0$

Therefore Statement - 1 is inaccurate.

80. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to :

(1) 3.6×10^{-3} m

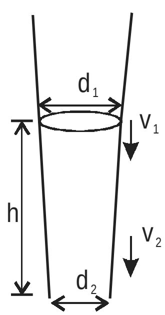
(2) 5.0×10^{-3} m

(3) 7.5×10^{-3} m

(4) 9.6×10^{-3} m

Key. 1

Soln. 80



By application of continutiy equation

$A_1v_1 = A_2v_2$

$\Rightarrow d_1^2v_1 = d_2^2v_2$ .... (1)

and by application of Banoulli's Equation

$$p_0 + \frac{1}{2}pv_1^2 + pgh = p_0 + \frac{1}{2}pv_2^2$$

$\Rightarrow v_1^2 = v_2^2 - 2gh$ ....(2)

Solving the above equations (1) & (2)

$$d_2 = d_1\sqrt{\frac{v_1}{v_2}}$$

$v_2 = 2.04$ m/s

$v_1 = 0.4$ m/s

$d_1 = 8 \times 10^{-3}$

$\therefore d_2 = 3.6 \times 10^{-3}$

81. A mass M, attached to a horizontal spring, executes S.H.M. with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together.

with amplitude A_2 The ratio of $\left(\frac{A_1}{A_2}\right)$ is :

(1) $\left(\frac{M + m}{M}\right)^{1/2}$

(2) $\frac{M}{M + m}$

(3) $\frac{M + m}{M}$

(4) $\left(\frac{M}{M + m}\right)^{1/2}$

Key. 1

Soln. 81

The speed of the block 'm' just before the other block is placed on it,

$v_1 = \omega_1 A_1 \qquad \omega_1 = \sqrt{\frac{K}{M}}$

Therefore, by momentum conservation, the speed just after the other block is placed,

$$v_2 = \frac{Mv_1}{m + M} \Rightarrow \left(\frac{M}{m + M}\right)\omega_1 A_1 = \omega_2 A_2$$

$$\omega_2 A_2 = \sqrt{\frac{K}{(M+m)}}$$

$$\frac{A_1}{A_2} = \frac{\omega_2}{\omega_1}\left(\frac{m+M}{M}\right)$$

$$\Rightarrow \left(\frac{A_1}{A_2}\right) = \sqrt{\frac{(M+m)}{M}}$$

82. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0 (X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is:

(1) $\frac{\pi}{6}$

(2) $\frac{\pi}{2}$

(3) $\frac{\pi}{3}$

(4) $\frac{\pi}{4}$

Key. 3

Soln. 82

The equations for the two SHMs can be given to be

$n_1 = A\sin(\omega t)$

$n_2 = A\sin(\omega t + \theta) + X_0$

Therefore, the separation,

$n_2 - n_1 = A[\sin(\omega t + \theta) - \sin(\omega t)] + X_0$

$\Rightarrow \ (n_2 - n_1) = 2A\sin\left(\frac{\theta}{2}\right)\cos(\omega t) + X_0$

Therefore the maximum separation,

$$2A\sin\left(\frac{\theta}{2}\right) + X_0 = A + X_0 \Rightarrow \frac{\theta}{2} = \frac{\pi}{6}$$

$$\theta = \frac{\pi}{3}$$

83. If wire is stretched to make it 0.1% longer, its resistance will :

(1) decrease by 0.05%

(2) increase by 0.05%

(3) increase by 0.2%

(4) decrease by 0.2%

Key. 3

Soln. 83

Given that the wire is streched by 0.1%

$\frac{\Delta\ell}{\ell} = 0.001$ (increase in length)

Therefore, its area of cross section, would reduce such that $\frac{\Delta A}{A} = 0.001$ (decrease)

Now, electrical resistance

$R = \rho\frac{\ell}{A}$

if the length increases by 0.1% and area of cross section decreases by 0.1% the resistance would increase by 0.2%

84. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is :

(1) $\pi\frac{v^2}{g^2}$

(2) $\pi\frac{v^2}{g}$

(3) $\pi\frac{v^4}{g^2}$

(4) $\frac{\pi}{2}\frac{v^4}{g^2}$

Key. 3

Soln. 84

By application of projectile motion concept, the range 'R' for a water droplet,

$$R = \frac{v^2 \sin 2\theta}{g}$$

$$R = \frac{v^2 \sin 2\theta}{g}$$

Therefore, maximum area covered

$$A_{max} = R_{max}^2 = \left(\frac{\pi v^4}{g^2}\right)$$

85. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by :

(1) $\frac{(\gamma-1)}{2R}Mv^2K$

(2) $\frac{(\gamma-1)}{2(\gamma+1)R}Mv^2K$

(3) $\frac{(\gamma-1)}{2\gamma R}Mv^2K$

(4) $\frac{\gamma Mv^2}{2R}K$

Key. 1

Soln. 85

When suddenly brought to rest, the Kinetic Energy of the vessel (by vitue of its translation) is converted into internal energy for the gas molecules of total no. of moles of gas in "n"

$$\frac{1}{2}(nM)v^2 = nC_v\Delta T$$

$$= n\frac{R}{(\gamma-1)}\Delta T$$

$$\Delta T = \frac{(\gamma-1)}{2R}Mv^2$$

86. A screw gauge gives the following reading when used to measure the diameter of a wire.

Main scale reading : 0mm

Circular scale reading : 52 divisions

Given that 1 mm on main scale corresponds to 100 divisions of the circular scale.

The diameter of wire from the above data is :

(1) 0.005 cm

(2) 0.52 cm

(3) 0.052 cm

(4) 0.026 cm

Key. 3

Soln. 86

MSR = 0

CSR = 52 div.

$\therefore$ measurement = MSR(mm)+ CSR(0.01mm)

= 0 + 52 x 0.01mm

= 0.052 cm

87. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is :

(1) $\frac{g}{3}$

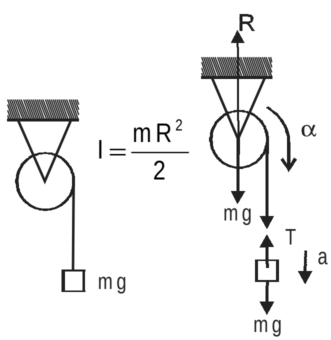
(2) $\frac{3}{2}g$

(3) g

(4) $\frac{2}{3}g$

Key. 4

Soln. 87



$mg - T = ma$ $\qquad TR = I\alpha$ $\qquad I = \frac{mR^2}{2}$

$a = R\alpha$ Solving, $a = \dfrac{mg}{(M + I/R^2)} = \dfrac{2}{3}g$

88. The transverse displacement y(x, t) of a wave on a string is given by

$y(x,\ t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}\ xt)}$

This represents a :

(1) standing wave of frequency $\dfrac{1}{\sqrt{b}}$

(2) wave moving in +x direction with speed $\sqrt{\dfrac{a}{b}}$

(3) wave moving in -x direction with speed $\sqrt{\dfrac{b}{a}}$

(4) standing wave of frequency $\sqrt{b}$

Key. 3

Soln. 88 $y(x,\ t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}\ xt)} = e^{-(\sqrt{a}x + \sqrt{b}t)^2}$

Therefore the above equation represents a travelling pulse $y(x, t) = f(kx \pm wt)$ with a velocity of

$v = (w/k) = \sqrt{\dfrac{b}{a}}$ along the "-ve" x- axis

89. A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is:

(1) 15 m/s

(2) $\dfrac{1}{10}$ m/s

(3) $\dfrac{1}{15}$ m/s

(4) 10 m/s

Key. 3

Soln. 89

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{t} \Rightarrow \frac{1}{v^2}\left(\frac{dv}{dt}\right) - \frac{1}{u^2}\left(\frac{du}{dt}\right) = 0$$

$$\Rightarrow \frac{dv}{dt} = \frac{v^2}{u^2}\frac{du}{dt}$$

u = 280 cm

t = 20 cm

$v = (280/15)\text{cm} \Rightarrow \left(\dfrac{dv}{dt}\right) = \dfrac{1}{15^2}\dfrac{du}{dt} = 15^{-1}\text{m/s}$

90. Let the x-z plane the boundary between two transparent media. Medium 1 in $\geq$ z o has a refractive index of $\sqrt{2}$ and medium 2 with z< 0 has refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\ \hat{i} + 8\sqrt{3}\ \hat{j} - 10\ \hat{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is

(1) 75°

(2) 30°

(3) 45°

(4) 60°

Key. 3

Soln.90 $m_1 = \sqrt{2}$

$m_2 = \sqrt{3}$

and the relaading sunface is the x-y plane, therefore the is along i.e. Hence for are incidant hean along A the angle of inci is such that

$$\text{Cos } i = \frac{\vec{A}.\ K^4}{|\vec{A}|} = \frac{10}{\sqrt{(36x3) + (64x3) + 100}} = \frac{10}{\sqrt{400}} = \frac{1}{2}$$

$\Rightarrow$ i =

$\mu_1 \sin i = \mu_2 \sin r \Rightarrow \sqrt{2} \sin (\text{II}/3) = \sqrt{3} \sin r => \sin r = \dfrac{1}{\sqrt{2}}$

$\Rightarrow$ r = 45