GATE - 2013

EC : ELECTRONICS AND COMMUNICATION ENGINEERING

Duration : Three Hours | Maximum Marks : 100

Read the following instructions carefully.

1. All questions in this paper are of objective type.
2. There are a total of 65 questions carrying 100 marks.
3. Questions 1 through 25 are 1-mark questions, question 26 through 55 are 2-mark questions.
4. Questions 48 and 51 (2 pairs) common data questions and question pairs (Q. 52 and Q.53) and (Q. 54 and Q.55) are linked answer questions. The answer to the second question of the above pair depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is un-attempted, then the answer to the second question in the pair will not be evaluated.
5. Questions 56 - 65 belong to general aptitude (GA). Questions 56 - 60 will carry 1-mark each, and questions 61-65 will carry 2-marks each. The GA questions will begin on a fresh page.
6. Un-attempted questions will carry zero marks.
7. Wrong answers will carry NEGATIVE marks. For Q.1 to Q.25 and Q.56 - Q.60, 1/3 mark will be deducted for each wrong answer. For Q. 26 to Q. 51, and Q.61 - Q.65, 2/3 mark will be deducted for each wrong answer. The question pairs (Q. 52, Q. 53) and (Q. 54, Q. 55) are questions with linked answers. There will be negative marks only for wrong answer to the first question of the linked answer question pair i.e. for Q. 52 and Q.54, 2/3 mark will be deducted for each wrong answer. There is no negative marking for Q. 53 and Q.55..

Q.1 to Q.25 carry one mark each.

1. A bulb in a staircase has two switches, one switch being at the ground floor and the other one at the first floor. The bulb can be turned ON and also can be turned OFF by any one of the switches irrespective of the state of the other switch. The logic of switching of the bulb resembles
   (a) an AND gate (b) an OR gate
   (c) an XOR gate (d) a NAND gate

2. Consider a vector field $\vec{A}(\vec{r})$. The closed loop line integral $\oint \vec{A} \bullet d\vec{l}$ can be expressed as
   (a) $\oiint (\nabla \times \vec{A}) \bullet d\vec{s}$ over the closed surface bounded by the loop
   (b) $\oiiint (\nabla \bullet \vec{A}) dv$ over the closed volume bounded by the loop
   (c) $\iiint (\nabla \bullet \vec{A}) dv$ over the open volume bounded by the loop
   (d) $\iint (\nabla \times \vec{A}) \bullet d\vec{s}$ over the open surface bounded by the loop

3. Two systems with impulse responses $h_1(t)$ and $h_2(t)$ are connected in cascade. Then the overall impulse response of the cascaded system is given by
   (a) product of $h_1(t)$ and $h_2(t)$
   (b) sum of $h_1(t)$ and $h_2(t)$
   (c) convolution of $h_1(t)$ and $h_2(t)$
   (d) subtraction of $h_2(t)$ from $h_1(t)$

4. In a forward biased pn junction diode, the sequence of events that best describes the mechanism of current flow is
   (a) injection, and subsequent diffusion and recombination of minority carriers
   (b) injection, and subsequent drift and generation of minority carriers
   (c) extraction, and subsequent diffusion and generation of minority carriers
   (d) extraction, and subsequent drift and recombination of minority carriers

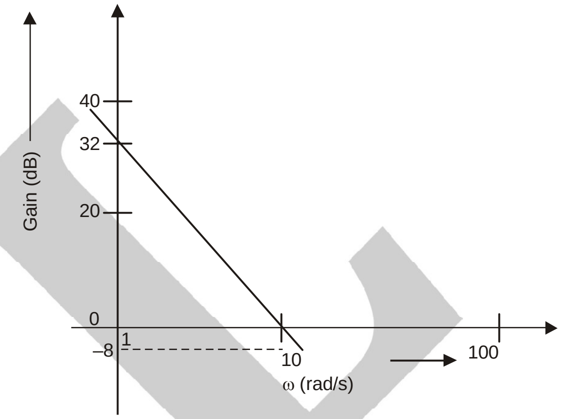
5. In IC technology, dry oxidation (using dry oxygen) as compared to wet oxidation (using steam or water vapor) produces
   (a) superior quality oxide with a higher growth rate
   (b) inferior quality oxide with a higher growth rate
   (c) inferior quality oxide with a lower growth rate
   (d) superior quality oxide with a lower growth rate

6. The maximum value of θ until which the approximation $\sin\theta \approx 0$ holds to within 10% error is
   (a) 10° (b) 18°
   (c) 50° (d) 90°

7. The divergence of the vector field $\vec{A} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ is
   (a) 0 (b) $\frac{1}{3}$
   (c) 1 (d) 3

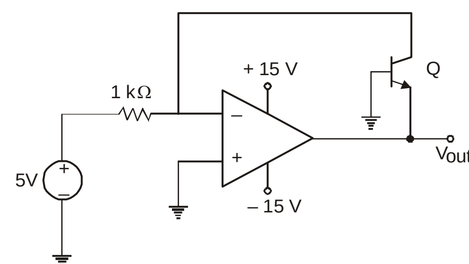
8. The impulse response of a system is h(t) = tu(t). For an input u(t - 1), the output is
   (a) $\frac{t^2}{2}u(t)$ (b) $\frac{t(t-1)}{2}u(t-1)$
   (c) $\frac{(t-1)^2}{2}u(t-1)$ (d) $\frac{t^2-1}{2}u(t-1)$

9. The Bode plot of a transfer function G(s) is shown in the figure below.



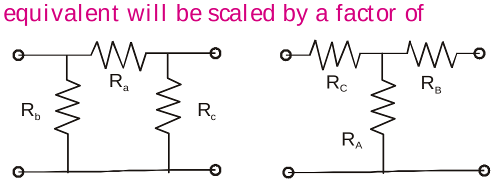
The gain (20 log| G(s)|) is 32 dB and - 8 dB at 1 rad/s and 10 rad/s respectively. The phase is negative for all ω. Then G (s) is
   (a) $\frac{39.8}{s}$ (b) $\frac{39.8}{s^2}$
   (c) $\frac{32}{s}$ (d) $\frac{32}{s^2}$

10. In the circuit shown below what is the output voltage (V_{out}) if a silicon transistor Q and an ideal op-amp are used?



   (a) - 15 V (b) - 0.7 V
   (c) +0.7 V (d) +15 V

11. Consider a delta connection of resistors and its equivalent star connection as shown below. If all elements of the delta connection are scaled by a factor K, K > O, the elements of the corresponding star equivalent will be scaled by a factor of



   (a) k^2 (b) k
   (c) l/k (d) $\sqrt{k}$

12. For 8085 microprocessor, the following program is executed.

```
     MVI A, 05H;
     MVI B, 05H;
PTR: ADD B;
     DCR B;
     JNZ PTR;
     ADI 03H;
     HLT;
```

At the end of program, accumulator contains

(a) 17H (b) 20H
(c) 23H (d) 05H

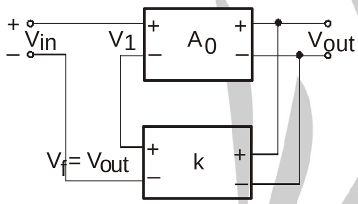
13. The bit rate of a digital communication system is R kbits/s. The modulation used is 32-QAM. The minimum bandwidth required for ISI free transmission is

(a) R/10 Hz (b) R/10 kHz
(c) R/5 Hz (d) R/5 kHz

14. For a periodic signal $v(t) = 30 \sin 100t + 10 \cos 300t + 6 \sin(500t + \pi/4)$, the fundamental frequency in rad/s is

(a) 100 (b) 300
(c) 500 (d) 1500

15. In a voltage-voltage feedback as shown below, which one of the following statements is TRUE if the gain k is increased?



(a) The input impedance increases and output impedance decreases.
(b) The input impedance increases and output impedance also increases.
(c) The input impedance decreases and output impedance also decreases.
(d) The input impedance decreases and output impedance increases.

16. A band-limited signal with a maximum frequency of 5kHz is to be sampled. According to the sampling theorem, the sampling frequency which is not valid is

(a) 5 kHz
(b) 12 kHz
(c) 15 kHz
(d) 20 kHz

17. In a MOSFET operating in the saturation region, the channel length modulation effect causes

(a) an increase in the gate-source capacitance
(b) a decrease in the transconductance
(c) a decrease in the unity-gain cutoff frequency
(d) a decrease in the output resistance

18. Which one of the following statements is NOT TRUE for a continuous time causal and stable LTI system?

(a) All the poles of the system must lie on the left side of the $j\omega$ axis.
(b) Zeros of the system can lie anywhere in the s-plane.
(c) All the poles must lie within $|s| = 1$.
(d) All the roots of the characteristic equation must be located on the left side of the $j\omega$ axis.

19. The minimum eigen value of the following matrix is

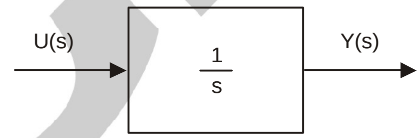
$$\begin{bmatrix} 3 & 5 & 2 \\ 5 & 12 & 7 \\ 2 & 7 & 5 \end{bmatrix}$$

(a) 0 (b) 1
(c) 2 (d) 3

20. A polynomial $f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x - a_0$ with all coefficients positive has

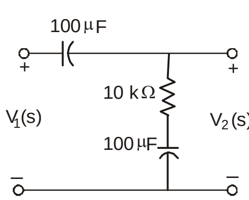
(a) no real roots
(b) no negative real root
(c) odd number of real roots
(d) at least one positive and one negative real root

21. Assuming zero initial condition, the response y(t) of the system given below to a unit step input u(t) is



(a) u(t) (b) tu(t)
(c) $\frac{t^2}{2}u(t)$ (d) $e^{-1}u(t)$

22. The transfer function $\frac{V_2(s)}{V_1(s)}$ of the circuit shown below is



(a) $\frac{0.5s+1}{s+1}$ (b) $\frac{3s+6}{s+2}$
(c) $\frac{s+2}{s+1}$ (d) $\frac{s+1}{s+2}$

23. A source $v_s(t) = V \cos 100\pi t$ has an internal impedance of $(4 + j3)\ \Omega$. If a purely resistive load connected to this source has to extract the maximum power out of the source, its value in Ω should be

(a) 3 (b) 4
(c) 5 (d) 7

24. The return loss of a device is found to be 20 dB. The voltage standing wave ratio (VSWR) and magnitude of reflection coefficient are respectively

(a) 1.22 and 0.1 (b) 0.81 and 0.1
(c) - 1.22 and 0.1 (d) 2.44 and 0.2

25. Let g(t) = $e^{-\pi t^2}$, and h(t) is a filter matched to g(t). If g(t) is applied as input to h(t), then the Fourier transform of the output is

(a) $e^{-\pi f^2}$ (b) $e^{-\pi f^2/2}$
(c) $e^{\pi |f|}$ (d) $e^{-2\pi f^2}$

26. Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively. The probability P(3V ≥ 2U) is

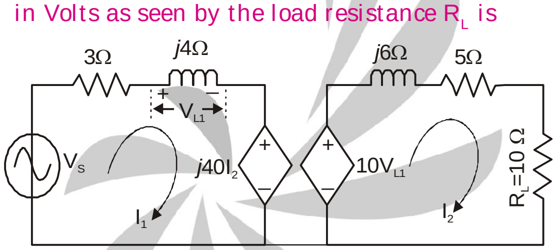
(a) $\frac{4}{9}$ (b) $\frac{1}{2}$
(c) $\frac{2}{3}$ (d) $\frac{5}{9}$

27. Let A be an m × n matrix and B an n × m matrix. It is given that determinant (I_m + AB) = determinant (I_n + BA), where I_k is the k × k identity matrix. Using the above property, the determinant of the matrix given below is

$$\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

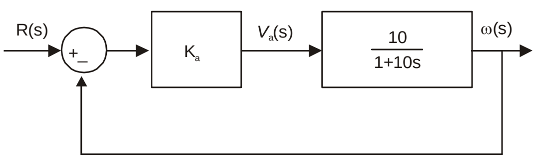
(a) 2 (b) 5
(c) 8 (d) 16

28. In the circuit shown below, if the source voltage V_s = 100∠53.13° V then the Thevenin's equivalent voltage in Volts as seen by the load resistance R_L is



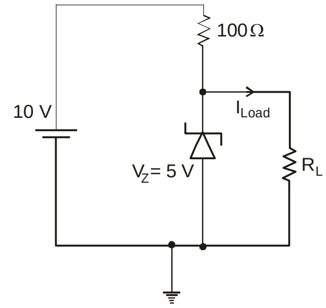
(a) 100∠90° (b) 800∠0°
(c) 800∠90° (d) 100∠60°

29. The open-loop transfer function of a dc motor is given as $\frac{w(s)}{V_a(s)} = \frac{10}{1+10s}$. When connected in feedback as shown below, the approximate value of K_a that will reduce the time constant of the closed loop system by one hundred times as compared to that of the open-loop system is



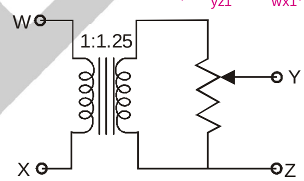
(a) 1 (b) 5
(c) 10 (d) 100

30. In the circuit shown below, the knee current of the ideal Zener diode is 10 mA. To maintain 5 V across R_L, the minimum value of R_L in Ω and the minimum power rating of the Zener diode in mW, respectively, are



(a) 125 and 125 (b) 125 and 250
(c) 250 and 125 (d) 250 and 250

31. The following arrangement consists of an ideal transformer and an attenuator which attenuates by a factor of 0.8. An ac voltage V_{wx1} = 100V is applied across WX to get an open circuit voltage V_{yz1} across YZ. Next, an ac voltage V_{yz2} = 100V is applied across YZ to get an open circuit voltage V_{wx2} across WX. Then, V_{yz1} / V_{wx1}, V_{wx2} / V_{yz2} are respectively,



(a) 125/100 and 80/100
(b) 100/100 and 80/100
(c) 100/100 and 100/100
(d) 80/100 and 80/100

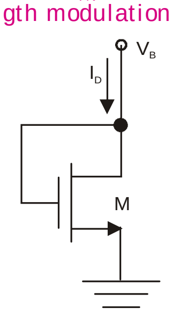
32. Two magnetically uncoupled inductive coils have Q factors q_1 and q_2 at the chosen operating frequency. Their respective resistances are R_1 and R_2. When connected in series, their effective Q factor at the same operating frequency is

(a) $q_1 + q_2$ (b) $(1/q_1)+(1/q_2)$
(c) $(q_1R_1 + q_2R_2) / (R_1 + R_2)$ (d) $(q_1 R_2 + q_2 R_1) / (R_1 + R_2)$

33. The impulse response of a continuous time system is given by h(t) = δ(t - 1) + δ(t - 3). The value of the step response at t = 2 is

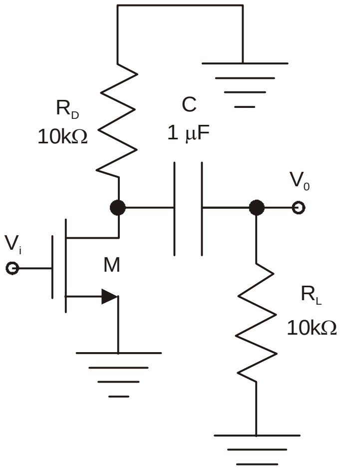
(a) 0 (b) 1
(c) 2 (d) 3

34. The small-signal resistance (i.e., dV_B/dI_D) in kΩ offered by the n-channel MOSFET M shown in the figure below, at a bias point of V_B = 2 V is (device data for M: device transconductance parameter $k_n = \mu_n C_{ox}'$ (W/L) = 40 μA/V², threshold voltage V_{TN} = 1 V, and neglect body effect and channel length modulation effects)



(a) 12.5 (b) 25
(c) 50 (d) 100

35. The ac schematic of an NMOS common-source stage is shown in the figure below, where part of the biasing circuits has been omitted for simplicity. For the n -channel MOSFET M, the transconductance g_m = 1 mA/V, and body effect and channel length modulation effect are to be neglected. The lower cutoff frequency in Hz of the circuit is approximately at



(a) 8 (b) 32
(c) 50 (d) 200

36. A system is described by the differential equation

$$\frac{d^2y}{dt^2}+5\frac{dy}{dt}+6y(t) = x(t).$$

Let x(t) be a rectangular pulse given by

$$x(t)=\begin{cases}1 & 0<t<2\\ 0 & \text{otherwise}\end{cases}$$

Assuming that y(0) = 0 and $\frac{dy}{dt}$ = 0 at t = 0, the Laplace transform of y(t) is

(a) $\frac{e^{-2s}}{s(s+2)(s+3)}$ (b) $\frac{1-e^{-2s}}{s(s+2)(s+3)}$

(c) $\frac{e^{-2s}}{(s+2)(s+3)}$ (d) $\frac{1-e^{-2s}}{(s+2)(s+3)}$

37. A system described by a linear, constant coefficient, ordinary, first order differential equation has an exact solution given by y(t) for t > 0, when the forcing function is x(t) and the initial condition is y(0) . If one wishes to modify the system so that the solution becomes - 2y(t) for t > 0, we need to

(a) change the initial condition to - y(0) and the forcing function to 2x(t)

(b) change the initial condition to 2y(0) and the forcing function to - x(t)

(c) change the initial condition to $j\sqrt{2}y(0)$ and the forcing function to $j\sqrt{2}x(t)$

(d) change the initial condition to - 2y(0) and the forcing function to - 2x(t)

38. Consider two identically distributed zero-mean random variables U and V. Let the cumulative distribution functions of U and 2V be F(x) and G(x) respectively. Then, for all values of x

(a) $F(x) - G(x) \le 0$ (b) $F(x) - G(x) \ge 0$

(c) $(F(x) - G(x)) \cdot x \le 0$ (d) $(F(x) - G(x)) \cdot x \ge 0$

39. The DFT of a vector $[a\ b\ c\ d]$ is the vector $[\alpha\ \ \beta\ \ \gamma\ \ \delta]$. Consider the product

$$[p\ \ q\ \ r\ \ s]=[a\ \ b\ \ c\ \ d]\begin{bmatrix}a & b & c & d\\ d & a & b & c\\ c & d & a & b\\ b & c & d & a\end{bmatrix}$$

The DFT of the vector $[p\ \ q\ \ r\ \ s]$ is a scaled version of

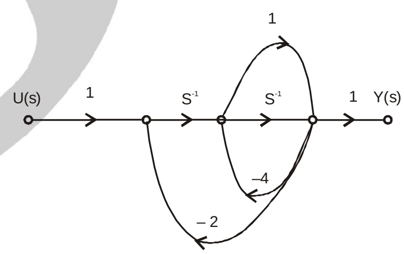
(a) $[\alpha^2\ \ \beta^2\ \ \gamma^2\ \ \delta^2]$

(b) $[\sqrt{\alpha}\ \ \sqrt{\beta}\ \ \sqrt{\gamma}\ \ \sqrt{\delta}]$

(c) $[\alpha+\beta\ \ \beta+\delta\ \ \delta+\gamma\ \ \gamma+\alpha]$

(d) $[\alpha\ \ \beta\ \ \gamma\ \ \delta]$

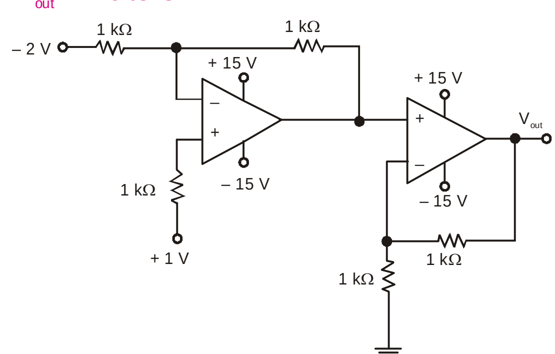
40. The signal flow graph for a system is given below. The transfer function $\frac{Y(s)}{U(s)}$ for this system is



(a) $\frac{s+1}{5s^2+6s+2}$ (b) $\frac{s+1}{s^2+6s+2}$

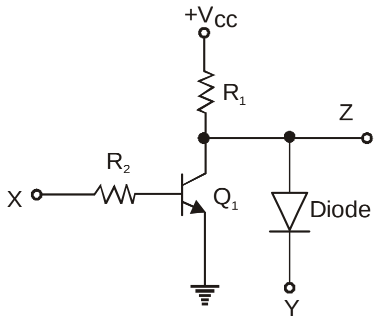
(c) $\frac{s+1}{s^2+4s+2}$ (d) $\frac{1}{5s^2+6s+2}$

41. In the circuit shown below the op-amps are ideal. Then V_{out} in Volts is



(a) 4 (b) 6
(c) 8 (d) 10

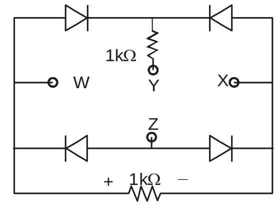
42. In the circuit shown below, Q_1 has negligible collector-to-emitter saturation voltage and the diode drops negligible voltage across it under forward bias. If V_{cc} is +5 V, X and Y are digital signals with 0 V as logic 0 and V_{cc} as logic 1, then the Boolean expression for Z is



(a) XY (b) $\bar{X}Y$

(c) $X\bar{Y}$ (d) $\overline{XY}$

43. A voltage 1000 sin ωt Volts is applied across YZ. Assuming ideal diodes, the voltage measured across WX in Volts, is



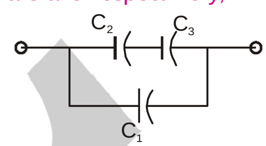
(a) sinωt

(b) (sin ωt + | sin ωt|)/2

(c) (sin ωt - | sin ωt|)/2

(d) 0 for all t

44. Three capacitors C_1, C_2 and C_3 whose values are 10μF, 5μF, and 2μF respectively, have breakdown voltages of 10V, 5V, and 2V respectively. For the interconnection shown below, the maximum safe voltage in Volts that can be applied across the combination, and the corresponding total charge in μC stored in the effective capacitance across the terminals are respectively,



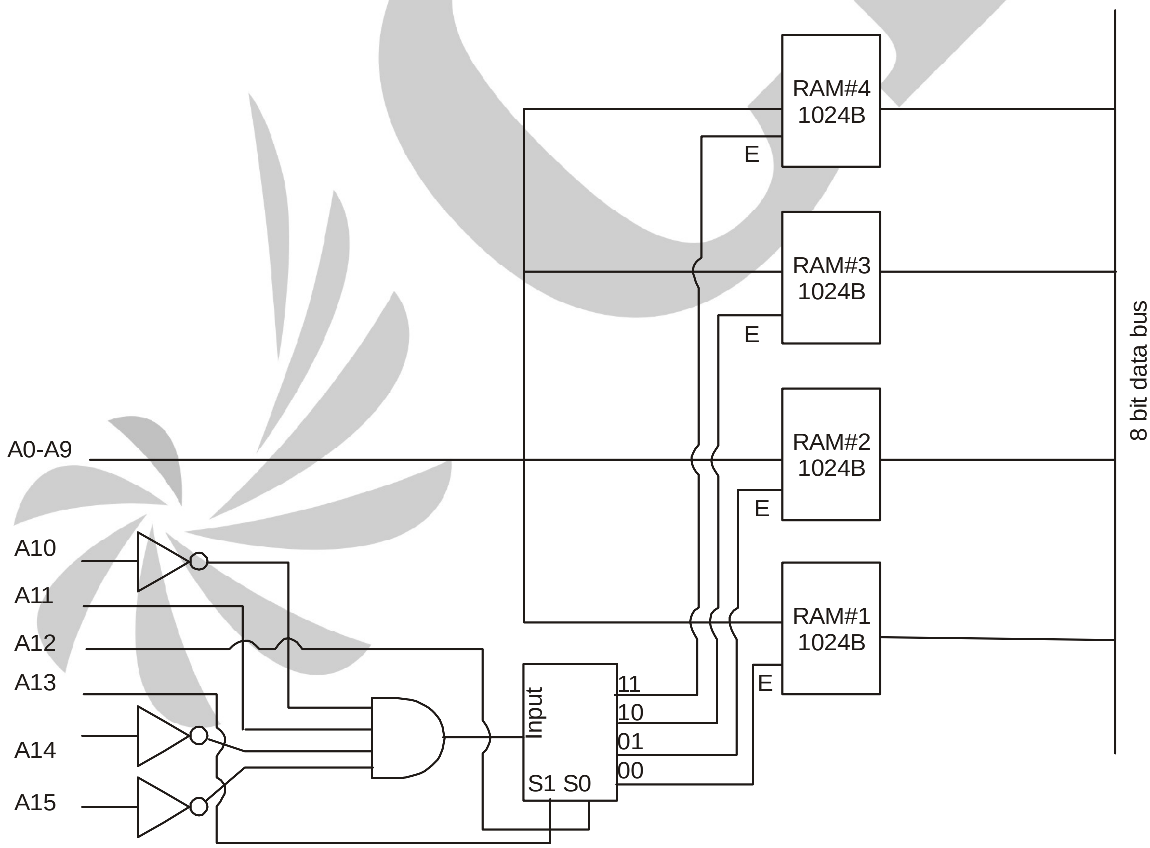
(a) 2.8 and 36

(b) 7 and 119

(c) 2.8 and 32

(d) 7 and 80

45. There are four chips each of 1024 bytes connected to a 16 bit address bus as shown in the figure below. RAMs 1, 2, 3 and 4 respectively are mapped to addresses



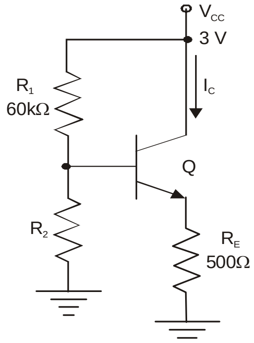
(a) 0C00H-0FFFH, 1C00H-1 FFFH, 2C00H-2FFFH, 3C00H-3FFFH

(b) 1800H-1FFFH, 2800H-2FFFH, 3800H-3FFFH,4800H-4FFFH

(c) 0500H-08FFH, 1500H-18FFH, 3500H-38FFH, 5500H-58FFH

(d) 0800H-0BFFH, 1800H-1BFFH, 2800H-2BFFH, 3800H-3BFFH

46. In the circuit shown below, the silicon npn transistor Q has a very high value of β. The required value of R_2 in kΩ to produce I_C = 1 mA is



(a) 20 (b) 30
(c) 40 (d) 50

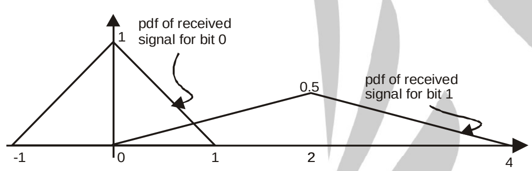
47. Let U and V be two independent and identically distributed random variables such that P(U = +1) = P(U = - 1) = $\frac{1}{2}$. The entropy H (U +V) in bits is

(a) $\frac{3}{4}$ (b) 1
(c) $\frac{3}{2}$ (d) $\log_2 3$

COMMON DATA QUESTIONS

Common Data for Questions 48 and 49:

Bits 1 and 0 are transmitted with equal probability. At the receiver, the pdf of the respective received signals for both bits are as shown below.



48. If the detection threshold is 1, the BER will be

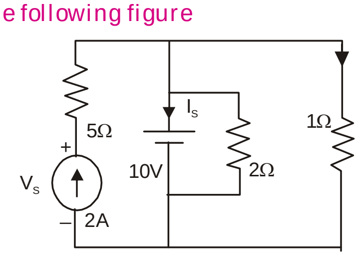
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) $\frac{1}{8}$ (d) $\frac{1}{16}$

49. The optimum threshold to achieve minimum bit error rate (BER) is

(a) $\frac{1}{2}$ (b) $\frac{4}{5}$
(c) 1 (d) $\frac{3}{2}$

Common Data for Questions 50 and 51:

Consider the following figure



50. The current I_s in Amps in the voltage source, and voltage V_s in Volts across the current source respectively, are

(a) 13, - 20 (b) 8, - 10
(c) - 8, 20 (d) - 13, 20

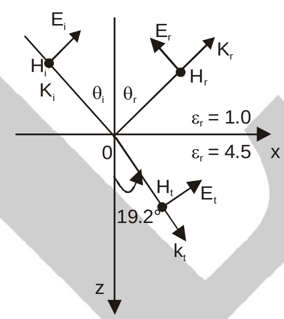
51. The current in the 1 Ω resistor in Amps is

(a) 2 (b) 3.33
(c) 10 (d) 12

LINKED ANSWER QUESTIONS

Statement for Linked Answer Questions 52 and 53 :

A monochromatic plane wave of wavelength λ = 600μm is propagating in the direction as shown in the figure below. $\vec{E}_i, \vec{E}_r$ and $\vec{E}_t$ denote incident, reflected, and transmitted electric field vectors associated with the wave.



52. The angle of incidence θ, and the expression for $\vec{E}_i$ are

(a) $60°$ and $\frac{E_0}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi\times10^4(x+z)}{3\sqrt{2}}}$ V / m

(b) $45°$ and $\frac{E_0}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi\times10^4 z}{3}}$ V / m

(c) $45°$ and $\frac{E_0}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi\times10^4(x+z)}{3\sqrt{2}}}$ V / m

(d) $60°$ and $\frac{E_0}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi\times10^4 z}{3}}$ V / m

53. The expression for $\vec{E}_r$ is

(a) $0.23\frac{E_0}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi\times10^4(x-z)}{3\sqrt{2}}}$ V / m

(b) $-\frac{E_0}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{j\frac{\pi\times10^4 z}{3}}$ V / m

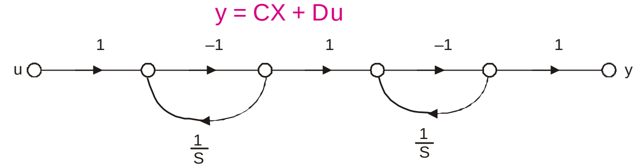
(c) $0.44\frac{E_0}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi\times10^4(x-z)}{3\sqrt{2}}}$ V / m

(d) $\frac{E_0}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi\times10^4(x+z)}{3}}$ V / m

Statement for Linked Answer Ques-tions 54 and 55 :

The state diagram of a system is shown below. A system is described by the state-variable equations

$$\dot{X} = AX + Bu;$$
$$y = CX + Du$$



54. The state-variable equations of the system shown in the figure above are

(a) $\dot{X} = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} X + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$, $y = \begin{bmatrix} 1 & -1 \end{bmatrix} X + u$

(b) $\dot{X} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} X + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$, $y = \begin{bmatrix} -1 & -1 \end{bmatrix} X + u$

(c) $\dot{X} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} X + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$, $y = \begin{bmatrix} -1 & -1 \end{bmatrix} X - u$

(d) $\dot{X} = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} X + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$, $y = \begin{bmatrix} 1 & -1 \end{bmatrix} X - u$

55. The state transition matrix e^{At} of the system shown in the figure above is

(a) $\begin{bmatrix} e^{-t} & 0 \\ te^{-t} & e^{-t} \end{bmatrix}$ (b) $\begin{bmatrix} e^{-t} & 0 \\ -te^{-t} & e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{-t} & 0 \\ e^{-t} & e^{-t} \end{bmatrix}$ (d) $\begin{bmatrix} e^{-t} & -te^{-t} \\ 0 & e^{-t} \end{bmatrix}$

GENERAL APTITUDE (GA) QUESTIONS

Q.56 to Q.60 carry one mark each.

56. Choose the grammatically CORRECT sentence:

(a) Two and two add four.
(b) Two and two become four.
(c) Two and two are four.
(d) Two and two make four.

57. Statement : You can always give me a ring whenever you need.

Which one of the following is the best inference from the above statement?

(a) Because I have a nice caller tune.
(b) Because I have a better telephone facility.
(c) Because a friend in need is a friend indeed.
(d) Because you need not pay towards the telephone bills when you give me a ring.

58. In the summer of 2012, in New Delhi, the mean temperature of Monday to Wednesday was 41°C and of Tuesday to Thursday was 43°C. If the temperature on Thursday was 15% higher than that of Monday, then the temperature in °C on Thursday was

(a) 40 (b) 43 (c) 46 (d) 49

59. Complete the sentence : Dare _____ mistakes.

(a) commit (b) to commit
(c) committed (d) committing

60. They were requested not to quarrel with others.

Which one of the following options is the closest in meaning to the word quarrel?

(a) make out (b) call out
(c) dig out (d) fall out

Q.61 to Q.65 carry two marks each.

61. A car travels 8 km in the first quarter of an hour, 6 km in the second quarter and 16 km in the third quarter. The average speed of the car in km per hour over the entire journey is

(a) 30 (b) 36
(c) 40 (d) 24

62. Find the sum to n terms of the series 10 + 84 + 734 + ...

(a) $\frac{9(9^n+1)}{10}+1$ (b) $\frac{9(9^n-1)}{8}+1$

(c) $\frac{9(9^n-1)}{8}+n$ (d) $\frac{9(9^n-1)}{8}+n^2$

63. Statement : There were different streams of freedom movements in colonial India carried out by the moderates, liberals, radicals, socialists, and so on.

Which one of the following is the best inference from the above statement?

(a) The emergence of nationalism in colonial India led to our Independence.
(b) Nationalism in India emerged in the context of colonialism.
(c) Nationalism in India is homogeneous.
(d) Nationalism in India is heterogeneous.

64. The set of values of p for which the roots of the equation $3x^2 + 2x + p(p - l) = 0$ are of opposite sign is

(a) $(-\infty, 0)$ (b) $(0, 1)$
(c) $(1, \infty)$ (d) $(0, \infty)$

65. What is the chance that a leap year, selected at random, will contain 53 Saturdays?

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$

(c) $\frac{1}{7}$ (d) $\frac{5}{7}$

ANSWERS

1. (c)	2. (d)	3. (c)	4. (a)	5. (d)	6. (b)	7. (d)	8. (c)	9. (b)	10. (b)
11. (b)	12. (a)	13. (b)	14. (a)	15. (a)	16. (a)	17. (d)	18. (c)	19. (a)	20. (c)
21. (b)	22. (d)	23. (c)	24. (a)	25. (d)	26. (c)	27. (b)	28. (c)	29. (c)	30. (b)
31. (b)	32. (c)	33. (b)	34. (b)	35. (a)	36. (b)	37. (d)	38. (d)	39. (a)	40. (a)
41. (c)	42. (b)	43. (d)	44. (c)	45. (d)	46. (c)	47. (c)	48. (c)	49. (b)	50. (d)
51. (c)	52. (c)	53. (c)	54. (a)	55. (a)	56. (d)	57. (c)	58. (c)	59. (b)	60. (b)
61. (c)	62. (d)	63. (d)	64. (b)	65. (a)					

EXPLANATIONS

1. Let us consider the switches A and B and bulb Y. Switches can be 2 positions up (0) or down (1)

Starting with both A and B in up position. Let the bulb be OFF. Now since B can operate independently when B goes down, the bulb goes ON

A	B	Y
up (0)	up (0)	OFF
up (0)	down (1)	ON

Now keeping A in down position when B goes down, the bulb will go OFF.

A	B	Y
down (1)	up (0)	ON
down (1)	down (1)	OFF

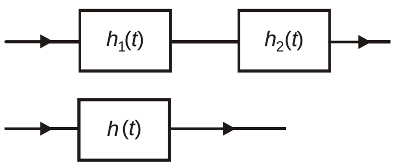
find truth table corresponds to XOR gate.

2. Hence, for a vector field $\vec{A}(\bar{r})$

$$\oint \vec{A}\cdot\vec{de} = \iint(\vec{\nabla}\times\vec{A})\cdot d\vec{s}$$

$\rightarrow$ according to Stokes theorem.

3.



$h(t) = h_1(t) \oplus h_2(t)$

4. Due to application of voltage (forward bias minority carrier are injected from either side of diode one subsequent diffusion takes place and finally recombination.

Injection and subsequent diffusion and recombination of minority carriers

5. As during dry oxidation, quality of oxide is superior as it does not contain ullter which is responsible for containmation/impurity but simultaneously lower the growth rate

6.
$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$
$$\lim_{\theta\to 0}\sin\theta = \theta$$

as within 10% of error. $=180° \times 1 = 18°$

7.
$$\bar{A} = xa_x + ya_y + za_z$$
$$\nabla\cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = 1 + 1 + 1 = 3$$

8. As h(t) = t a(f)

input response

$\delta(t) \rightarrow t\,a(t)$

$$u(t) \rightarrow \int_{-\infty}^{t} +a(I)dt = \int_0^t t\,dt = \frac{t^2}{2}a(t)$$

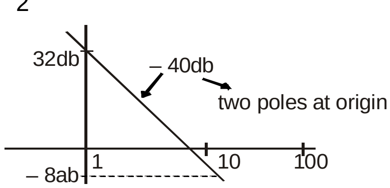
$$u(t-1) \rightarrow \frac{(t-1)^2}{2}a(t-1)$$

9. 20 log k = 32

$k = 10^{1.6}$

$= 39.8$

$$T(s) = \frac{39.8}{s^2}$$



10. Using the concept of "virtual ground" in an operational amplifier, we can set the voltage at the point to zero volts since the non inverting terminal is grounded.

Once $V_A = 0$, V_C will also be zero.

We know that for a silicon n-p-n transistor,

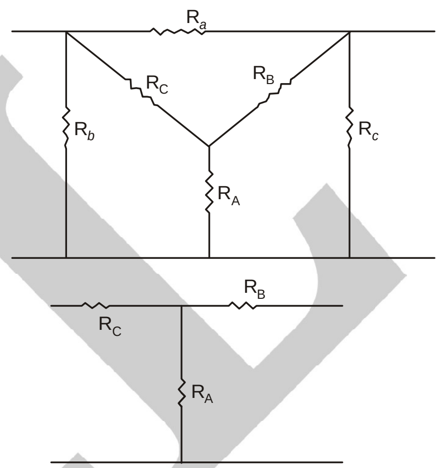
$V_{BE} = V_B - V_E = 0.7$ V

Since, $V_B = 0 \Rightarrow V_E = -0.7$ V

Hence the output voltage is the same as the emitter voltage

so, $V_{out} = -0.7$ V

11.



$$R_c = \frac{R_a \cdot R_b}{R_a + R_b + R_c}$$ as R_a is scaled by facts k

$$R_c' = \frac{R_a' \cdot R_b'}{R_a' + R_b + R_c} = \frac{k^2 R_a \cdot R_b}{k(R_a + R_b + R_c)} = k\cdot\frac{R_a \times R_b}{R_a + R_b + c}$$

so elements corresponding to star equivalence will be seated by facts k.

12. Accumulator changes as follows

(05 + 05 + 04 + 03 + 02 + 01)H

At the end of Loop accumulator contains = 14H

ADI 03H $\rightarrow$ A = (14 + 03) = 17H

13. Bit rate given = R Kbits/second

Modulation = 32-QAM

No. of bits/symbol = 5[$\log_2 32$]

Symbol rate = $\frac{R}{5}$ k symbols/second

Finally we are transmitting symbols.

$E_T \rightarrow$ transmission bandwidth

$$B_T = \frac{R(\text{symbol rate})}{(1+\alpha)} = \frac{R}{5(1+\alpha)}$$

For B_T to be minimum, α has to be maximum

$$\Rightarrow B_T = \frac{R}{5\times 2} = \frac{R}{10}$$

Maximum value of α is '1' which is a roll off factor

14. LCM of T_1, T_2, T_3 will be: $= \frac{\text{LCM of numerator}}{\text{HCF of denominator.}} = \frac{2\pi}{100}$

$\therefore$ Overall time period = $\frac{2\pi}{100}$ sec.

Harmonic frequency = 100 rad/sec.

15. Input Independence of a voltage-voltage feedback circuit

$= 2i\,(1 + A_0 k)$

Z_i = initial input impedance (without feed output Impedance of a voltage-voltage feedback circuit

$= Z_0/(1+A_0 k)$

Z_0 = initial output impedance (without feedback)

Hence, As K is increased, the input impedance will increase and output impedance will decrease.

16. Here $f_m = 5\text{KHZ}$

$\therefore \quad f_s \geq 2f_m = 10\text{ KHZ}$

B,C,D options are greater than 10 KHZ

17. $I_D = K_n\left(\frac{\omega}{2}\right)(V_{GS}-V_t)^2(1+\lambda V_{DS})$ output resistance is given by→

$$r_0 = \frac{1}{\partial I_D/\partial V_{DS}} \quad ...(1)$$

$$\frac{\partial I_D}{\partial V_{DS}} = \lambda k' n\frac{\omega}{2}(V_{GS}-V_t)^2 \quad ...(2)$$

$$r_0 = \frac{1}{\partial I_D/dV_{DS}} = \frac{1}{\lambda k' n\frac{\omega}{L}(V_{Gs}-V_t)^2}$$

For Ideal cases

$$I_D = k' n\left(\frac{\omega}{L}\right)(V_{Gs}-V_t)^2$$

$$\frac{\partial ID}{\partial V_{DS}} = 0$$

$$r = \infty$$

18. Consider option A: In which all the pols lie on the left of jω axis which satisfy casual stable LT1 system.

Option B: For a stable casual system, there are no restriction for the position of zeroes on s plane.

Option C: text true.

Option D: Roots of characteristic equation are all closed loop poles and they all line on the left side of the jω axis.

19. $|A| = 3[60-49] - 5[25-14] + 2[35-24] = 0$

$|A| = a_1 \times a_2 \times a_3 = 0$

which, implies either of the above eigen values equal to zero. It may be one or two negative eigen values.

20. $f(n) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x - a_0$

If complex roots are in even no. (in pair) then the real roots will also be even.

→ option (c) is wrong.

from the equation,

Product of roots = $\frac{-a_0}{a_4}(<0)$

As no. of roots = 4

$\therefore$ Products of 4 nos . < 0

⇒ either 1 or 3 nos < 0.

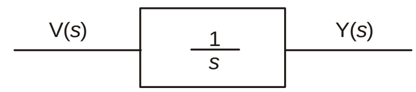
$\therefore$ B is

Product of roots:

$Z_1Z_1^*, Z_2Z_2^* < 0$

$|Z_1|^2|Z_2|^2 < 0$ which is not possible

21. $H(s) = \frac{1}{s}$

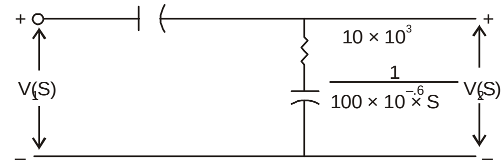


$h(t) = u(t)$

$u(t)$ = input

output = $u(t) \oplus h(t) = u(t) \oplus u(t) = r(t) = +u(t)$

22. Taking Laplace transformation of the circuit,



By applying voltage devider rule:

$$V_2(s) = \frac{10\times10^3 + \frac{10^4}{s}}{10\times10^3 + \frac{10^4}{s} + \frac{10^4}{s}} \times V_1(s)$$

$$\frac{V_2(s)}{V_1(s)} = \frac{1+\frac{1}{s}}{1+\frac{2}{s}} = \frac{s+1}{s+2}$$

23. $Z_i = (4 + j\,3)\ \Omega$

$Z_L = \sqrt{4^2+3^2}\ \Omega = 5\ \Omega$

24. The reflection co-efficient is $-20\log\Gamma = 20\text{dB}$

⇒ $\log\Gamma = -1\text{dB}$; ⇒ $\Gamma = 10^{-1}$ ⇒ $\Gamma = 0.1$

Relation between Γ and VSWR is

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.1}{1-0.1} = \frac{1.1}{0.9} = 1.22$$

25. $g(t) = e^{-\pi t^2}$

$h(t) = g(-t) = e^{-(-t)^2} = e^{-t^2}$

As we know $e^{-\pi t^2} \rightleftharpoons e^{-\pi f^2}$

$F[r(t)] = F[h(t) \oplus g(t)]$

$= H(s)\cdot G(s)$

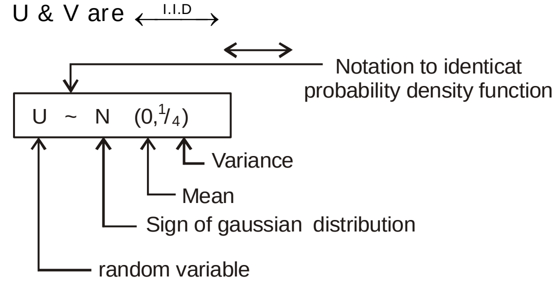
$= e^{-\pi f^2}\cdot e^{-\pi f^2}$

$= e^{-2\pi f^2}$

26. Given:

U & V be two independent & identical, distributed random variable

U & V are $\xleftrightarrow{\text{I.I.D}}$



$V \sim N(0, 1/9)$.

To find: $P(3V \geq 2U)$

Solution: Rearrange the problem

$$P\left[\left(\frac{V-O}{1/3}\right) \geq \left(\frac{U-O}{1/2}\right)\right]$$

Let denote $X = \dfrac{V - O}{1/3}$, $Y = \dfrac{U - O}{1/2}$

$$\begin{cases} E[X] = E\left[\dfrac{V}{1/3}\right] = \dfrac{1}{1/3}E[V] = O \\ E[1\times 1^2] = E\left[\dfrac{|V|^2}{1/9}\right] = \dfrac{1}{1/9}E[|V|] \\ \quad = 9 \times \dfrac{1}{9} \\ \quad = 1. \end{cases}$$

then X & Y become

$X \sim N(0, 1)$.

normalised distribution (Gaussian)

$Y \sim N(0, 1)$

normalised Gaussian distributed.

You can see clearly. How they became normalised distribution, as we know that normalised distribution has zero-mean & variance '1'.

$$E[|y|] = E\left[\frac{|U|}{1/2}\right] = \frac{1}{1/2}E[|U|] = 0$$

$$E[|y|^2] = E\left[\frac{|U|^2}{1/4}\right] = \frac{1}{1/4}E[|U|^2] = 4 \times \frac{1}{4} = 1.$$

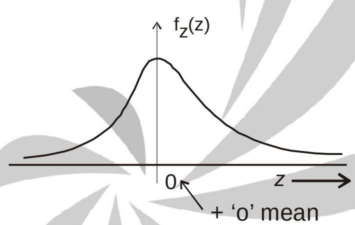
Question reduce to:

$$P(X \geq Y) = P(X - Y \geq O)$$

As let, Z = X - Y, it is just linear combination e random variable which is gaussion distribution with mean - O & variance - 1.

$\therefore$ their Linear combination is also gaussian = $P(Z \geq O)$

Note: By using central limit theorm, note here mean will be $\boxed{E[|Z|] = 0}$ as mean remain same & variance become 'n' times of resulting random variable, so figure will became



27. Given: A : m × n matrix

B : n × m matrix

$\det(I_m + AB) = \det(I_n + BA)$

I_K : K × K identify matrix

To find:

$$\det\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} = \det(M) \qquad \{\because \text{ say}\}$$

Analysis: We will break matrix m to match $(I_m + AB)$

Plan:

1. As per analysis part we will break matrix m into sum of I_m and AB
2. Then use $\det(I_m + AB) = \det(I_m + BA)$

Carrying out plan:

$$\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad ...(1)$$

Now we will break second matrix in RHS of above as follows

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}_{4\times 4} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}_{4\times 1} \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}_{1\times 4} \quad ...(2)$$

Using (2) into (1), we get

$$\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} = I_4 + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \quad ...(3)$$

Let $A = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}_{4\times 1}$ and $B = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}_{1\times 4}$

$$\therefore \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} = I_4 + A_{4\times 1}B_{1\times 4} \Rightarrow \boxed{\begin{matrix} m = 4 \\ n = 1 \end{matrix}}$$

But we are given that $\det(I_m + AB) = \det(I_n + BA)$

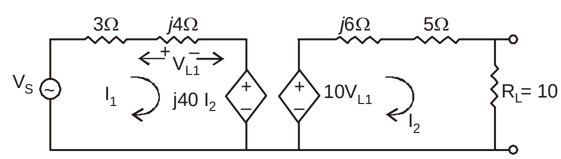
$$\therefore \det(I_4 + A_{4\times 1}B_{1\times 4}) = \det\left(I_1 + \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}\right)$$

= det (1 + 4) $\qquad \{\because I_1 = 1\}$

= det (5)

= 5 $\qquad$ {$\because$ determinant of a scalar is the same scalar}

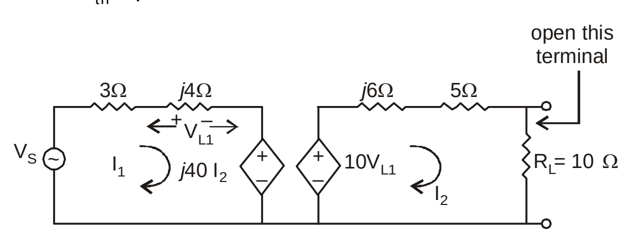
28. Given: $V_s = 100\angle 53.13°V$



To find: Thevenin's voltage across Load resistonce

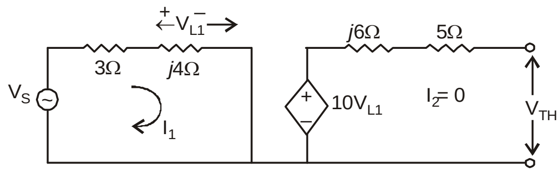
Solution

* For V'_{th} open it.



* Opening, then $I_2 = 0$
* When $I_2 = 0$, then $j40I_2 = 0$ {voltage source will short circuit}

$\therefore$ Circuit became



$\therefore$ $\boxed{I_1 = \dfrac{V_s}{3+4j}}$ $\quad\therefore$ $\boxed{V_{L1} = j4\times\dfrac{V_s}{3+4j}}$

* $V_{TH} = 10V_{L1}$ because no-current flowing through circuit.

$$V_{TH} = \frac{10\times j4\times V_s}{3+4j} = \frac{40j\ V_s}{3+4j}$$

From rectangular domain to polar domain.

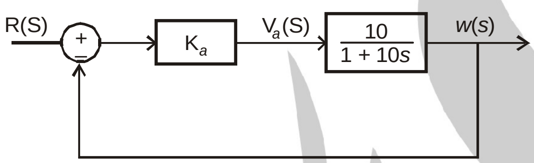
$$= \frac{40\angle 90^o}{5\angle 53\cdot 13}\times 100\angle 53\cdot 13^o$$

$$\boxed{V_{TH} = 800\angle 90^o}$$

29. Given:

Open loop transfer function of a dc motor as

$$\frac{w(s)}{V_a(s)} = \frac{10}{1+10s}$$



Topic : P controller with unity feed back

Formula: For first order system loop transfer function

is $\dfrac{C(s)}{R(s)} = \dfrac{K}{1+sT}$ comparing with $\dfrac{w(s)}{V_a(s)} = \dfrac{10}{1+10s}$ $T_{\text{open loop}} = 10$

Now for closed loop overall transfer function is given by

$$\frac{w(s)}{R(s)} = \frac{K_a\left(\dfrac{10}{1+10s}\right)}{1+K_a\left(\dfrac{10}{1+10s}\right)}$$

$$= \frac{K_a 10}{1+10s+K_a 10} = \frac{10K_a}{10s+(10K_a+1)}$$

Dividing numerator and denominator by $10K_a + 1$

Now $$\frac{w(s)}{R(s)} = \frac{\dfrac{10K_a}{10K_a+1}}{1+\left(\dfrac{10}{10K_a+1}\right)s}$$

So $T_{\text{closed loop}} = \dfrac{10}{10K_a+1}$ (By comparing from formula)

In Question given that time constant of closed loop system is $\dfrac{1}{100}$ times of time constant of open loop system

so $\dfrac{10}{10K_a+1} = \dfrac{1}{100}10\left(T_{\text{closed loop}} = \dfrac{1}{100}T_{\text{openloop}}\right)$

$10K_a + 1 = 100$

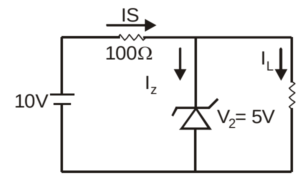
$10K_a = 99$

$K_a = 9.9 \simeq 10$

$\therefore$ $\boxed{K_a = 10}$ approximate value

30. $I_s = I_z + I_L$

$I_s - I_z = I_L$



Two extreme condition:

If I_z (min), then I_L (max)

If I_z (max) then I_L (min) = 0

$I_z(\text{max}) = I_s = \dfrac{10-5}{10} = 50$ mA

$I_z(\text{min}) = I_s - I_L(\text{max})$

$I_L(\text{max}) = I_s - I_z(\text{min}) = I_s - I_z = (50 - 10) = 40$mA

$R_L(\text{min}) = \dfrac{V}{I_L(\text{max})} = \dfrac{5}{40}K = 125\Omega$

$P_z = V_z \times I_z(\text{max}) = 5 \times 50$ mA $= 250$ mw

31. 1st case :

$V_{wx1} = 100$ V

So, $Vy'z_1 = \dfrac{M_2}{M_1}Vwx_1 = 1.25\times 100 = 125V$

$\therefore$ $Vy'z_1 = Vy'z_1 \times x = 125 \times 0.8 = 100$ v.

$\therefore$ $Vyz_1/Vwx_1 = \dfrac{100}{100}$.

2nd case:

$Vyz_2 = 100$ V

$\therefore$ $Vy't_2 = \dfrac{100}{\alpha} = \dfrac{100}{0.8} = 125v$

Now, $Vwx_2 = \dfrac{M_1}{M_2}Vy^1z_2 = \dfrac{1}{1.25}\times 125 = 100v$

$\therefore$ $Vwx_2/Vyz_2 = \dfrac{100}{100}$

32. Let the effective Q factor is q_1 then it can be written using inductance and resistance of equivalent circuit.

$$q = \frac{\omega Leq\cdot}{Req\cdot} = \frac{\omega(L_1+L_2)}{R_1+R_2}$$

Now we substitute the value of L_1 and L_2 in terms of q_1 and q_2

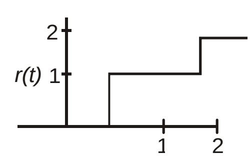
$$\therefore \quad q = \omega\left[\frac{q_1R_1}{\omega}+\frac{q_2R_2}{\omega}\right] = \left(q_1R_1+q_2R_2\right)/\left(R_1+R_2\right)$$

33. As $h(t) = \delta(t - 1) + \delta(t - 3)$

$r(t) = h(t) \oplus u(t)$

$= [\delta(t - 1) + \delta(t - 3)] \oplus u(t)$

$= u(t - 1) + u(t - 3)$



34. $$I_D = \frac{\mu}{2}\cos\frac{w}{L}(V_{as}-V_T)^2$$

as $V_B = V_{as}$

$$\Rightarrow \quad \frac{\partial I_D}{\partial V_{as}} = \frac{\partial ID}{\partial V_{as}} = \frac{\mu}{2}\cos\frac{w}{L}\times 2(V_{as}-V_T)$$

$$= \mu\cos\frac{W}{L}\left(V_{as}-V_T\right)$$

$$= 40\times 10^{-6}(2-1) = 40\times 10^{-6}$$

$$\Rightarrow \quad \frac{dV_B}{\partial I_D} = \frac{\partial V_{as}}{\partial I_D} = \frac{1}{40\times 10^{-6}} = 25K$$

35.
$$w_L = \frac{1}{\tau} = \frac{1}{CR_e q}$$
$$w_L = \frac{1}{1\times10^{-6}\times(10k+10k)}$$
$$2\pi f_L = \frac{1}{1\times10^{-6}\times 20\times10^3}$$
$$f_L = \frac{1}{2\pi\times1\times10^{-6}\times10^3} \cong 8$$

36. Since n(t) can be written in function of t using step function x(t) = 4 (t) = 4 (t - L)

we need to x (s) laplace transform x(t)

$$x(s) = \frac{1}{s} - \frac{1}{s}e^{-2s}$$

$\therefore$ $$L\left(\frac{d^2y(t)}{d(t)}\right) = s^2Y(s)$$

$\therefore$ $$L\left[\frac{d^2y(t)}{dt} + s\frac{dy}{dt} + 6y = x\right]$$

$\Rightarrow$ $s^2y(s) +\ ssy(s) + 6y(s) = x(s)$

$\therefore$ $$y(s) = \frac{x(s)}{s^2+5s+6} = \frac{1-e^{-2s}}{s(s+2)(s+3)}$$

37. Given:

$\Rightarrow$ A linear constant coefficient, first order differential equation of y(t)

$\Rightarrow$ forcing function - x(t)

$\Rightarrow$ initial condition : y(0)

$\Rightarrow$ y(t) > 0

Find out:

- 2y(t), by changing the value of x(t) and y(0)

Analysis:

Lets the differential equation is

$$\frac{dy}{dt} + py = x(t)$$

$\because$ y(t) = function of x(t) and y(0) = f(x(t), y(0))

Let find the relation between y(t) and x(t)

define integrating factor IF.

$$I.F. = e^{\int Pdt}$$

Then $$y(t) = \frac{1}{I.F.}\int x(t)\,I.F.\,dt + y(0)$$

I.F. is independent of n(t) and y(0)

So for find - 2y(t)

$$-2y(t) = -2\frac{1}{I.F.}\int n(t)\,I.F.\,dt + (-2)\,y(0)$$

$$-2y(t) = \frac{1}{I.F.}\int[-2n(t)]\,I.F.\,dt + [-2y(0)]$$

So for get - 2y(t), change the n(t) by - 2x(t), and y(0) with - 2y(0)

38. Given:

Two random variables (R.V.) U and V; Identically distributed.

Mean:

E (U) = 0 and E(V) = 0

Cumulative distribution function (CDF):

$F_U(x) = F(x)$

$F_{2V}(x) = G(x)$

To find: F(x) - G(x) and (F(x) - G(x)) x

Analysis:

Identically distributed R.V. U and V means their probability density function(pol) will be some. And CDF is the integration of polf. So their CDF will also be some

Hence $F_U(x) = F_V(x)$

Plan:

1. First find F(x) - G(x); Then check its positivity or negativity.
2. Secondly find (F(x) - G(x))x; Then check its positivity or negativity.

Carrying out plan:

We know that $F_U(x) = F(x) = Pr(U \le x)$ ...(1)

and $F_{2V} = G(x) = Pr(2V \le x)$ ...(2)

Now, consider $F_V(x) - F_{2V}(x)$ {From (1) and (2)}

$= F(x) - G(x)$ {From (1) and (2)}

$= Pr(U \le x) - Pr(2V \le x)$ {From (1) and (2)}

Now, $P(2V \le x) = P\left(V \le \frac{x}{2}\right)$ ...(3)

Use this result into above expression

$\Rightarrow$ $F(x) - G(x) = Pr(U \le x) - Pr\left(V \le \frac{x}{2}\right)$ ....(4)

We know that $Pr(U \le x) = F_U(x)$ ...(5)

and $Pr\left(V \le \frac{x}{2}\right) = F_V\left(\frac{x}{2}\right)$ ...(6)

From analysis section we have $F_U(x) = F_V(x)$ ...(7)

From (6) and (7) $F_V\left(\frac{x}{2}\right) = F_U\left(\frac{x}{2}\right)$ ...(8)

Using (5) and (6) into (4)

$F(x) - G(x) = F_u(x) - F_v\left(\frac{x}{2}\right)$ ....(9)

From (8) and (9), we have

$\Rightarrow$ $F(x) - G(x) = F_U(x) - F_U\left(\frac{x}{2}\right)$

Using (1) into above

$\Rightarrow$ $F(x) - G(x) = F(x) - F\left(\frac{x}{2}\right)$ ...(10)

Property of CDF: CDF is a non-decreasing function.

Case 1: x is +ve ($x \ge 0$)

$\Rightarrow$ $x \ge \frac{x}{2}$

$\Rightarrow$ $F(x) \ge F\left(\frac{x}{2}\right)$ {$\because$ Non-decreasing function}

$\Rightarrow$ $F(x) - F\left(\frac{x}{2}\right) \ge 0$

So from (10) $\boxed{F(x) - G(x) \ge 0}$ ...(11)

Case 2: x is - ve ($x \le 0$)

$\Rightarrow$ $x \le \frac{x}{2}$

$\Rightarrow$ $F(x) \le F\left(\frac{x}{2}\right) \Rightarrow F(x) - F\left(\frac{x}{2}\right) \le 0$

{$\because$ Non-Negative function}

So from (10) $\boxed{F(x) - G(x) \le 0}$ ...(12)

So from case 1 and case 2, result our answer depends on positivity and negativity of x. So for $x \geq 0$ our answer is option (B); But for $x \leq 0$ our answer is option (A). But not always (A) and (B) so we reject these two options.

Now we have to find some compact form for our answer which will be true for any value of x. i.e. for positive and negative x.

Case 3: $x \geq 0$

From (11) $F(x) - G(x) \geq 0$

Multiply by x, we get

$$\boxed{(F(x)-G(x))x \geq 0} \qquad ...(13)$$

Case 4: $x \leq 0$

From (12) $F(x) - G(x) \leq 0$

Multiply by x, we get

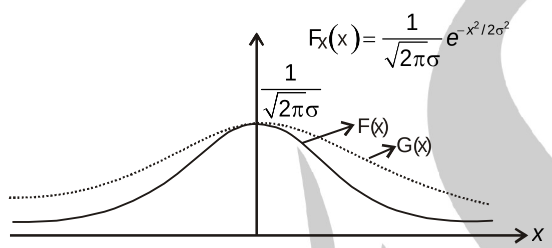
$$\boxed{F(x)-G(x)x \geq 0} \qquad ...(14)$$

So from (13) and (14) we have

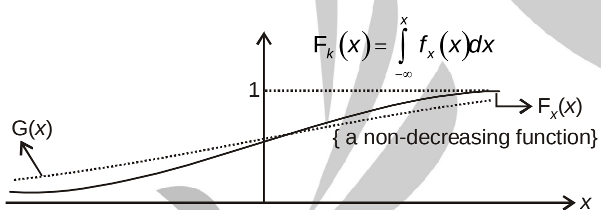
$$\boxed{(F(x)-G(x))x \geq 0 \quad \text{for all } x}$$

$\therefore$ Option (D) is correct answer.

Example: A Gaussion random variable with mean ZERO has following probability density function (pelf)



The CDF will be (approximately)



Conclusion:

The important point to remember in question is that "CDF function is a non-decreasing function."

39. Given:

DFT [a b c d] = [α β η δ] ...(1)

$$[p \quad q \quad r \quad s] = [a \quad b \quad c \quad d]\begin{bmatrix} a & b & c & d \\ d & a & b & c \\ c & d & a & b \\ b & c & d & a \end{bmatrix} \qquad ...(2)$$

To find:

DFT [p q r s]?

Analysis: We know that

DFT $[x_1(n) \otimes x_2(n)] = x_1(K)\, x_2(K)$ ...(3)

where $\otimes$ represent circular convolution

and $x_1(k) = \text{DFT}[x_1(n)]$ ...(4)

$x_2(k) = \text{DFT}[x_2(n)]$ ...(5)

we know that circular convolution can be found by "MATRIX METHOD"

MATRIX METHOD: Let two sequences

$x_1(n) = \{x_1(0), x,(1), ...x_1(N)\}^T$ {a colum vector}

$x_2(n) = \{x_2(0), x_2(1), ...x_2(N)\}^T$ {a colum vector}

Design a matrix say by using sequence $x_2(n)$

$$M = \begin{bmatrix} x_2(0) & x_2(N) & x_2(N-1) & .... & x_2(1) \\ x_2(1) & x_2(0) & x_2(N) & & . \\ . & x_2(1) & x_2(0) & & . \\ . & . & x_2(1) & & . \\ . & . & . & & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & & x_2(N-1) \\ x_2(N-1) & x_2(N-2) & x_2(N-3) & & x_2(N) \\ x_2(N) & x_2(N-1) & x_2(N-2) & .... & x_2(0) \end{bmatrix} \qquad ...(6)$$

Now circular convolution is given by

$x_1(n) * x_2(n) = M \times x_1(n)$ ...(7)

Plan:

1. Design matrix M by using vactor [a b c d]
2. Find circular convolution using equation 7

Carrying our plan:

Using vector [a b c d] design matrix M

$$M = \begin{bmatrix} a & d & c & b \\ b & a & d & c \\ c & b & a & d \\ d & c & b & a \end{bmatrix}$$

Now circular convolution of [a b c d] with it self.

$$[a \quad b \quad c \quad d]^T \otimes [a \quad b \quad c \quad d]^T = \begin{bmatrix} a & d & c & b \\ b & a & d & c \\ c & b & a & d \\ d & c & b & a \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Take transpose on both sides

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \otimes \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = [a \quad b \quad c \quad d]\begin{bmatrix} a & d & c & b \\ b & a & d & c \\ c & b & a & d \\ d & c & b & a \end{bmatrix} \qquad ...(8)$$

$\Rightarrow$ from 8 and 2, we get

$$[p \quad q \quad r \quad s] = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \otimes \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Take DFT both sides

$$\text{DFT } [p\ q\ r\ s] = \text{DFT}\left\{\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \oplus \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}\right\}$$

= DFT [a b c d] $\cdot$ DFT [a b c d] = [α β γ δ] $\cdot$ [α β γ δ]

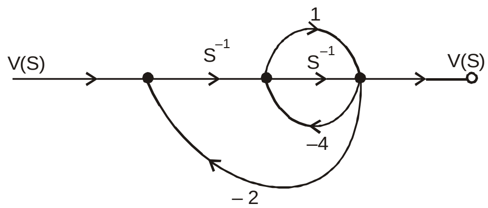
$$\boxed{\text{DFT}[p \quad q \quad r \quad s] = [\alpha^2 \quad \beta^2 \quad \gamma^2 \quad \delta^2]} \left\{\begin{array}{l}\text{Element wise} \\ \text{multiplication}\end{array}\right\} \qquad ...(9)$$

$\therefore$ Option (A) is the correct answer

Conclusion:

The important point to be noted is that sequences can be written as vectors. So vector and sequences were same thing.

40.



Forward path =

$P_1 = S^{-1}, S^{-1} = S^{-2}$ | $\Delta_1 = 1$

$P_2 = S^{-1}$ | $\Delta_2 = 1$

Loop =

$L_1 = -4S^{-1}$

$L_2 = -2S^{-1}S^{-1} = -2S^{-2}$

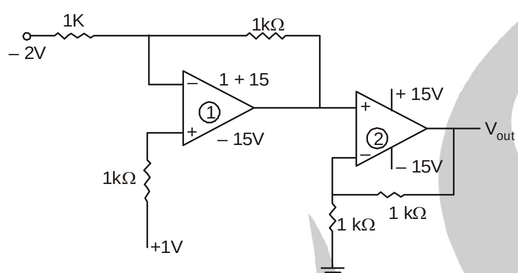
$L_3 = -2S^{-1}$

$L_4 = -4$

$$T(S) = \frac{S^{-2}+S^{-1}}{1+4+2S^{-1}+2S^{-2}+4S^{-1}}$$

$$= \frac{S^{-1}+S^{-2}}{S+6S^{-1}+2S^{-2}} = \frac{S+1}{5S^2+6S+2}$$

41.

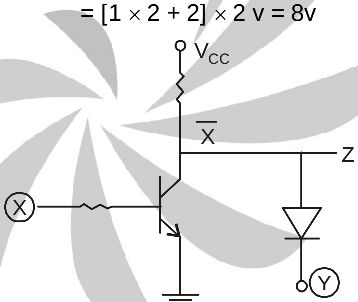


$$V\ out = \left[+1v\left(1+\frac{1}{1}\right) - 2v\left(\frac{-1}{1}\right)\right]\left[1+\frac{1}{1}\right]$$

⇑ Gain of non inverty amp (1) ⇑ Gain of inverty amp (1) ⇑ Gain of non inverty amp (2)

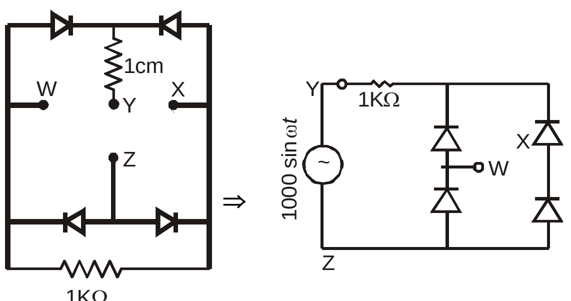
= [1 × 2 + 2] × 2 v = 8v

42.



$z = \bar{x}.y$

43. 'D' 0 for all +



Note:

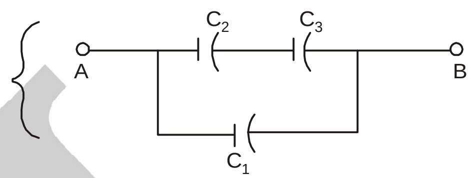
⇒ All diode conducts only done negative half.

⇒ XW is at symmetrical point so voltage across XW is zero for all time.

44. Given values:

	Capacitance Value	Voltage Breakdown
C_1	10μF	10V
C_2	5μF	5V
C_3	2μF	2V.

Given circuit:



To find:

Max. safe voltage in volts that can be applied across the combination and Corresponding total charge is μC.

Devising a plan:

(a) We will first calculate equivalent capacitance of given figure.

(b) By calculating capacitance, we can check which answer can be the solution, it reduce your effort in solving problem.

(c) Then apply the remaining answer for remaining options.

Solving:

Equivalent capacitance, as C_2 & C_3 are in series and we know that when capacitance are in series equivalent capacitance = $\frac{C_2C_3}{C_2+C_3}$

& when two capacitor are in parallel then their addition

$$= C_1 + \frac{C_2C_3}{C_2+C_3}$$

* $C = C_1 + \frac{C_2C_3}{C_2+C_3} = 10\mu F + \frac{5\mu F \times 2\mu F}{5\mu F + 2\mu F}. = 11.4285\mu F$

Now check the option by using formula

CV = θ

(a) 2.8 × 11.4285 = 32 μC ≠ 36 μC (option (a) is wrong)

hence 'c' option can be right

Now when 7V

(b) 7V × 11.4285 μF = 80μc ≠ 119μC { B is

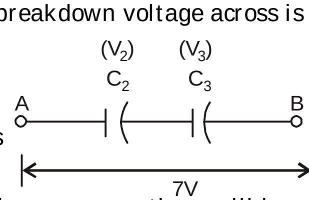
hence 'd' option is right.

only 'c' & 'd' are left.

⇒ Now apply 7V first in given circuit,

when voltage across AB is 7V then C_1 can handle it broz max breakdown voltage across is 10Y.

and across



as they are in series so charge across them will be same.

$Q_2 = Q_3$

$C_2V_2 = C_3V_3$

$C_2V_2 = C_3V_3 \quad [V_2 + V_3 = 7V]$

$C_2V_2 = C_3(7 - V_2)$

$C_2V_2 = 7C_3 - C_3V_2$

$$V_2 = \frac{7C_3}{C_2 + C_3}$$

$$V_2 = \frac{7 \times 2}{2+5} = 2V \qquad \left\{\boxed{V_2 = 2V}\right.$$

and $V_3 = 7 - V_2 = 7 - 2$

$$\boxed{V_3 = 5V}$$

As when we take '7V' then for that V_2 = 2V & V_3 = 5V, but max voltage across V_3 can be 2V [because above which it breakdown].

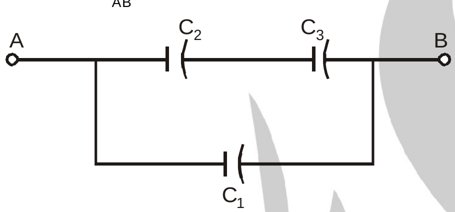
as $\boxed{V_3 \le 2V}$

But, when we take 7V then V_3 have to be 5V which is not possible hence 'd' is also wrong.

$\Rightarrow$ Only option left is 'c'

But, I will show that it is also right.

When $V_{AB} = 2.\delta\, V$



voltage across C_1 = 2.8 V [possible as breakdown voltage is 10 V].

$$V_2 = \frac{C_3 \times 2.8}{C_2 + C_3}$$

$\boxed{V_2 = \cdot 8V}$ [possible as breakdown voltage across C_2 is 5V)]

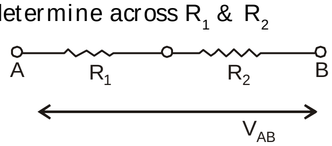
$$V_3 = \left(\frac{C_2}{C_2 + C_3}\right) \times 2.8$$

= 2 V [possible as breakdown voltage across C_3 is 2V]

hence option 'D' is correct

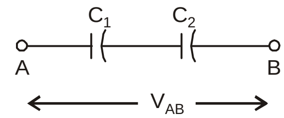
*Verification: As I already showed that only 's' is true no one else.

Conclusion: $\boxed{\text{voltage across method capacitance}}$ is not same as $\boxed{\text{voltage across method resistance.}}$ For determine voltage across resistance we just do as we want to determine across R_1 & R_2



$$\boxed{V_{(R_1)} = \frac{R_1}{R_1 + R_2} V_{AB}}$$

But in capacitor



$$\boxed{V_{(C_1)} = \frac{C_1}{C_1 + C_2} V_{AB}}$$

So, don't apply resistance voltage method in to capacitor one, if you do that then you will obtain 'D' as answer, but which is wrong.

45. s1 s0

A_{15}	A_{14}	A_{13}	A_{12}	A_{11}	A_{10}	A_9	A_8	A_7	A_6	A_5	A_4	A_3	A_2	A_1	A_0	
0	0	0	0	1	0	×	×	×	×	×	×	×	×	×	×	RAM 1
0	0	0	1	1	0	×	×	×	×	×	×	×	×	×	×	RAM 2
0	0	1	0	1	0	×	×	×	×	×	×	×	×	×	×	RAM 3
0	0	1	1	1	0	×	×	×	×	×	×	×	×	×	×	RAM 4

RAM1 - 0800H- 0BFF H

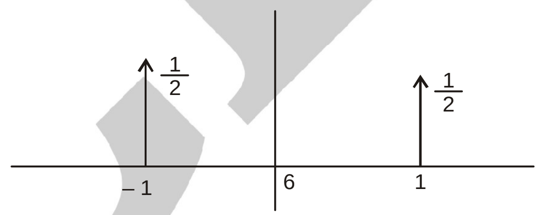
RAM2 - 1800H- 1BFF H

RAM3 - 2800H - 23FF H

RAM4 - 3800H- 3BFF H

Let's plan to solve problem

47. U X V are two independent and identically distribution random variable



so $p_u(u) = \frac{1}{2}\delta(u+1) + \frac{1}{2}\delta(u-1)$

similarly $p_v(v) = \frac{1}{2}\delta(v+1) + \frac{1}{2}\delta(v-1)$

$Z = (U + V)$

$P_z(z) = P_u(u) \oplus P_v(u)$

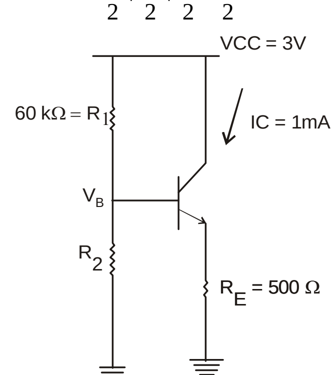
$= \frac{1}{4}\delta(z+2) + \frac{1}{2}\delta(z) + \frac{1}{4}\delta(z-2)$

So entropy H(z) = H (U + V)

$= P_1 \log + P_2 \log \frac{1}{P_2} + P_3 4 \frac{1}{P_3}$

$= \frac{1}{4}\log_2 4 + \frac{1}{2}\log 2 + \frac{1}{4}\log_2 4$

$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$



As $I_C = I_E$

since $B \to \infty$

$I_B = 0$

$V_B = V_{BE} + R_E I_E = V_{BE} + R_E I_C$

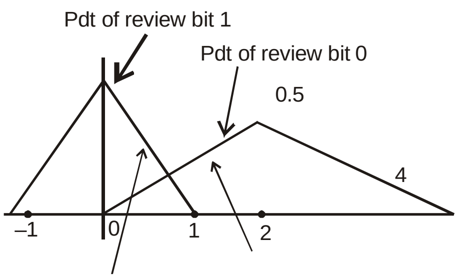
$V_B = .7 + 1 \times 10^{-3} \times 500 = 1.2$

$V_B = 1.2V = \frac{R_2}{R_1 + R_2} = \frac{R_2}{60k + R_2} \times 3V$

$1.2\,(60 + R_2) = 3R_2$

$R_2 = \frac{1.2 \times 60}{1.8} = 40k$

48.



$y = -x + 1$ ...(i)

$y = \frac{.5}{2} \times x$

$y = \frac{x}{4}$ ...(ii)

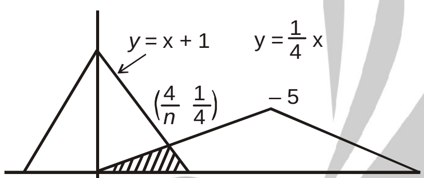
Solving (i) and (ii)

$-x + 1 = \frac{x}{4} \Rightarrow 1 = \frac{x}{4} + 1 = \frac{5x}{4} \Rightarrow x = \frac{4}{5}$

$\therefore \quad y = \frac{1}{5}$

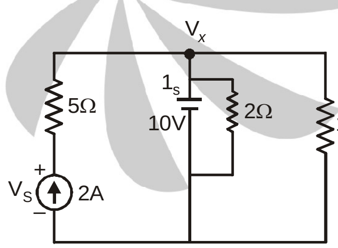
So optimum ratio to achieve minimum bit rate (BER) $\frac{4}{5}$

49.



Bit error rate = $\frac{1}{2} \times 1 \times \frac{1}{4} = \frac{1}{8}$

50- 51



$\Rightarrow$ Applying KCK at mode (V_x)

$2 - I_s - \frac{10}{2} - \frac{10}{1} = 0$

$I_s = -5 - 10 + 2 = -13$

$\Rightarrow$ Current through $1\Omega = \frac{10}{1} = 10$ A

$\Rightarrow$ $V_s - 5 \times 2 - 10V = 0$

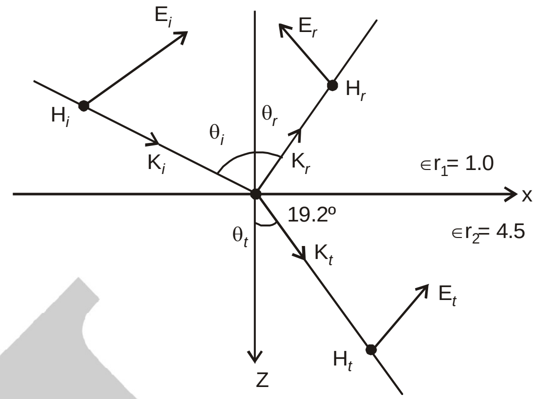
$V_s = 20V$

50. (C) Current through 1Ω = 10A]

51. (D) [I_s = - 13A, V_s = 20]

52. Given:

Electric field vectors of a monochromatic plane wave.



$\lambda = 600\ \mu m$

$K_0 = \frac{2\pi}{\lambda} = \frac{\pi}{3} \times 10^4$ rad/mt

Topic:

Plane wave propagation (Electro magnetics)

Formula:

Snell's law, $\sqrt{Er_1} \sin\theta_i = -\sqrt{Er_2} \sin\theta_t$

Solution:

To find the angle of incidence use snell's law.

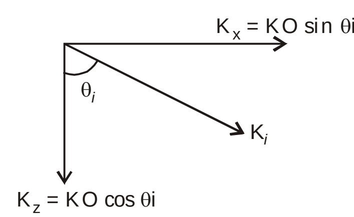
$\sqrt{Er_1} \sin\theta_i = \sqrt{Er_2} \sin\theta_t$

$\Rightarrow \quad \sqrt{1.0} \sin\theta_i = \sqrt{4.5} \sin 19.2$

$\Rightarrow \quad \theta i = 44.3 \simeq 45°$, is the desired incidence angle.

To find the incidence Electric filed vector, we need to find the propagation constant first.

We can resolve K_i into two components as shown in the figure.



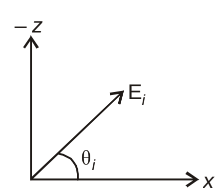
Kx = KO sin θi ; K_w = KO cos Qi

$= \frac{KO}{\sqrt{2}} \qquad = \frac{KO}{\sqrt{2}}$

Now the phase term can be given by

$e^{-j(\beta_x x + \beta_z z)} = e^{-j\left(\frac{2\pi}{\lambda}\cos\theta_i x + \frac{2\pi}{\lambda}\sin\theta_i z\right)} = e^{-j\frac{\pi \times 10^4}{3\sqrt{2}}(x+z)}$ ...(1)

The components consituting the amplitude part of Ei are



$$Eio = Exa\hat{x} - Eza\hat{z} = E_0\cos\theta_i\ a\hat{x} - E_0\sin\theta_i a\hat{z}$$

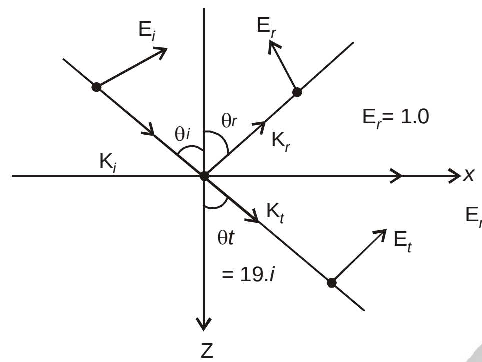
$$= \frac{E_0}{\sqrt{2}}\left(\hat{a}_x - \hat{a}_z\right) \text{ v/m} \qquad ...(2)$$

Therefore, the expression for $\bar{E}_i$ can be obtained from (1) and (2) equations

$$Ei = \frac{E_0}{\sqrt{2}}\left(\hat{a}_x - \hat{a}_z\right)e^{-j\frac{\pi\times10^4}{3\sqrt{2}}(x+z)} \text{ v/mt}$$

53. Given:

Electric field vectors of a monochromatic plane wave.



$\lambda = 600\ \mu m$

$$K_0 = \frac{2\pi}{\lambda} = \frac{\pi}{3}\times10^4 \text{ rad/m}$$

Topic:

Plane wave propagation (Electromagnetics)

Formula:

For an oblique incidence the reflection coefficient is

$$T = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = \frac{\sqrt{\frac{\mu_2}{\epsilon_2}}\cos\theta_t - \sqrt{\frac{\mu_1}{\epsilon_1}}\cos\theta_i}{\sqrt{\frac{\mu_2}{\epsilon_2}}\cos\theta_t - \sqrt{\frac{\mu_1}{\epsilon_1}}\cos\theta_i}$$

Here $\mu_1 = \mu_2 = \mu_0 = 4\pi\times10^{-7}$ H/mt

$\epsilon_1 = \epsilon_0;\ \epsilon_2 = 4.5\ \epsilon_0$

Solution:

First we find the angle of incidnece θi from snell's law as follows

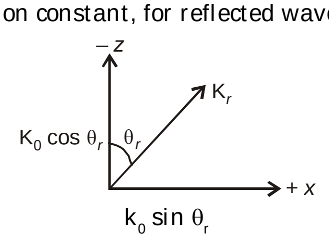
$$\sqrt{\epsilon r_1}\sin\theta_i = \sqrt{\epsilon r_2}\sin\theta_t$$

On substituting the values we get $\theta_i \simeq 45°$

The reflection coefficient is,

$$T = \frac{\frac{1}{\sqrt{4.5}}\cos 19.2 - \frac{1}{\sqrt{1}}\cos 45^o}{\frac{1}{\sqrt{4.5}}\cos 19.2 + \frac{1}{\sqrt{1}}\cos 45^o} = -0.23$$

To find the reflected field vector $\bar{E}_r$, we need to find the propagation constant, for reflected wave in medium.

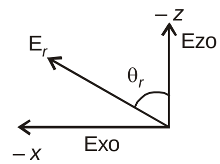


$$\boxed{\theta_i + \theta_r = 90^o}$$

The phase term will be

$$e^{-j(\beta_x x + \beta_z(-z))} = e^{-j\left(\frac{2\pi}{\lambda}\cos\theta_r x - \frac{2\pi}{\lambda}\sin\theta_r z\right)} = e^{-j\frac{\pi\times10^4}{3\sqrt{2}}(x-z)} \qquad ...(1)$$

The amplitude term can be obtained by resolving $\bar{E}_r$ into two components



$$E_{r0} = \left(-Ex_0 a\hat{x} - Ez_0 a\hat{z}\right)\times \text{reflection coefficient}$$

$$= \left(-E_0\sin\theta_r a\hat{x} - E_0\cos\theta_r a\hat{z}\right)\times T$$

$$= \frac{0.23E_0}{\sqrt{2}}\left(\hat{a}_x + \hat{a}_z\right) \text{ v/m} \qquad ...(2)$$

Thus the expression for $\bar{E}_r$ can be obtained form (1) and (2) as,

$$\bar{E}_r = 0.23\frac{E_0}{\sqrt{2}}\left(\hat{a}_x + \hat{a}_z\right)e^{-j\frac{\pi\times10^4}{3\sqrt{2}}(x-z)} \text{ V/m}$$

55. Given:

Using data of previous question

State transtition matrix:

$$A = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$$

Find out:

e^{At}, which is given as

$L^{-1}[[SI - A]^{-1}]$

Solution

$$SI - A = S\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} S+1 & 0 \\ -1 & S+1 \end{bmatrix}$$

$$[SI - A]^{-1} = \frac{1}{(S+1)^2}\begin{bmatrix} (S+1) & 0 \\ +1 & (S+1) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{S+1} & 0 \\ \frac{1}{(S+1)^2} & \frac{1}{S+1} \end{bmatrix}$$

taking Laplace Inverse

$$L^{-1}[SI - A]^{-1} = L^{-1}\begin{bmatrix} \frac{1}{S+1} & 0 \\ \frac{1}{(S+1)^2} & \frac{1}{S+1} \end{bmatrix}$$

$$= \begin{bmatrix} L^{-1}\left(\frac{1}{S+1}\right) & 0 \\ L^{-1}\left(\frac{1}{(S+1)^2}\right) & L^{-1}\left(\frac{1}{S}+1\right) \end{bmatrix}$$

$$= \begin{bmatrix} e^{-t} & 0 \\ te^{-t} & e^{-t} \end{bmatrix}$$

■■