

# GATE - 2013

## IN : INSTRUMENTATION ENGINEERING

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Duration : Three Hours

Maximum Marks : 100

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Read the following instructions carefully.

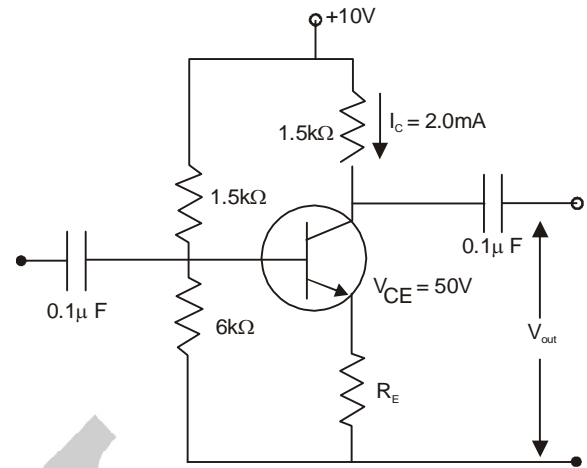
1. All questions in this paper are of objective type.
2. There are a total of 65 questions carrying 100 marks.
3. Questions 1 through 25 are 1-mark questions, question 26 through 55 are 2-mark questions.
4. Questions 48 to 51 (2 pairs) common data questions and question pairs (Q. 52 and Q.53) and (Q. 54 and Q.55) are linked answer questions. The answer to the second question of the above pair depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is un-attempted, then the answer to the second question in the pair will not be evaluated.
5. Questions 56 - 65 belong to general aptitude (GA). Questions 56 - 60 will carry 1-mark each, and questions 61-65 will carry 2-marks each. The GA questions will begin on a fresh page.
6. Un-attempted questions will carry zero marks.
7. Wrong answers will carry NEGATIVE marks. For Q.1 to Q.25 and Q.56 - Q.60, 1/3 mark will be deducted for each wrong answer. For Q. 26 to Q. 51, and Q.61 - Q.65, 2/3 mark will be deducted for each wrong answer. The question pairs (Q. 52, Q. 53) and (Q. 54, Q. 55) are questions with linked answers. There will be negative marks only for wrong answer to the first question of the linked answer question pair i.e. for Q. 52 and Q.54, 2/3 mark will be deducted for each wrong answer. There is no negative marking for Q. 53 and Q.55..

**Q 1 to Q 25 carry one mark each**

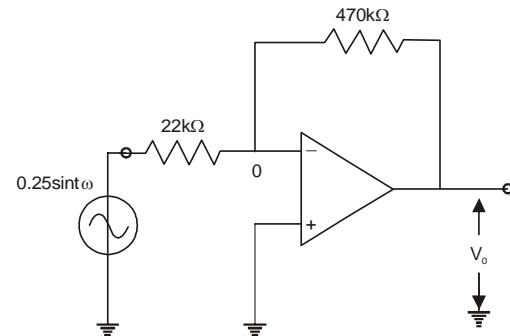
1. The dimension of the null space of the matrix

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix} \text{ is}$$

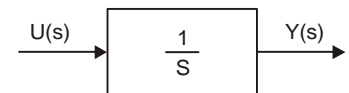
- (a) 0 (b) 1  
(c) 2 (d) 3
2. If the A-matrix of the state space model of a SISO linear time Invariant system is rank deficient, the transfer function of the system must have
- (a) a pole with a positive real part  
(b) a pole with a negative real part  
(c) a pole with a positive imaginary part  
(d) a pole at the origin
3. Two systems with impulse responses  $h_1(t)$  and  $h_2(t)$  are connected in cascade. Then the overall impulse response of the cascaded system is given by
- (a) product of  $h_1(t)$  and  $h_2(t)$   
(b) sum of  $h_1(t)$  and  $h_2(t)$   
(c) convolution of  $h_1(t)$  and  $h_2(t)$   
(d) subtraction of  $h_2(t)$  from  $h_1(t)$
4. The complex function  $\tanh(s)$  is analytic over a region of the imaginary axis of the complex s-plane if the following is TRUE everywhere in the region for all integers n
- (a)  $\operatorname{Re}(s) = 0$   
(b)  $\operatorname{Im}(s) \neq n\pi$   
(c)  $\operatorname{Im}(s) \neq \frac{n\pi}{3}$   
(d)  $\operatorname{Im}(s) \neq \frac{(2n+1)\pi}{2}$
5. For a vector  $E$ , which one of the following statements is NOT TRUE?
- (a) If  $\nabla E = 0$ ,  $E$  is called solenoidal  
(b) If  $\nabla \times E = 0$ ,  $E$  is called conservative  
(c) If  $\nabla \times E = 0$ ,  $E$  is called irrotational  
(d) If  $\nabla E = 0$ ,  $E$  is called irrotational
6. For a periodic signal  $v(t) = 30\sin 100t + 10\cos 300t + 6\sin(500t + \pi/4)$ , the fundamental frequency in rad/s is
- (a) 100 (b) 300  
(c) 500 (d) 1500
7. In the transistor circuit as shown below, the value of resistance  $R_E$  in  $k\Omega$  is approximately,
- (a) 1.0 (b) 1.5  
(c) 2.0 (d) 2.5



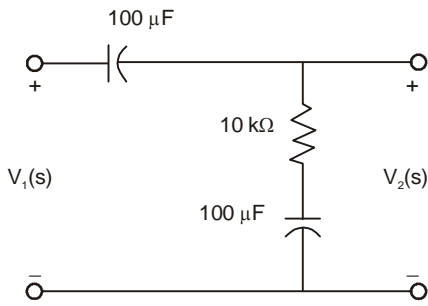
8. A source  $v_s(t) = V \cos 100\pi t$  has an internal impedance of  $4 + j3 \Omega$ . If a purely resistive load connected to this source has to extract the maximum power out of the source, its value in  $\Omega$  should be
- (a) 3 (b) 4  
(c) 5 (d) 7
9. Which one of the following statements is NOT TRUE for a continuous time causal and stable LTI system?
- (a) All the poles of the system must lie on the left side of the  $j\omega$  axis  
(b) Zeros of the system can lie anywhere in the s-plane  
(c) All the poles must lie within  $|s| = 1$   
(d) All the roots of the characteristic equation must be located on the left side of the  $j\omega$  axis
10. The operational amplifier shown in the circuit below has a slew rate of  $0.8 \text{ Volts}/\mu\text{s}$ . The input signal is  $0.25 \sin(\omega t)$ . The maximum frequency of input in kHz for which there is no distortion in the output is



- (a) 23.84 (b) 25.0  
(c) 50.0 (d) 46.60
11. Assuming zero initial condition, the response  $y(t)$  of the system given below to a unit step input  $u(t)$  is
- (a)  $u(t)$   
(b)  $t u(t)$   
(c)  $\frac{t^2}{2} u(t)$   
(d)  $e^{-1} u(t)$



12. The transfer function  $\frac{V_2(s)}{V_1(s)}$  of the circuit shown below is



- (a)  $\frac{0.5s+1}{s+1}$       (b)  $\frac{3s+6}{s+2}$   
 (c)  $\frac{s+2}{s+1}$       (d)  $\frac{s+1}{s+2}$

13. The type of the partial differential equation

$$\frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial x^2}$$

- (a) Parabolic      (b) Elliptic  
 (c) Hyperbolic      (d) Nonlinear

14. The discrete-time transfer function  $\frac{1-2z^{-1}}{1-0.5z^{-1}}$  is

- (a) non-minimum phase and unstable  
 (b) minimum phase and unstable  
 (c) minimum phase and stable  
 (d) non-minimum phase and stable

15. Match the following biomedical instrumentation techniques with their applications

- |   |                          |   |                                |
|---|--------------------------|---|--------------------------------|
| P | Otoscopy                 | U | Respiratory volume measurement |
| Q | Ultrasound Technique     | V | Ear diagnostics                |
| R | Spirometry               | W | Echo-cardiography              |
| S | Thermodilution Technique | X | heart volume measurement       |

- (a) P-U, Q-V, R-X, S-W  
 (b) P-V, Q-U, R-X, S-W  
 (c) P-V, Q-W, R-U, S-X  
 (d) P-V, Q-W, R-X, S-U

16. A continuous random variable X has a probability density function  $f(x) = e^{-x}$ ,  $0 < x < \infty$ . Then  $P\{X > 1\}$  is

- (a) 0.368      (b) 0.5  
 (c) 0.632      (d) 1.0

17. A band-limited signal with a maximum frequency of 5 kHz is to be sampled. According to the sampling theorem, the sampling frequency in kHz which is not valid is

- (a) 5      (b) 12  
 (c) 15      (d) 20

18. The differential pressure transmitter of a flow meter using a venture tube reads  $2.5 \times 10^5$  Pa for a flow rate of  $0.5 \text{ m}^3/\text{s}$ . The approximate flow rate in  $\text{m}^3/\text{s}$  for a differential pressure  $0.9 \times 10^5$  Pa is

- (a) 0.30      (b) 0.18  
 (c) 0.83      (d) 0.60

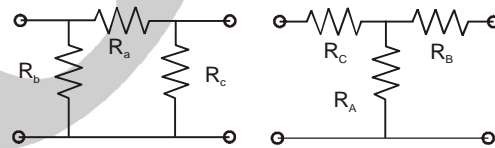
19. A bulb in a staircase has two switches one switch being at the ground floor and the other one at the first floor. The bulb can be turned ON and also can be turned OFF by any one of the switches irrespective of the state of the other switch. The logic of switching of the bulb resembles

- (a) an AND gate      (b) an OR gate  
 (c) an XOR gate      (d) a Nand gate

20. The impulse response of a system is  $h(t) = t u(t)$ . For an input  $u(t-1)$ , the output is

- (a)  $\frac{t^2}{2} u(t)$       (b)  $\frac{t(t-1)}{2} u(t-1)$   
 (c)  $\frac{(t-1)^2}{2} u(t-1)$       (d)  $\frac{t^2-1}{2} u(t-1)$

21. Consider a delta connection of resistors and its equivalent star connection as shown below. If all elements of the delta connection are scaled by a factor k,  $k > 0$ , the elements of the corresponding star equivalent will be scaled by a factor of

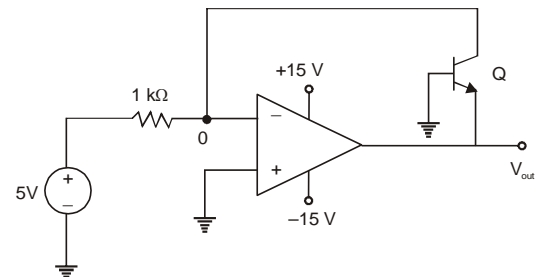


- (a)  $k^2$       (b) k  
 (c)  $1/k$       (d)  $\sqrt{k}$

22. An accelerometer has input range of 0 to 10g. natural frequency 30 Hz and mass 0.001 kg. The range of the secondary displacement transducer in mm required to cover the input range is

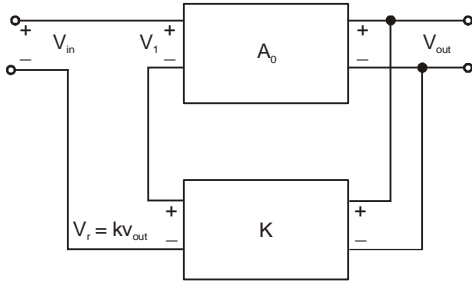
- (a) 0 to 2.76      (b) 0 to 9.81  
 (c) 0 to 11.20      (d) 0 to 52.10

23. In the circuit shown below what is the output voltage ( $V_{out}$ ) in Volts if a silicon transistor Q and an ideal op-amp are used?



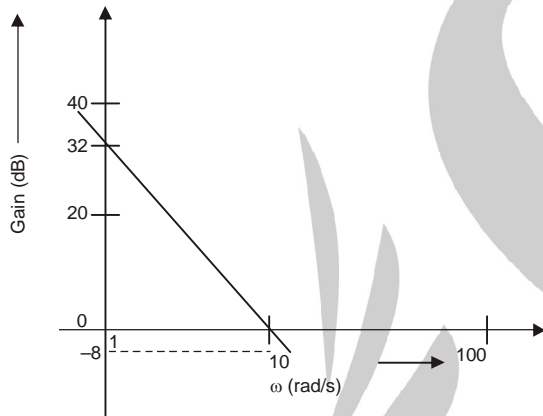
- (a) -15      (b) -0.7  
 (c) +0.7      (d) +15

24. In the feedback network shown below, if the feedback factor  $k$  is increased, then the



- (a) input impedance increases and output impedance decreases
- (b) input impedance increases and output impedance also increases
- (c) input impedance decreases and output impedance also decreases
- (d) input impedance decreases and output impedance increases

25. The Bode plot of a transfer function  $G(s)$  is shown in the figure below.



The gain  $(20 \log |G(s)|)$  is 32 dB and -8 dB at 1 rad/s and 10 rad/s respectively. The phase is negative for all  $\omega$ . Then  $G(s)$  is

- (a)  $\frac{39.8}{s}$
- (b)  $\frac{39.8}{s^2}$
- (c)  $\frac{32}{s}$
- (d)  $\frac{32}{s^2}$

Q.26 to 55 carry two marks each

26. While numerically solving the differential equation

$$\frac{dy}{dx} + 2xy^2 = 0, y(0) = 1$$

using Euler's predictor corrector (improved Euler-Cauchy) method with a step size of 0.2, the value of  $y$  after the first step is

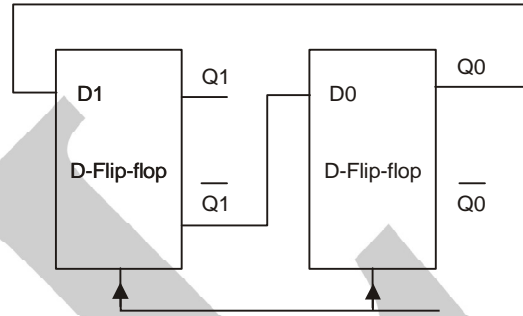
- (a) 1.00
- (b) 1.03
- (c) 0.97
- (d) 0.96

27. One part of eigenvectors corresponding to the two

eigenvalues of the matrix  $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$  is

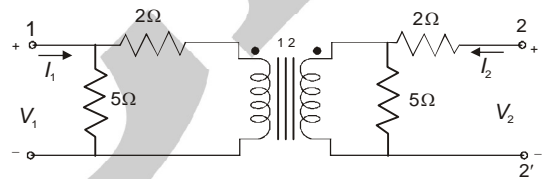
- (a)  $\begin{bmatrix} 1 \\ -J \end{bmatrix}$
- (b)  $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
- (c)  $\begin{bmatrix} 1 \\ J \end{bmatrix}$
- (d)  $\begin{bmatrix} 1 \\ J \end{bmatrix}$

28. The digital circuit shown below uses two negative edge-triggered D-flip-flops. Assuming initial condition of Q1 and Q0 as zero, the output Q1Q0 of this circuit is



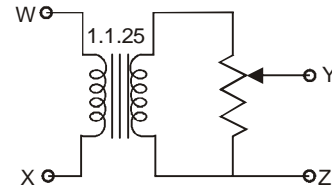
- (a) 00,01, 10, 11,00
- (b) 00,01,11,10,00
- (c) 00,11,10,01,00
- (d) 00,01,11,11,00

29. Considering the transformer to be ideal, the transmission parameter 'A' of the 2-port network



- (a) 1.3
- (b) 1.4
- (c) 0.5
- (d) 2.0

30. The following arrangement consists of an ideal transformer and an attenuator which attenuates by a factor of 0.8. An ac voltage  $V_{wx1} = 100V$  is applied across WX to get an open circuit voltage  $V_{yz1}$  across YZ. Next, an ac voltage  $V_{yz2} = 100V$  is applied across YZ to get an open circuit voltage  $V_{wx2}$  across WX. Then,  $V_{yz1} / V_{wx1} / V_{wx2} / V_{yz2}$  are respectively.

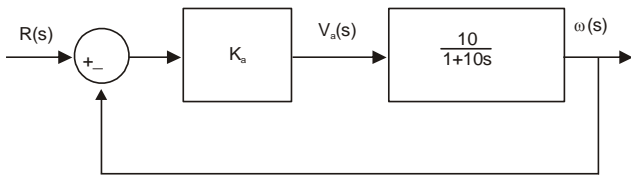


- (a) 125/100 and 80/100
- (b) 100/100 and 80/100
- (c) 100/100 and 100/100
- (d) 80/100 and 80/100

31. The open-loop transfer function of a dc motor is given

as  $\frac{\omega(s)}{V_a(s)} = \frac{10}{1+10s}$ . When connected in feedback as

shown below, the approximate value of  $K_a$  that will reduce the time constant of the closed loop system by one hundred times as compared to that of the open-loop system is

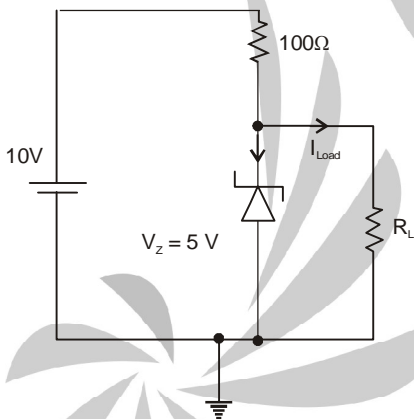


- (a) 1
- (b) 5
- (c) 10
- (d) 100

32. Two magnetically uncoupled inductive coils have Q factors  $q_1$  and  $q_2$  at the chosen operating frequency. Their respective resistances are  $R_1$  and  $R_2$ . When connected in series, the effective Q factor of the series combination at the same operating frequency is

- (a)  $q_1 + q_2$
- (b)  $(1/q_1) + (1/q_2)$
- (c)  $(q_1 R_1 + q_2 R_2)/(R_1 + R_2)$
- (d)  $(q_1 R_2 + q_2 R_1)/(R_1 + R_2)$

33. For the circuit shown below, the knee current of the ideal Zener diode is 10 mA. To maintain 5 V across  $R_L$ , the minimum value of the load resistor  $R_L$  in  $\Omega$  and the minimum power rating of the Zener diode in mW respectively are



- (a) 125 and 125
- (b) 125 and 250
- (c) 250 and 125
- (d) 250 and 250

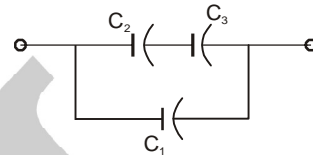
34. The impulse response of a continuous time system is given by  $h(t) = \delta(t - 1) + \delta(t - 3)$ . The value of the step response at  $t = 2$  is

- (a) 0
- (b) 1
- (c) 2
- (d) 3

35. Signals from fifteen thermocouples are multiplexed and each one is sampled once per second with a 16-bit ADC the digital samples are converted by a parallel to serial converter to generate a serial PCM signal. This PCM signal is frequency modulated with FSK modulator with 1200 Hz as 1 and 960 Hz as 0. The minimum band allocation required for faithful re-production of the signal by the FSK receiver without considering noise is

- (a) 840 Hz to 1320 Hz
- (b) 960 Hz to 1200Hz
- (c) 1080 Hz to 1320 Hz
- (d) 720 Hz to 1440 Hz

36. Three capacitors  $C_1, C_2$  and  $C_3$ , whose values are  $10\mu F, 5\mu F$ , and  $2\mu F$  respectively, have breakdown voltages of 10V, 5V, and 2V respectively. For the interconnection shown below the maximum safe voltage in Volts that can be applied across the combination and the corresponding total charge in  $\mu C$  stored in the effective capacitance across the terminals are respectively

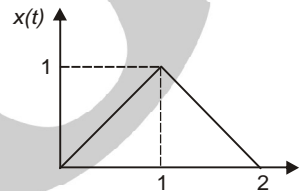


- (a) 2.8 and 36
- (b) 7 and 119
- (c) 2.8 and 32
- (d) 7 and 80

37. The maximum value of the solution  $y(t)$  of the differential equation  $y(t) + y(t) = 0$  with initial conditions  $y(0) = 1$  and  $y'(0) = 1$  for  $t \geq 0$  is

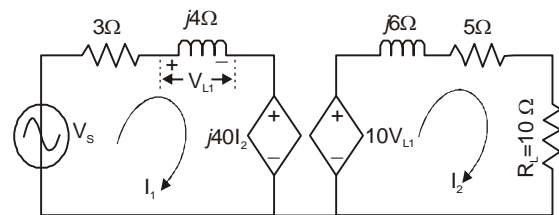
- (a) 1
- (b) 2
- (c)  $\pi$
- (d)  $\sqrt{2}$

38. The Laplace Transform representation of the triangular pulse shown below is



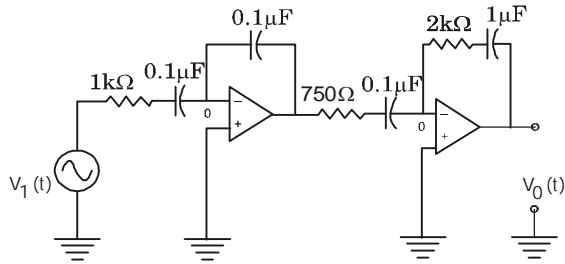
- (a)  $\frac{1}{s^2} [1 + e^{-2s}]$
- (b)  $\frac{1}{s^2} [1 - e^{-s} + e^{-2s}]$
- (c)  $\frac{1}{s^2} [1 - e^{-s} + 2e^{-2s}]$
- (d)  $\frac{1}{s^2} [1 - 2e^{-s} + e^{-2s}]$

39. In the circuit shown below, if the source voltage  $V_s = 100 \angle 53.13^\circ$  volts then the Thevenin's equivalent voltage in volts as seen by the load resistance  $R_L$  is



- (a)  $100 \angle 90^\circ$
- (b)  $800 \angle 0^\circ$
- (c)  $800 \angle 90^\circ$
- (d)  $100 \angle 60^\circ$

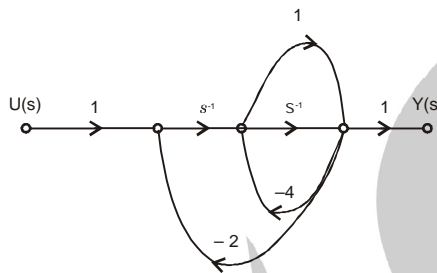
40. A signal  $V_1(t) = 10 + 10\sin 100\pi t + 10 \sin 4000 \pi t + 10 \sin 10000 \pi t$  is supplied to a filter circuit (shown below) made up of ideal op-amps. The least attenuated frequency component in the output will be



- (a) 0 Hz  
(b) 50 Hz  
(c) 2 k Hz  
(d) 50 kHz

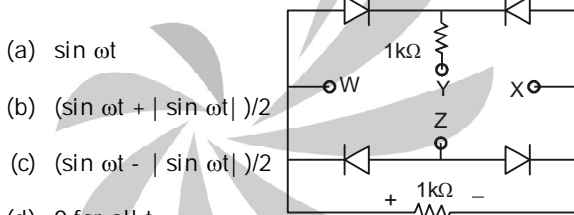
41. The signal flow graph for a system is given below. The

transfer function  $\frac{Y(s)}{U(s)}$  for this system is



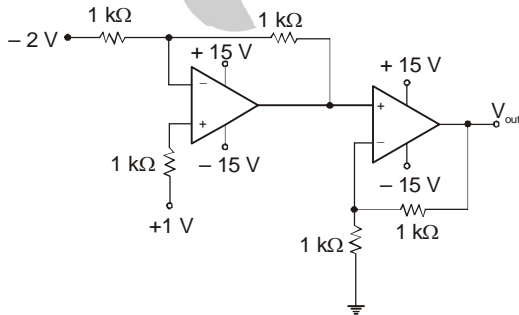
- (a)  $\frac{s+1}{5s^2+6s+2}$   
(b)  $\frac{s+1}{s^2+6s+2}$   
(c)  $\frac{s+1}{s^2+4s+2}$   
(d)  $\frac{1}{5s^2+6s+2}$

42. A voltage  $1000 \sin \omega t$  Volts is applied across YZ. Assuming ideal diodes, the voltage measured across WX in Volts, is



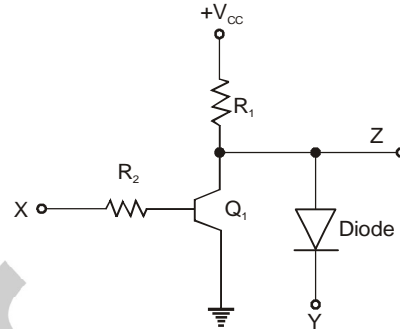
- (a)  $\sin \omega t$   
(b)  $(\sin \omega t + |\sin \omega t|)/2$   
(c)  $(\sin \omega t - |\sin \omega t|)/2$   
(d) 0 for all t

43. In the circuits shown below the op-amps are ideal. Then  $V_{out}$  in Volts is



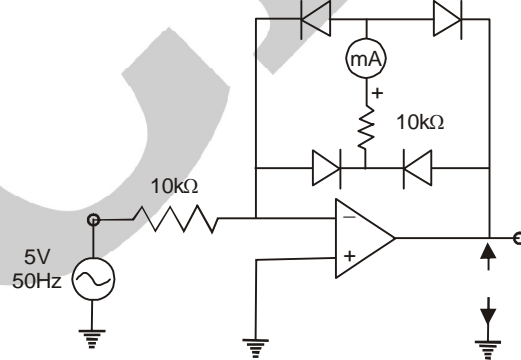
- (a) 4  
(b) 6  
(c) 8  
(d) 10

44. In the circuit shown below,  $Q_1$  has negligible collector-to-emitter saturation voltage and the diode drops negligible voltage across it under forward bias. If  $V_{cc}$  is +5 V, X and Y are digital signals with 0 V as logic 0 and  $V_{cc}$  as logic 1, then the Boolean expression for Z is



- (a)  $X Y$   
(b)  $\bar{X} Y$   
(c)  $X \bar{Y}$   
(d)  $\bar{X} \bar{Y}$

45. The circuit below incorporates a permanent magnet moving coil milli-ammeter of range 1 mA having a series resistance of 10 kΩ. Assuming constant diode forward resistance of 50 Ω, a forward diode drop of 0.7 V and infinite reverse diode resistance for each diode, the reading of the meter in mA is



- (a) 0.45  
(b) 0.5  
(c) 0.7  
(d) 0.9

46. Measurement of optical absorption of a solution is disturbed by the additional stray light falling at the photo-detector. For estimation of the error caused by stray light the following data could be obtained from controlled experiments.

Photo-detector output without solution and without stray light is 500 μW  
Photo-detector output without solution and with stray light is 600 μW  
Photo-detector output with solution and with stray light is 200 μW  
The percent error in computing absorption coefficient due to stray light is

- (a) 12.50  
(b) 31.66  
(c) 33.33  
(d) 94.98

47. Two ammeters  $A_1$  and  $A_2$  measure the same current and provide reading  $I_1$  and  $I_2$  respectively. The ammeter errors can be characterized as independent zero mean Gaussian random variables of standard deviations  $\sigma_1$  and  $\sigma_2$  respectively. The value of the current is computed as

$$I = \mu I_1 + (1 - \mu) I_2$$

The value of  $\mu$  which gives the lowest standard deviation of  $I$  is

- (a)  $\frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$       (b)  $\frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}$   
 (c)  $\frac{\sigma_2}{\sigma_1 + \sigma_2}$       (d)  $\frac{\sigma_1}{\sigma_1 + \sigma_2}$

**COMMON DATA QUESTIONS**

Common Data for Questions 48 and 49

A tungsten wire used in a constant current hot wire anemometer has the following parameters

Resistance at  $0^\circ\text{C}$  is  $10\ \Omega$ , Surface area is  $10^{-4}\text{m}^2$ , Linear temperature coefficient of resistance of the tungsten wire is  $4.8 \times 10^{-3}/^\circ\text{C}$ , Convective heat transfer coefficient is  $25.2\text{ W/m}^2/^\circ\text{C}$ , flowing air temperature is  $30^\circ\text{C}$ , wire current is  $100\text{ mA}$ , mass-specific heat product is  $2.5 \times 10^{-5}\text{ J}/^\circ\text{C}$ .

48. The thermal time constant of the hot wire under flowing air condition in ms is  
 (a) 24.5      (b) 12.25  
 (c) 6.125      (d) 3.0625
49. At steady state, the resistance of the wire in  $\Omega$  is  
 (a) 10.000      (b) 10.144  
 (c) 12.152      (d) 14.128

Common Data for Questions 50 and 51

A piezo-electric force sensor, connected by a cable to a voltage amplifier, has the following parameters.

Crystal properties Stiffness  $10^9\text{ N/m}$ , Damping ratio 0.01, Natural frequency  $10^5\text{ rad/s}$ , Force-to-Charge sensitivity  $10^{-9}\text{ C/N}$ , Capacitance  $10^{-9}\text{ F}$  with its loss angle assumed negligible.

Cable properties Capacitance  $2 \times 10^{-9}\text{ F}$  with its resistance assumed negligible

Amplifier properties Input impedance  $1\text{ M}\Omega$ , Bandwidth  $1\text{ MHz}$ , Gain 3

50. The maximum frequency of a force signal in Hz below the natural frequency within its useful midband range of measurement, for which the gain amplitude is less than 1.05, approximately is,  
 (a) 35      (b) 350  
 (c) 3500      (d)  $16 \times 10^3$
51. The minimum frequency of a force signal in Hz within its useful mid-band range of measurement, for which the gain amplitude is more than 0.95, approximately is,  
 (a) 16      (b) 160  
 (c) 1600      (d)  $16 \times 10^3$

**LINKED ANSWER QUESTIONS**

Statement for Linked Answer Questions 52 and 53

Consider a plant with the transfer function  $G(s) = 1/(s + 1)^3$ . Let  $K_u$  and  $T_u$  be the ultimate gain and ultimate period corresponding to the frequency response based closed loop Ziegler-Nichols cycling method, respectively. The Ziegler-Nichols tuning rule for a P-controller is given as  $K = 0.5K_u$

52. The values of  $K_u$  and  $T_u$ , respectively, are

- (a)  $2\sqrt{2}$  and  $2\pi$       (b) 8 and  $2\pi$   
 (c) 8 and  $2\pi/\sqrt{3}$       (d)  $2\sqrt{2}$  and  $2\pi/\sqrt{3}$

53. The gain of the transfer function between the plant output and an additive load disturbance input of frequency  $2\pi/T_u$  in closed loop with a P-controller designed according to the Ziegler-Nichols tuning rule as given above is

- (a) -10      (b) 0.5  
 (c) 1.0      (d) 2.0

Statement for Linked Answer Questions 54 and 55

A differential amplifier with signal terminals X, Y, Z is connected as shown in Fig.(a) below for CMRR measurement where the differential amplifier has an additional constant offset voltage in the output. The observations obtained are when  $V_1 = 2\text{V}$ ,  $V_o = 3\text{mV}$ , and when  $V_1 = 3\text{V}$ ,  $V_o = 4\text{mV}$

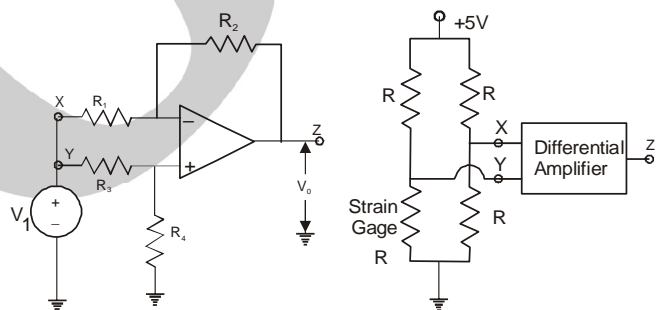


Fig. (a)

Fig. (b)

54. Assuming its differential gain to be 10 and the op-amp to be otherwise ideal, the CMRR is  
 (a)  $10^2$       (b)  $10^3$   
 (c)  $10^4$       (d)  $10^5$
55. The differential amplifier is connected as shown in Fig. (b) above to single strain gage bridge. Let the strain gage resistance vary around its no-load resistance  $R$  by  $\pm 1\%$ . Assume the input impedance of the amplifier to be high compared to the equivalent source resistance of the bridge, and the common mode characteristic to be as obtained above. The output voltage in mV varies approximately from  
 (a) +128 to -128  
 (b) +128 to -122  
 (c) +122 to -122  
 (d) +99 to -101

GENERAL APTITUDE (GA) QUESTIONS

Q. 56 to Q. 60 carry one mark each.

56. **Statement :** you can always give me a ring whenever you need

Which one of the following is the best inference from the above statement?

- (a) Because I have a nice caller tune  
 (b) Because I have a better telephone facility  
 (c) Because a friend in need is a friend indeed  
 (d) Because you need not pay towards the telephone bills when you give me a ring

57. **Complete the sentence**

Dare \_\_\_\_\_ mistakes

- (a) Commit  
 (b) to commit  
 (c) Committed  
 (d) Committing

58. **Choose the grammatically CORRECT sentence**

- (a) Two and two add four  
 (b) Two and two become four  
 (c) Two and two are four  
 (d) Two and two make four

59. **They were requested not to quarrel with others**

Which one of the following options is the closest in meaning to the word quarrel?

- (a) make out  
 (b) call out  
 (c) dig out  
 (d) fall out

60. **In the summer of 2012, in New Delhi, the mean temperature of Monday to Wednesday was 41°C and of Tuesday to Thursday was 43°C. If the temperature on Thursday was 15% higher than that of Monday, then the temperature in °C on Thursday was**

- (a) 40 (b) 43  
 (c) 46 (d) 49

**SOLVED PAPER-2013 (INSTRUMENTATION ENGINEERING - IN)**

Q. 61 to Q. 65 carry two marks each.

61. **Find the sum to n terms of the series 10 + 84 + 734 +**

- (a)  $\frac{9(9^n + 1)}{10} + \tilde{1}$   
 (b)  $\frac{9(9^n - 1)}{8} + 1$   
 (c)  $\frac{9(9^n - 1)}{8} + n$   
 (d)  $\frac{9(9^n - 1)}{8} + n^2$

62. **The set of values of p for which the roots of the equation  $3x^2 + 2x + p(p - 1) = 0$  are of opposite sign is**

- (a)  $(-\infty, 0)$  (b)  $(0, 1)$   
 (c)  $(1, \infty)$  (d)  $(0, \infty)$

63. **A car travels 8 km in the first quarter of an hour, 6 km in the second quarter and 16 km in the third quarter. The average speed of the car in km per hour over the entire journey is**

- (a) 30 (b) 36  
 (c) 40 (d) 24

64. **What is the chance that a leap year, selected at random, will contain 53 Saturdays?**

- (a)  $\frac{2}{7}$  (b)  $\frac{3}{7}$   
 (c)  $\frac{1}{7}$  (d)  $\frac{5}{7}$

65. **Statement :** There were different streams of freedom movements in colonial India carried out by the moderates, liberals, radicals, socialists, and so on

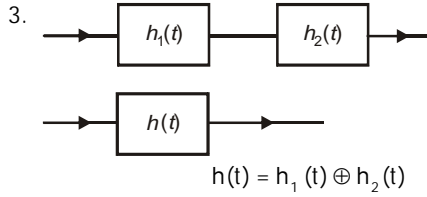
Which one of the following is the best inference from the above statement?

- (a) The emergence of nationalism in colonial India led to our Independence  
 (b) Nationalism in India emerged in the context of colonialism  
 (c) Nationalism in India is homogeneous  
 (d) Nationalism In India is heterogeneous

ANSWERS

1. (b) 2. (d) 3. (c) 4. (d) 5. (d) 6. (a) 7. (a) 8. (c) 9. (c) 10. (a)  
 11. (b) 12. (d) 13. (d) 14. (d) 15. (c) 16. (a) 17. (a) 18. (a) 19. (c) 20. (c)  
 21. (b) 22. (a) 23. (b) 24. (a) 25. (b) 26. (a) 27. (a) 28. (b) 29. (a) 30. (b)  
 31. (c) 32. (c) 33. (b) 34. (b) 35. (d) 36. (c) 37. (d) 38. (d) 39. (c) 40. (a)  
 41. (a) 42. (d) 43. (c) 44. (b) 45. (a) 46. (b) 47. (c) 48. (b) 49. (b) 50. (d)  
 51. (b) 52. (c) 53. (b) 54. (c) 55. (b) 56. (c) 57. (b) 58. (d) 59. (b) 60. (c)  
 61. (d) 62. (b) 63. (c) 64. (a) 65. (d)

EXPLANATIONS



4.  $\tan h(s)$  is analytic  
 If  $e^s + e^{-s} \neq 0$   
 $\Rightarrow e^{2s} \neq -1 \Rightarrow s \neq$   
 $\Rightarrow \text{Im}(s) \neq$

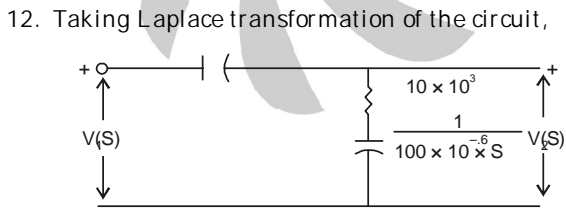
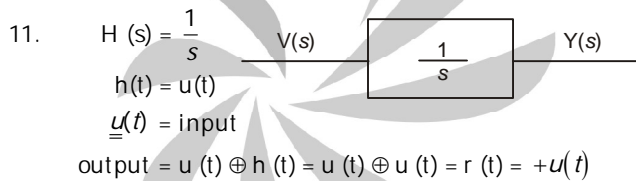
5.  $E = 0$ ,  $E$  is called irrotational, which is not true.

6.  $W_0 = 100$  rad/sec fundamental  
 $3W_0 = 300$  rad/sec third harmonic  
 $5W_0 = 500$  rad/sec fifth harmonic

7.  $V_B = \frac{10 \times 6k}{(6+15)k} = 2.8V$   
 $\Rightarrow VE = 2.8 - 0.7 = 2.1V$   
 $\Rightarrow RE = \frac{VE}{IE} = \frac{2.1}{2mA} = 1k\Omega$

8.  $Z_1 = (4 + j 3) \Omega$   
 $Z_L = \sqrt{4^2 + 3^2} \Omega = 5 \Omega$

9. Consider option A : In which all the poles lie on the left of  $j\omega$  axis which satisfy casual stable LT1 system.  
 Option B : For a stable casual system, there are no restriction for the position of zeroes on  $s$  plane.  
 Option C : text true.  
 Option D : Roots of characteristic equation are all closed loop poles and they all lie on the left side of the  $j\omega$  axis.



By applying voltage divider rule:  
 $V_2(s) = \frac{10 \times 10^3 + \frac{10^4}{s}}{10 \times 10^3 + \frac{10^4}{s} + \frac{10^4}{s}} \times V_1(s)$   
 $\frac{V_2(s)}{V_1(s)} = \frac{1 + \frac{1}{s}}{1 + \frac{2}{s}} = \frac{s+1}{s+2}$

13.  $\frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial x^2}$

$\therefore B^2 - 4AC = 0$

$\therefore$  The equation is parabolic

14.  $H(z) = \frac{1-2z^{-1}}{1-0.5z^{-1}}$

For minimum phase system, all poles and zeros must lie inside the unit circle.

The stable system all poles must lie inside the unit circle.

17. Here  $f_m = 5KHZ$   
 $\therefore f_s \geq 2f_m = 10 KHZ$   
 B,C,D options are greater than 10 KHZ

18. Let switches =  $P_1, P_2$

$P_1$	$P_2$	2(O/P)
OFF	OFF	OFF
OFF	ON	ON
ON	OFF	ON
ON	ON	OFF

$\therefore$  From truth table it can be verified that Ex - OR logic is implemented

19. Let us consider the switches A and B and bulb Y. Switches can be 2 positions up (0) or down (1) Starting with both A and B in up position. Let the bulb be OFF. Now since B can operate independently when B goes down, the bulb goes ON

A	B	Y
up (0)	up (0)	OFF
up (0)	down (1)	ON

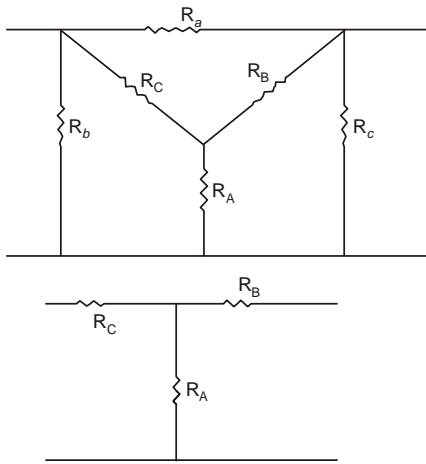
Now keeping A in down position when B goes down, the bulb will go OFF.

A	B	Y
down (1)	up (0)	ON
down (1)	down (1)	OFF

find truth table corresponds to XOR gate.

20. As  $h(t) = t a(t)$   
 input response  
 $\delta(t) \rightarrow t a(t)$   
 $u(t) \rightarrow \int_{-\infty}^t +a(I)dt = \int_0^t tdt = \frac{t^2}{2} a(t)$   
 $u(t-1) \rightarrow \frac{(t-1)^2}{2} a(t-1)$

21.



$$R_c = \frac{R_a \cdot R_b}{R_a + R_b + R_c} \text{ as } R_a \text{ is scaled by factor } k$$

$$R'_c = \frac{R'_a \cdot R'_b}{R'_a + R'_b + R'_c} = \frac{k^2 R_a \cdot R_b}{k(R_a + R_b + R_c)} = k \cdot \frac{R_a \cdot R_b}{R_a + R_b + R_c}$$

so elements corresponding to star equivalence will be scaled by factor k.

23. Using the concept of "virtual ground" in an operational amplifier, we can set the voltage at the point to zero volts since the non inverting terminal is grounded.

Once  $V_A = 0$ ,  $V_C$  will also be zero.

We know that for a silicon n-p-n transistor,

$$V_{BE} = V_B - V_E = 0.7 \text{ V}$$

$$\text{Since, } V_B = 0 \Rightarrow V_E = -0.7 \text{ V}$$

Hence the output voltage is the same as the emitter voltage so,

$$V_{out} = -0.7 \text{ V}$$

24. Input Impedance of a voltage-voltage feedback circuit =  $2i(1 + A_o k)$

$Z_i$  = initial input impedance (without feedback)

Impedance of a voltage-voltage feedback circuit =  $Z_o / (1 + A_o k)$

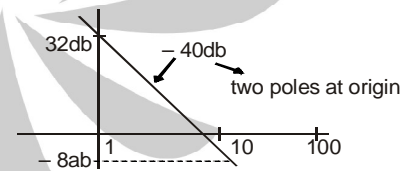
$Z_o$  = initial output impedance (without feedback)

Hence, As K is increased, the input impedance will increase and output impedance will decrease.

25.  $20 \log k = 32$

$$k = 10^{1.6} = 39.8$$

$$T(s) = \frac{39.8}{s^2}$$



26.  $\frac{dy}{dx} = -2xy^2, x_0 = 6, y_0 = 1$

$$h = 0.2$$

$$f(x, y); y^{p_1} = y_0 + h + (x_0, y_0) = 0.96$$

is the value of y after step.

27.  $\lambda = j, -j$  are eigen values

$$(A - jI)x = 0 \Rightarrow \begin{bmatrix} -j & -1 \\ 1 & -j \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\therefore \text{Eigen vector for } j = \begin{bmatrix} j \\ 1 \end{bmatrix}$$

$$\text{and } (A + jI)x = 0 \Rightarrow \begin{bmatrix} j & -1 \\ 1 & j \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\therefore \text{Eigen vector for } -j \text{ is } \begin{bmatrix} 1 \\ j \end{bmatrix}$$

30. 1st case :

$$V_{wx1} = 100 \text{ V}$$

$$\text{So, } Vy'z_1 = \frac{M_2}{M_1} V_{wx1} = 1.25 \times 100 = 125 \text{ V}$$

$$\therefore Vy'z_1 = Vy'z_1 \times x = 125 \times 0.8 = 100 \text{ v.}$$

$$\therefore Vy'z_1 / V_{wx1} = \frac{100}{100}$$

2nd case:

$$Vy'z_2 = 100 \text{ V}$$

$$\therefore Vy't_2 = \frac{100}{\alpha} = \frac{100}{0.8} = 125 \text{ v}$$

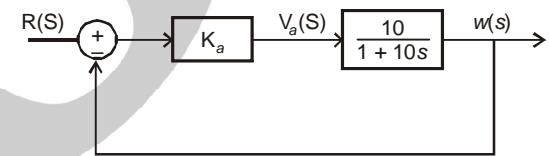
$$\text{Now, } V_{wx2} = \frac{M_1}{M_2} Vy't_2 = \frac{1}{1.25} \times 125 = 100 \text{ v}$$

$$\therefore V_{wx2} / Vy'z_2 = \frac{100}{100}$$

31. Given:

Open loop transfer function of a dc motor as

$$\frac{w(s)}{V_a(s)} = \frac{10}{1+10s}$$



Topic : P controller with unity feed back

Formula: For first order system loop transfer function

$$\text{is } \frac{C(s)}{R(s)} = \frac{K}{1+sT} \text{ comparing with } \frac{w(s)}{V_a(s)} = \frac{10}{1+10s} \quad T_{open \text{ loop}} = 10$$

Now for closed loop over all transfer function is given by

$$\begin{aligned} \frac{w(s)}{R(s)} &= \frac{K_a \left( \frac{10}{1+10s} \right)}{1 + K_a \left( \frac{10}{1+10s} \right)} \\ &= \frac{K_a 10}{1+10s + K_a 10} = \frac{10K_a}{10s + (10K_a + 1)} \end{aligned}$$

Dividing numerator and denominator by  $10K_a + 1$

$$\text{Now } \frac{w(s)}{R(s)} = \frac{\frac{10K_a}{10K_a + 1}}{1 + \left( \frac{10}{10K_a + 1} \right) s}$$

$$\text{So } T_{closed \text{ loop}} = \frac{10}{10K_a + 1} \text{ (By comparing from formula)}$$

In Question given that time constant of closed loop system

is  $\frac{1}{100}$  times of time constant of open loop system

$$\text{So } \frac{10}{10K_a + 1} = \frac{1}{100} 10 \left( T_{\text{closed loop}} = \frac{1}{100} T_{\text{open loop}} \right)$$

$$10 K_a + 1 = 100$$

$$10 K_a = 99$$

$$K_a = 9.9 \approx 10$$

∴  $K_a = 10$  approximate value

32. Let the effective Q factor is  $q_1$  then it can be written using inductance and resistance of equivalent circuit.

$$q = \frac{\omega L e q \cdot}{\text{Re } q \cdot} = \frac{\omega(L_1 + L_2)}{R_1 + R_2}$$

Now we substitute the value of  $L_1$  and  $L_2$  in terms of  $q_1$  and  $q_2$

$$\therefore q = \omega \left[ \frac{q_1 R_1}{\omega} + \frac{q_2 R_2}{\omega} \right] = (q_1 R_1 + q_2 R_2) / (R_1 + R_2)$$

33.

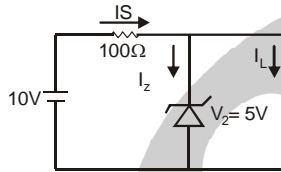
$$I_s = I_z + I_L$$

$$I_s - I_z = I_L$$

Two extreme condition:

If  $I_z$  (min), then  $I_L$  (max)

If  $I_z$  (max) then  $I_L$  (min) = 0



$$I_z(\text{max}) = I_s = \frac{10 - 5}{10} = 50 \text{ mA}$$

$$I_z(\text{min}) = I_s - I_L(\text{max})$$

$$I_L(\text{max}) = I_s - I_z(\text{min}) = I_s - I_z = (50 - 10) = 40 \text{ mA}$$

$$R_L(\text{min}) = \frac{V}{I_L(\text{max})} = \frac{5}{40} \text{ K} = 125 \Omega$$

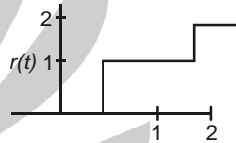
$$P_z = V_z \times I_z(\text{max}) = 5 \times 50 \text{ mA} = 250 \text{ mw}$$

34. As  $h(t) = \delta(t - 1) + \delta(t - 3)$

$$r(t) = h(t) \oplus u(t)$$

$$= [\delta(t - 1) + \delta(t - 3)] \oplus u(t)$$

$$= u(t - 1) + u(t - 3)$$



35.

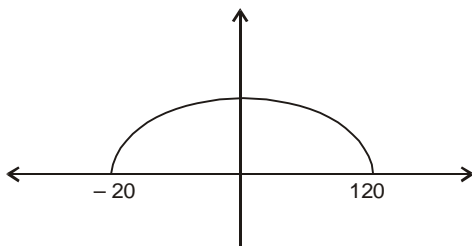
Data rate from ADC is  $16 \times 15$  bits/second

$$= 240 \text{ bits/second}$$

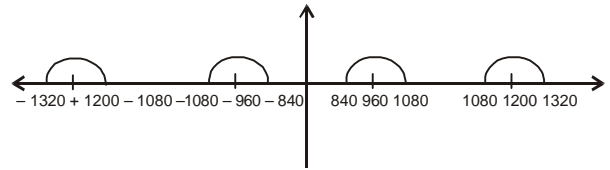
The bandwidth required for transmitting 240 bits/second

$$= 120 \text{ Hz (half of bits rates)}$$

Now when each pulse representing '1' and '0' sets FSK modulator with 960Hz and 1200 Base band signal before modulation.



The spectrum of signal after modulation

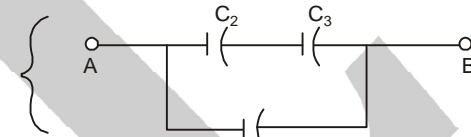


∴ all frequency from 840 to 1320 in required for allocation.

36. Given values:

	Capacitance Value	Voltage Breakdown
$C_1$	$10 \mu\text{F}$	10V
$C_2$	$5 \mu\text{F}$	5V
$C_3$	$2 \mu\text{F}$	2V.

Given circuit:



To find:

Max. safe voltage in volts that can be applied across the combination and Corresponding total charge is  $\mu\text{C}$ .

Devising a plan:

(a) We will first calculate equivalent capacitance of given figure.

(b) By calculating capacitance, we can check which answer can be the solution, it reduce your effort in solving problem.

(c) Then apply the remaining answer for remaining options.

Solving:

Equivalent capacitance, as  $C_2$  &  $C_3$  are in series and we know that when capacitance are in series equivalent

$$\text{capacitance} = \frac{C_2 C_3}{C_2 + C_3}$$

& when two capacitor are in parallel then their addition

$$= C_1 + \frac{C_2 C_3}{C_2 + C_3}$$

$$* C = C_1 + \frac{C_2 C_3}{C_2 + C_3} = 10 \mu\text{F} + \frac{5 \mu\text{F} \times 2 \mu\text{F}}{5 \mu\text{F} + 2 \mu\text{F}} = 11.4285 \mu\text{F}$$

Now check the option by using formula

$$CV = \theta$$

(a)  $2.8 \times 11.4285 = 32 \mu\text{C} \neq 36 \mu\text{C}$  (option (a) is wrong) hence 'c' option can be right

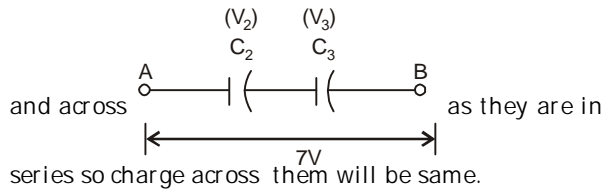
Now when 7V

(b)  $7V \times 11.4285 \mu\text{F} = 80 \mu\text{C} \neq 119 \mu\text{C}$  { B is hence 'd' option is right.

only 'c' & 'd' are left.

⇒ Now apply 7V first in given circuit,

when voltage across AB is 7V then  $C_1$  can handle it broz max breakdown voltage across is 10V.



$$Q_2 = Q_3$$

$$C_2 V_2 = C_3 V_3$$

$$C_2 V_2 = C_3 V_3 \quad [V_2 + V_3 = 7V]$$

$$C_2 V_2 = C_3 (7 - V_2)$$

$$C_2 V_2 = 7C_3 - C_3 V_2$$

$$V_2 = \frac{7C_3}{C_2 + C_3}$$

$$V_2 = \frac{7 \times 2}{2 + 5} = 2V \quad \boxed{V_2 = 2V}$$

and  $V_3 = 7 - V_2 = 7 - 2$

$$\boxed{V_3 = 5V}$$

As when we take '7V' then for that  $V_2 = 2V$  &  $V_3 = 5V$ , but max voltage across  $V_3$  can be 2V [because above which it breakdown].

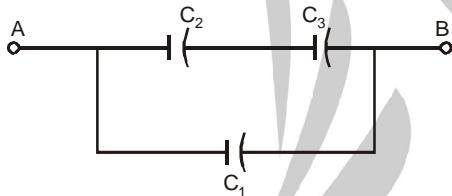
as  $\boxed{V_3 \leq 2V}$

But, when we take 7V then  $V_3$  have to be 5V which is not possible hence 'd' is also wrong.

⇒ Only option left is 'c'

But, I will show that it is also right.

When  $V_{AB} = 2.8V$



voltage across  $C_1 = 2.8V$  [possible as breakdown voltage is 10V].

$$V_2 = \frac{C_3 \times 2.8}{C_2 + C_3}$$

$\boxed{V_2 = -8V}$  [possible as breakdown voltage across  $C_2$  is 5V]

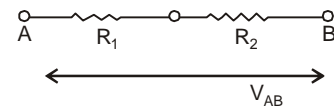
$$V_3 = \left( \frac{C_2}{C_2 + C_3} \right) \times 2.8$$

$$= 2V \text{ [possible as breakdown voltage across } C_3 \text{ is } 2V]$$

hence option 'D' is correct

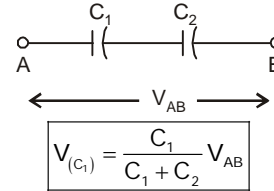
\* Verification: As I already showed that only 's' is true no one else.

Conclusion: voltage across method capacitance is not same as voltage across method resistance. For determine voltage across resistance we just do as we want to determine across  $R_1$  &  $R_2$



$$V_{(R_1)} = \frac{R_1}{R_1 + R_2} V_{AB}$$

But in capacitor



So, don't apply resistance voltage method in to capacitor one, if you do that then you will obtain 'D' as answer, but which is wrong.

37.  $y(t) + y(t) = 0$

Taking laplace on both the sides

we get

$$(s^2 + 1) y(s) = 2s + 1 = y(s) = \frac{2s}{s^2 + 1} + \frac{1}{s^2 + 1}$$

∴  $y(t) = 2 \cos t + \sin t$

∴ maximum value of  $y(t) = \sqrt{2^2 + 1^2}$

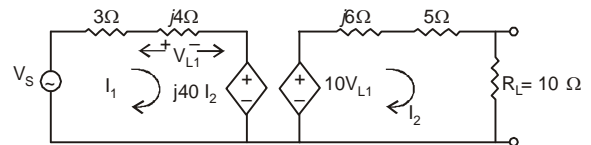
$= \sqrt{5}$

38.  $n(t) = t$  for  $0 < t < 1$   
 $= 2 - t$  for  $1 < t < 2$

$$L [n(t)] = \int_0^1 e^{-st} \times t dt + \int_1^2 e^{-st} (2t) dt$$

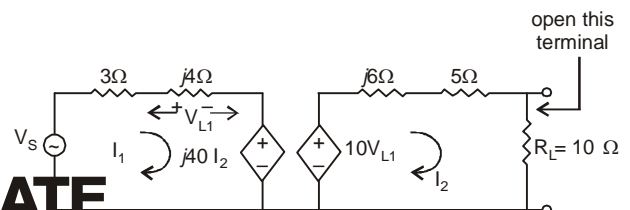
$$= \frac{1}{s^2} [1 - 2e^{-s} + e^{-2s}]$$

39. Given:  $V_s = 100 \angle 53.13^\circ V$

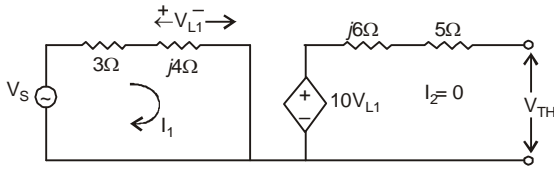


To find: Thevenin's voltage across Load resistance  
 Solution

\* For  $V_{th}'$  open it.



- \* Opening, then  $I_2 = 0$
- \* When  $I_2 = 0$ , then  $j40I_2 = 0$  (voltage source will short circuit)
- ∴ Circuit became



$$\therefore I_1 = \frac{V_s}{3+4j} \quad \therefore V_{L1} = j4 \times \frac{V_s}{3+4j}$$

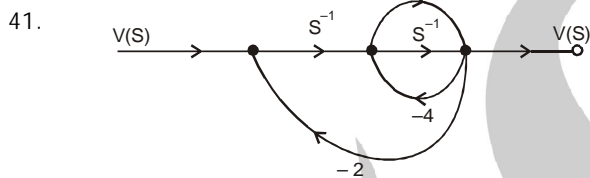
\*  $V_{TH} = 10V_{L1}$  because no-current flowing through circuit.

$$V_{TH} = \frac{10 \times j4 \times V_s}{3+4j} = \frac{40j V_s}{3+4j}$$

From rectangular domain to polar domain.

$$= \frac{40 \angle 90^\circ}{5 \angle 53.13^\circ} \times 100 \angle 53.13^\circ$$

$$V_{TH} = 800 \angle 90^\circ$$



Forward path =

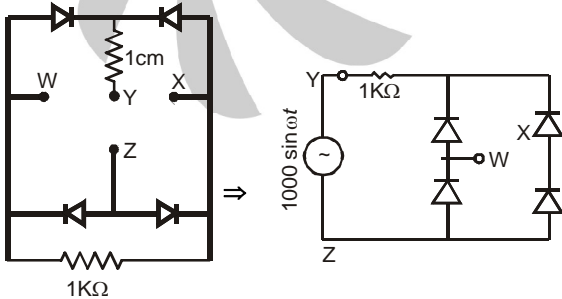
$$\begin{matrix} P_1 = S^{-1}, S^{-1} = S^{-2} & \Delta_1 = 1 \\ P_2 = S^{-1} & \Delta_2 = 1 \end{matrix}$$

Loop =

$$\begin{matrix} L_1 = -4S^{-1} \\ L_2 = -2S^{-1}S^{-1} = -2S^{-2} \\ L_3 = -2S^{-1} \\ L_4 = -4 \end{matrix}$$

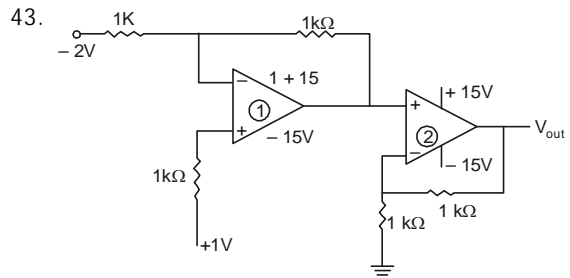
$$T(S) = \frac{S^{-2} + S^{-1}}{1 + 4 + 2S^{-1} + 2S^{-2} + 4S^{-1}} = \frac{S^{-1} + S^{-2}}{S + 6S^{-1} + 2S^{-2}} = \frac{S+1}{5S^2 + 6S + 2}$$

42. 'D' 0 for all +



Note:

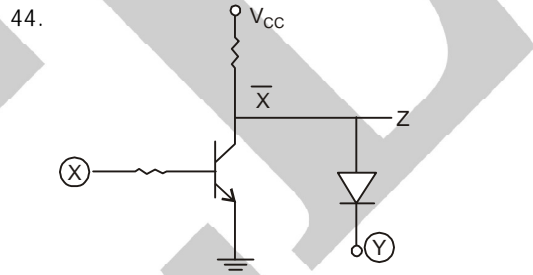
- ⇒ All diode conducts only done negative half.
- ⇒ XW is at symmetrical point so voltage across XW is zero for all time.



$$V_{out} = \left[ +1V \left( 1 + \frac{1}{1} \right) - 2V \left( \frac{-1}{1} \right) \right] \left[ 1 + \frac{1}{1} \right]$$

Gain of non inverting amp (1)      Gain of inverting amp (2)      Gain of non inverting amp (2)

$$= [1 \times 2 + 2] \times 2 v = 8v$$



$$z = \bar{x} \cdot y$$

54.

$$\Delta V_o = \frac{\Delta V}{CMRR} \left( 1 + \frac{R_2}{R_1} \right)$$

$$\Rightarrow CMRR = \frac{1V}{1mV} (10) = 10^4$$

60. Let the temperature of Monday be  $T_M$  sum of temperature of Tuesday and Wednesday = 1 and temperature of Thursday =  $T_{Th}$

$$T_m + T = 41 \times 3 = 123$$

and  $T_{th} + T = 43 \times 3 = 129$

$$\therefore T_{th} - T_m = 6, T_{Th} = 1, 155 \text{ cm}$$

$$\therefore 0.15T_m = 6 \Rightarrow T_m = 40$$

$$\therefore \text{Temp of thursday} = 40 + 6 = 46^\circ\text{C}$$

61.  $10 + 84 + 734 + \dots + n$  term

$$= (9 + 1) + (9^2 + 3) + (9^3 + 5) + (9^4 + 7) + \dots n \text{ term.}$$

$$= \frac{9(9^n - 1)}{9 - 1} + n^2$$

$$\left( S_n = \frac{a(r^n - 1)}{r - 1} (r > 1) \text{ at Sum of first n odd number in } n^2 \right)$$

$$62. \therefore \frac{P(P-1)}{3} < 0 \Rightarrow P(P-1) < 0 \Rightarrow (P-0)(P-1)$$

$$< 0 \Rightarrow 0 < P < 1$$

$\therefore$  the required set of valusion is (0, 1)

$$63. \text{ Average speed} = \frac{\text{Total distance}}{\text{total time}} = \frac{8+6+6}{\frac{1}{4} + \frac{1}{4} + \frac{1}{4}}$$

$$= 40 \text{ km/hv.}$$

64. There are 52 complete week in a calender

$$\text{Year} = 52 \times 7 = 364 \text{ days}$$

$\therefore$  No. of days in a leap year = 366

$\therefore$  Probability of 53 Saturdays =  $\frac{2}{7}$

■ ■

