MATHEMATICS

PART I

SECTION – I

Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

1. If 0 < x < 1, then $\sqrt{1+x^2}\,[\{x\cos(\cot^{-1}x)+\sin(\cot^{-1}x)\}^2-1]^{\frac{1}{2}} =$

(A) $\dfrac{x}{\sqrt{1+x^2}}$ (B) x (C) $x\sqrt{1+x^2}$ (D) $\sqrt{1+x^2}$

Sol. (C)

The given expression $\sqrt{1+x^2}\left[\left\{\dfrac{x\times x}{\sqrt{1+x^2}}+\dfrac{1}{\sqrt{1+x^2}}\right\}^2-1\right]^{\frac{1}{2}}$

$$= \sqrt{1+x^2}\left[\frac{(x^2+1)^2}{x^2+1}-1\right]^{\frac{1}{2}}$$

$$= \sqrt{1+x^2}\left[x^2+1-1\right]^{\frac{1}{2}} = x\sqrt{1+x^2}.$$

2. The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors $\hat{a}, \hat{b}, \hat{c}$ such that $\hat{a}\cdot\hat{b}=\hat{b}\cdot\hat{c}=\hat{c}\cdot\hat{a}=\dfrac{1}{2}$.

Then, the volume of the parallelopiped is

(A) $\dfrac{1}{\sqrt{2}}$ (B) $\dfrac{1}{2\sqrt{2}}$ (C) $\dfrac{\sqrt{3}}{2}$ (D) $\dfrac{1}{\sqrt{3}}$

Sol. (A)

$$\because \vec{a}=\hat{i},\quad \hat{b}=\frac{1}{2}\hat{i}+\frac{\sqrt{3}}{2}\hat{j},\quad \vec{c}=\frac{1}{2}\hat{i}+\frac{1}{2\sqrt{3}}\hat{j}+\frac{\sqrt{2}}{\sqrt{3}}\hat{k}$$

$$\text{Volume} = \begin{vmatrix} 1 & 0 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{1}{2} & \frac{1}{2\sqrt{3}} & \frac{\sqrt{2}}{\sqrt{3}} \end{vmatrix} = \frac{1}{\sqrt{2}}$$

3. Consider the two curves

$C_1 : y^2 = 4x$

$C_2 : x^2 + y^2 - 6x + 1 = 0$

Then,

(A) C_1 and C_2 touch each other only at one point

(B) C_1 and C_2 touch each other exactly at two points

(C) C_1 and C_2 intersect (but do not touch) at exactly two points

(D) C_1 and C_2 neither intersect nor touch each other

Sol. (B)

$\therefore y^2 = 4x$ ....(i) and $x^2 + y^2 - 6x + 1 = 0$...... (ii)

Solving (i) and (ii)

$x^2 + 4x - 6x + 1 = 0 \Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow (x - 1)^2 = 0$

$\Rightarrow x = 1$

$\therefore y = \pm 2.$

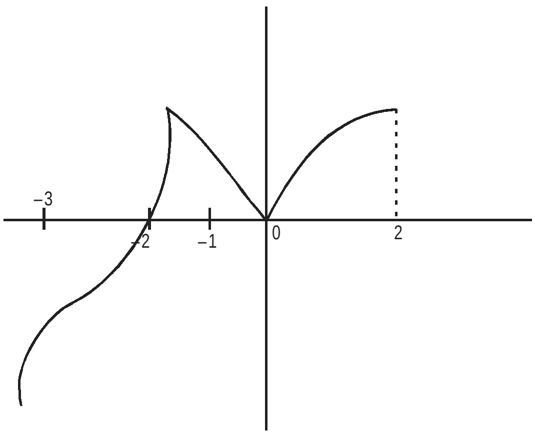
$\therefore$ C_1 and C_2 touch each other exactly at two points.

4. The total number of local maxima and local minima of the function $f(x) = \begin{cases} (2+x)^3, & -3 < x \le -1 \\ x^{2/3}, & -1 < x \le 2 \end{cases}$ is

(A) 0 (B) 1 (C) 2 (D) 3

Sol. (C)

The graph of the function is



There is one local maxima and one local minima.

5. Let a and b non-zero real numbers. Then, the equation $(a x^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$ represents

(A) four straight lines, when c = 0 and a, b are of the same sign

(B) two straight lines and a circle, when a = b, and c is of sign opposite to that of a

(C) two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a

(D) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a

Sol. (B) $x^2 - 5xy + 6y^2 = 0$

$\Rightarrow x^2 - 3xy - 2xy + 6y^2 = 0$

$\Rightarrow x(x - 3y) - 2y(x - 3y) = 0$

$\Rightarrow (x - 3y)(x - 2y) = 0 \Rightarrow$ two straight lines.

and when a = b and sign of c is opposite of a the equation $ax^2 + by^2 + c = 0$ represent circle.

6. Let $g(x)=\dfrac{(x-1)^n}{\log\cos^m(x-1)}; 0<x<2,$ m and n are integers, $m\neq 0,$ $n>0,$ and let p be the left hand derivative of $|x-1|$ at x = 1.

If $\lim\limits_{x\to 1+} g(x)=p,$ then

(A) n = 1, m = 1
(B) n = 1, m = – 1
(C) n = 2, m = 2
(C) n > 2, m = n

Sol. (C)

$\because$ p = –1,

$$\underset{x\to 1^+}{\text{Lt}}\, g(x)=-1 \Rightarrow \underset{x\to 1^+}{\text{Lt}}\frac{(x-1)^n}{\log\cos^m(x-1)}=-1$$

$$\Rightarrow \underset{x\to 1^+}{\text{Lt}}\frac{n(x-1)^{n-1}}{\dfrac{1}{\cos^m(x-1)}\times m\cos^{m-1}(x-1)\times(-\sin(x-1)}=-1$$

$$\Rightarrow \underset{x\to 1^+}{\text{Lt}}\frac{n(x-1)^{n-2}\cdot(x-1)}{-\sin(x-1)\times\dfrac{1}{\cos(x-1)}\times m}=-1. \quad \Rightarrow \underset{x\to 1^+}{\text{Lt}}\frac{n(x-1)^{n-2}}{m}=-1$$

Limit to be exist $n-2=0\Rightarrow n=2.$ and m = 2

SECTION - II

Multiple Correct Answers Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** is/are correct.

7. Let $S_n = \sum_{k=1}^{n} \frac{n}{n^2 + kn + k^2}$ and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2}$, for n = 1, 2, 3, …. Then,

(A) $S_n < \frac{\pi}{3\sqrt{3}}$

(B) $S_n > \frac{\pi}{3\sqrt{3}}$

(C) $T_n < \frac{\pi}{3\sqrt{3}}$

(D) $T_n > \frac{\pi}{3\sqrt{3}}$

Sol. (A, D)

$$\because S_n = \sum_{k=1}^{n} \frac{n}{\frac{3n^2}{4} + \left(k + \frac{n}{2}\right)^2}$$

$$= \sum_{k=1}^{n} \frac{1}{n} \frac{1}{\frac{3}{4} + \left(\frac{1}{2} + \frac{k}{n}\right)^2}$$

$$S_\infty = \underset{n\to\infty}{\text{Lt}}\, S_n = \underset{n\to\infty}{\text{Lt}}\, \frac{1}{n}\sum_{k=1}^{n} \frac{1}{\frac{3}{4} + \left(\frac{1}{2} + \frac{k}{n}\right)^2}$$

$$= \int_0^1 \frac{dx}{\frac{3}{4} + \left(\frac{1}{2} + x\right)^2}$$

$$= \frac{1}{\frac{\sqrt{3}}{2}} \left[\tan^{-1} \frac{\frac{1}{2} + x}{\frac{\sqrt{3}}{4}} \right]_0^1 = \frac{\pi}{3\sqrt{3}}$$

Similarly we can calculate T_∞

$\because S_1 < S_2 < S_3 < \ldots S_n < \ldots S_\infty$ and $T_1 > T_2 > T_3 > \ldots T_n > \ldots T_\infty$

$\Rightarrow S_n < \dfrac{\pi}{3\sqrt{3}}$ and $T_n > \dfrac{\pi}{3\sqrt{3}}$.

8. Let f(x) be a non-constant twice differentiable function defined on $(-\infty, \infty)$ such that $f(x) = f(1-x)$ and $f'\left(\frac{1}{4}\right) = 0$. Then,

(A) $f''(x)$ vanishes at least twice on [0,1]

(B) $f'\left(\frac{1}{2}\right) = 0$

(C) $\displaystyle\int_{-1/2}^{1/2} f\left(x + \frac{1}{2}\right) \sin x \, dx = 0$

(D) $\displaystyle\int_0^{1/2} f(t)\, e^{\sin \pi t}\, dt = \int_{1/2}^{1} f(1-t)\, e^{\sin \pi t}\, dt$

Sol. (A, B, C, D)

(a, b, c, d)

f(x) = f(1 – x)

$f'(x) = -f'(1-x)$

At $x = \frac{1}{2}$, $f'\left(\frac{1}{2}\right) = -f'\left(\frac{1}{2}\right) \Rightarrow f'\left(\frac{1}{2}\right) = 0$

At $x = \frac{1}{4}$, $f'\left(\frac{1}{4}\right) = -f'\left(\frac{3}{4}\right) \Rightarrow f'\left(\frac{3}{4}\right) = 0$

$\therefore f'\left(\frac{1}{4}\right) = f'\left(\frac{1}{2}\right) = f'\left(\frac{3}{4}\right) = 0$

$\therefore$ By Rolle's theorem, there exists.

c_1, c_2 between $\left(\frac{1}{4}, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, \frac{3}{4}\right)$ respectively.

Such that $f''(c_1) = f''(c_2) = 0$

$$\int_{-1/2}^{1/2} f\left(x+\frac{1}{2}\right)\sin x\,dx$$

$$= \int_{-1/2}^{1/2} f\left(-x+\frac{1}{2}\right)\sin(-x)\,dx$$

$$= -\int_{-1/2}^{1/2} f\left(x+\frac{1}{2}\right)\sin x\,dx$$

$$\left(\text{as } f(x) = f(1-x) \text{ let } x = x+\frac{1}{2} \Rightarrow f\left(x+\frac{1}{2}\right) = f\left(\frac{1}{2}-x\right)\right)$$

$$\Rightarrow \int_{-1/2}^{1/2} f\left(x+\frac{1}{2}\right)\sin x\,dx = 0$$

$$\int_{0}^{1/2} f(t)e^{\sin \pi t}dt \quad \text{let } t = 1-x$$

$dt = -dx$

$$-\int_{1}^{1/2} f(1-x)e^{\sin \pi(1-x)}dx$$

$$= \int_{1/2}^{1} f(1-x)e^{\sin \pi x}dx$$

9. A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcirle, then

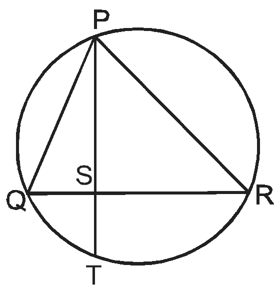
(A) $\frac{1}{PS}+\frac{1}{ST}<\frac{2}{\sqrt{QS\times SR}}$

(B) $\frac{1}{PS}+\frac{1}{ST}>\frac{2}{\sqrt{QS\times SR}}$

(C) $\frac{1}{PS}+\frac{1}{ST}<\frac{4}{QR}$

(D) $\frac{1}{PS}+\frac{1}{ST}>\frac{4}{QR}$

Sol. (B,D)



$\because$ PS × ST = QS × SR

Apply GM > HM for PS and ST

$$\sqrt{PS\times ST}>\frac{2}{\frac{1}{PS}+\frac{1}{ST}}$$

$$\Rightarrow \frac{1}{PS}+\frac{1}{ST}>\frac{2}{\sqrt{QS\times SR}}$$

and apply AM > GM on QS, SR

$$\frac{QR+SR}{2}>\sqrt{QS\times SR}$$

$$\frac{QR}{2}>\sqrt{QS\times SR}$$

$$\frac{1}{\sqrt{QS\times SR}}>\frac{2}{QR}$$

$$\Rightarrow \frac{1}{PS}+\frac{1}{ST}>\frac{2}{\sqrt{QS\times SR}}>\frac{4}{QR}$$

10. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$, $y_1 < 0$, $y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are

(A) $x^2 + 2\sqrt{3}\,y = 3 + \sqrt{3}$

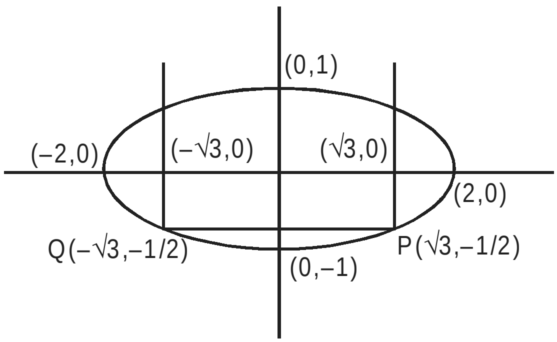
(B) $x^2 - 2\sqrt{3}\,y = 3 + \sqrt{3}$

(C) $x^2 + 2\sqrt{3}\,y = 3 - \sqrt{3}$

(D) $x^2 - 2\sqrt{3}\,y = 3 - \sqrt{3}$

Sol. (B, C)

$$\frac{x^2}{4} + \frac{y^2}{1} = 1$$



Focus $\left(0, \frac{-1}{2}\right)$

Directrix $y = \frac{-1}{2} \pm \sqrt{3}$

$\therefore$ Equation of parabola becomes

$$(x-0)^2 + \left(y + \frac{1}{2}\right)^2 = \left(y + \frac{1}{2} \mp \sqrt{3}\right)^2$$

$$= \left(y + \frac{1}{2}\right)^2 + 3 \pm 2\sqrt{3}\left(y + \frac{1}{2}\right)$$

$$x^2 = 3 \pm \sqrt{3}(2y+1)$$

$$x^2 \mp 2\sqrt{3}y = 3 \pm \sqrt{3}$$

ie. $x^2 - 2\sqrt{3}y = 3 + \sqrt{3}$

or $x^2 + 2\sqrt{3}y = 3 - \sqrt{3}$

SECTION - III

Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

11. Let f and g be real valued functions defined on interval (– 1, 1) such that $g''(x)$ is continuous, $g(0) \neq 0, g'(0) = 0, g''(0) \neq 0,$ and f(x) = g(x) sin x.

STATEMENT - 1 : $\lim_{x \to 0} [g(x)\cot x - g(0)\operatorname{cosec} x] = f''(0)$.

and

STATEMENT - 2 : $f'(0) = g(0)$.

(A) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 **is** a correct explanation for STATEMENT - 1

(B) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 **is NOT** a correct explanation for STATEMENT - 1

(C) STATEMENT - 1 is True, STATEMENT - 2 is False

(D) STATEMENT - 1 is False, STATEMENT - 2 is True

Sol. (B)

f(x) = g(x) sinx

$f'(x) = g(x)\cos x + g'(x)\sin x$

$f'(0) = g(0)$ ....(II)

$f''(x) = -g(x)\sin x + g'(x)\cos x + g'(x)\cos x + g''(x)\sin x$

$= -g(x)\sin x + 2g'(x)\cos x + g''(x)\sin x$

$f''(0) = 0$

$$\lim_{x \to 0} \frac{g(x)\cos x - g(0)}{\sin x}$$

$\frac{0}{0}$ form

$$\lim_{x \to 0} \frac{g'(x)\cos x - g(x)\sin x}{\cos x} = 0 = f''(0)$$

12. Consider three planes

$P_1 : x - y + z = 1$
$P_2 : x + y - z = -1$
$P_3 : x - 3y + 3z = 2.$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, and P_1 and P_2, respectively.

STATEMENT - 1 : At least two of the lines L_1, L_2 and L_3 are non-parallel.

and

STATEMENT - 2: The three planes do not have a common point.

(A) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 **is** a correct explanation for STATEMENT - 1

(B) STATEMENT - 1 s True, STATEMENT - 2 True; STATEMENT - 2 **is NOT** a correct explanation for STATEMENT - 1

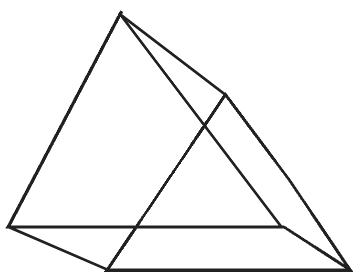
(C) STATEMENT - 1 is True, STATEMENT - 2 s False

(D) STATEMENT - 1 is False, STATEMENT - 2 is True

Sol. (D)

Gives L_1, L_2, L_3 have direction ratios 0 :1 :1
all are parallel
Here $D = 0 = D_1$
$D_2 = D_3 = -2$

No solution



13. Consider the system of equations

ax + by = 0, cx + dy = 0, where $a, b, c, d \in \{0, 1\}$.

STATEMENT - 1 : The probability that the system of equation has a unique solution is $\frac{3}{8}$.

and

STATEMENT - 2 : The probability that the system of equations has a solution is 1.

(A) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 **is** a correct explanation for STATEMENT - 1

(B) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 **is NOT** a correct explanation for STATEMENT - 1

(C) STATEMENT - 1 is True, STATEMENT - 2 is False

(C) STATEMENT - 1 is False, STATEMENT - 2 is True

Sol. (B)

There are 16 determinats of entry 0 and 1.

and $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ are six non-zero determinats.

$\therefore$ Probability that system has unique solution is $\frac{6}{16} = \frac{3}{8}$.

$\Rightarrow$ Statement 1 is correct.

and $\because$ It is a homogeneous equation

$\Rightarrow$ System has a solution $\Rightarrow$ Probability is 1.

14. Consider the system of equations

$$x - 2y + 3z = -1$$
$$-x + y - 2z = k$$
$$x - 3y + 4z = 1.$$

STATEMENT - 1 : The system of equation has no solution for $k \neq 3$.

and

STATEMENT - 2 : The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$.

(A) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 **is** a correct explanation for STATEMENT - 1

(B) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 **is NOT** a correct explanation for STATEMENT - 1

(C) STATEMENT - 1 is True, STATEMENT - 2 is False

(D) STATEMENT - 1 is False, STATEMENT - 2 is True

Sol. (A)

$$\because D = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = 0$$

$$D_1 = \begin{vmatrix} -1 & -2 & 3 \\ k & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$$

$= -(4 - 6) + 2(4k + 2) + 3(-3k - 1)$

$= 2 + 8k + 4 - 9k - 3$

$= -k + 3 \neq 0, k \neq 3.$

$\Rightarrow$ System has no solution for $K \neq 3$

$\Rightarrow$ Statement 1 is correct.

Again

$$-D_2 = \begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & K \\ 1 & 4 & 1 \end{vmatrix}$$

$\Rightarrow -D_2 = 1(-2 - 4K) - 3(-1 - K) - 1(-4 + 2)$

$= -2 - 4K + 3 + 3K + 4 - 2$

$= -K + 3 \neq 0, k \neq 0$

$\Rightarrow$ Statement 2 is true and correct explanation of statement 1

Section – IV

Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which only one is correct.

Paragraph for question no. 15 to 17.

Let A, B, C be three sets of complex numbers as defined below.

$A = \{z : \operatorname{Im} z \geq 1\}$

$B = \{z : |z - 2 - i| = 3\}$

$C = \{z : \operatorname{Re}(1 - i)z = \sqrt{2}\}$

15. The number of elements in the set $A \cap B \cap C$ is

(A) 0 (B) 1 (C) 2 (D) ∞

Sol. (B)

To answer this question, are needs to draw the area/region as marked by A, B & C

For $A = \{z : \operatorname{Im}(z) \geq 1\}$

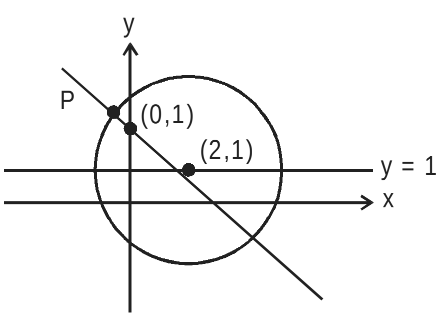
A denotes area/region (in Argand plane) beyond $y \geq 1$

For $B = \{z : |z - 2 - i| = 3\}$

B denotes point on the circle (in Argand plane) with centre at (2, 1) and radius of 3 units.

For $C = \{z : R(1 - i)z = \sqrt{2}\}$

C denotes points on the line $x + y = \sqrt{2}$



Hence, $A \cap B \cap C$ will be only one point P which is intersection of line $x + y = \sqrt{2}$ and circle : $(x - 2)^2 + (y - 1)^2 = 9$.

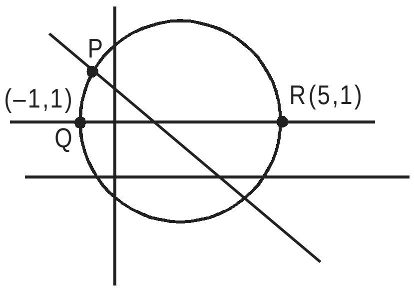
16. Let z be any point in $A \cap B \cap C$. Then, $|z+1-i|^2 + |z-5-i|^2$ lies between

(A) 25 and 29 (B) 30 and 34 (C) 35 and 39 (D) 40 and 44

Sol. (C)

z is the point P as per solution of question 15.

Now, if we look carefully at $|z+1-i|^2 + |z-5-i|^2$, we can see that it is nothing but sum of square of distance between P and Q(–1, 1) and P and R(5, 1).



As is evident that Q and R are on circle itself at two ends of diameters.

Hence,

$PQ^2 + PR^2 = QR^2$

As QR = 6

$\therefore$ $QR^2 = 36 = PQ^2 + QR^2$

17. Let z be any point in $A \cap B \cap C$ and let w be any point satisfying $|\omega - 2 - i| < 3$. Then, $|z| - |\omega| + 3$ lies between

(A) –6 and 3 (B) –3 and 6 (C) –6 and 6 (D) –3 and 9

Sol. (B)

Now ω is defined by $|\omega - 2 - i| < 3$,

Which means ω is all the points inside circle represented by $(x-2)^2 + (y-1)^2 = 9$ in the Argand plane.

Now, we have to find out coordinates of point P. to find the range of $|z| - |\omega| + 3$

To find coordinates of point P we have to solve the following equations simultaneously.

$$x + y = \sqrt{2}$$

$$(x-2)^2 + (y-1)^2 = 9$$

on solving, we get

$$2x^2 - 2\left(1+\sqrt{2}\right)x - 2\left(1+\sqrt{2}\right) = 0$$

$$x^2 - 2\left(1+\sqrt{2}\right)x - 2\left(1+\sqrt{2}\right) = 0$$

$$x = \frac{\left(1+\sqrt{2}\right) - \sqrt{7+6\sqrt{2}}}{2}$$ (+ve, sign to be ignored as P is in second quadrant)

$$x = \frac{2.4 - \sqrt{15.4}}{2} = -0.76$$

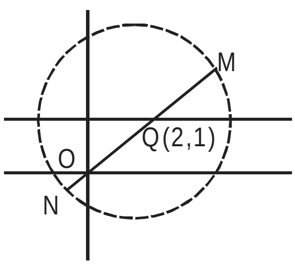
as $x + y = \sqrt{2} \Rightarrow y = 2.16$

$$P \equiv (-0.76, 2.16)$$

$$|z| = \sqrt{(0.76)^2 + (2.16)^2} = \sqrt{0.58 + 4.67} = \sqrt{5.25} = 2.3$$

$|z| + 3 = 5.3$

Range of $|\omega|$ can be found by finding OM and ON



OM = distance before O(0, 0) to centre (2, 1) + Radius of circle denoted by $(x-2)^2 + (y-1)^2 = 9$

$\therefore OM = \sqrt{4+1} + 3 = \sqrt{5} + 3$

$\therefore OM = 5.3$

Further ON = MN – OM = 2×3 – 3

$\therefore ON = 0.7$

Hence $0.7 < |z| - |\omega| + 3 < 5.3$

Paragraph for question no. 18 to 20

Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line.

If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real valued differentiable function y = f(x).

If $x \in (-2, 2)$, the equation implicitly defines a unique real valued differentiable function y = g(x) satisfying g(0) = 0.

18. If $f\left(-10\sqrt{2}\right) = 2\sqrt{2}$, then $f''\left(-10\sqrt{2}\right) =$

(A) $\frac{4\sqrt{2}}{7^3 3^2}$ (B) $-\frac{4\sqrt{2}}{7^3 3^2}$ (C) $\frac{4\sqrt{2}}{7^3 3}$ (D) $-\frac{4\sqrt{2}}{7^3 3}$

Sol. (B)

$$y^3 - 3y + x = 0$$

$$\therefore 3y^2 y' - 3y' + 1 = 0, \quad y' = \frac{1}{3\left(1 - y^2\right)}$$

$$\Rightarrow 3y'' y^2 - 3y'' + 6y(y')^2 = 0$$

$$\Rightarrow y''\left(y^2 - 1\right) = -2y(y')^2$$

$$\Rightarrow y'' = \frac{2y}{\left(1 - y^2\right)} \frac{1}{\left[3\left(1 - y^2\right)\right]^2} = \frac{2y}{9\left(1 - y^2\right)^3}$$

$$\because f\left(-10\sqrt{2}\right) = 2\sqrt{2}$$

$$\therefore f''\left(-10\sqrt{2}\right) = \frac{2.2\sqrt{2}}{9\left(1 - \left(2\sqrt{2}\right)^2\right)^3} = \frac{4\sqrt{2}}{7^3 . 3^2}$$

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19. The area of the region bounded by the curve y = f(x), the x-axis and the lines x = a and x = b, where $-\infty < a < b < -2$ is

(A) $\int_a^b \frac{x}{3\left(\left(f(x)\right)^2-1\right)}dx + b\,f(b) - af(a)$

(B) $-\int_a^b \frac{x}{3\left(\left(f(x)\right)^2-1\right)}dx + b\,f(b) - af(a)$

(C) $\int_a^b \frac{x}{3\left(\left(f(x)\right)^2-1\right)}dx - b\,f(b) + af(a)$

(D) $-\int_a^b \frac{x}{3\left(\left(f(x)\right)^2-1\right)}dx - b\,f(b) + af(a)$

Sol. (A)

$\because \int f(x)\,dx = x\,f(x) - \int x\,f'(x)\,dx$

$\therefore$ Required area = $\int_a^b f(x)\,dx$

$= \left[x\,f(x)\right]_a^b - \int_a^b \frac{x}{3\left(1-f(x)^2\right)}\,dx$

$= \int_a^b \frac{x}{3\left(\left(f(x)\right)^2-1\right)}\,dx + b\,f(b) - a\,f(a)$

20. $\int_{-1}^{1} g'(x)\,dx =$

(A) 2g(–1) (B) 0 (C) –2g(1) (D) 2g(1)

Sol. (D)

$\int_{-1}^{1} g'(x)\,dx = \left[g(x)\right]_{-1}^{1}$

= g(1) – g(–1)

= g(1) – (–g(1))

$= 2g(1)$

Since given curve $y^3 - 3y + x = 0$ is symmetric about origin

$\therefore$ if y = g(x)

$\Rightarrow$ –y = g(–x)

$\Rightarrow$ g(–1) = –g(1)

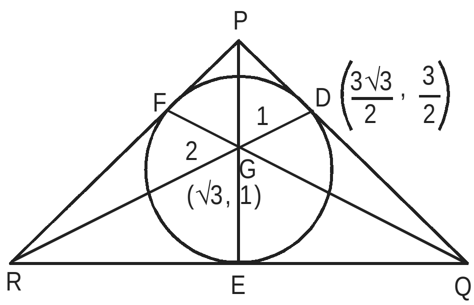
Paragraph for question no. 21 to 23

21. A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F, respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

(A) $\left(x - 2\sqrt{3}\right)^2 + (y-1)^2 = 1$ (B) $\left(x - 2\sqrt{3}\right)^2 + \left(y + \frac{1}{2}\right)^2 = 1$

(C) $\left(x - \sqrt{3}\right)^2 + (y+1)^2 = 1$ (D) $\left(x - \sqrt{3}\right)^2 + (y-1)^2 = 1$

Sol. (D)



Equation of PQ $\sqrt{3}x + y - 6 = 0$

Radius of circle = 1

$r = \frac{a}{2\sqrt{3}}$, a = length of side of equilateral triangle.

$\Rightarrow a = 2\sqrt{3}$

Equation of DR is $x - \sqrt{3}y + k = 0$

$$\Rightarrow \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} + k = 0 \quad \Rightarrow k = 0$$

$\therefore$ Equation of DR is $x - \sqrt{3}y = 0$

$\therefore$ Co-ordinates of $G \equiv \left(\sqrt{3}, 1\right)$ as origin and centre of circle lie in the same side.

$\therefore$ Equation of circle $\left(x - \sqrt{3}\right)^2 + (y-1)^2 = 1$

22. Points E and F are given by

(A) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\sqrt{3}, 0\right)$

(B) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), \left(\sqrt{3}, 0\right)$

(C) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(D) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Sol. (A)

$\because$ Slope of PQ is = $-\sqrt{3}$

$$\therefore \cos\theta = -\frac{1}{2}, \sin\theta = \frac{\sqrt{3}}{2}$$

$$\therefore x = \frac{3\sqrt{3}}{2} \pm \sqrt{3}\left(-\frac{1}{2}\right) = \frac{3\sqrt{3}}{2} \mp \frac{\sqrt{3}}{2}$$

$$= \sqrt{3}, 2\sqrt{3}$$

$$y = \frac{3}{2} \pm \sqrt{3} \times \frac{\sqrt{3}}{2}$$

$= \frac{3}{2} \pm \frac{3}{2} = 3, 0$

$\therefore$ Co-ordinates of P and Q are $\left(\sqrt{3}, 3\right)$ and $\left(2\sqrt{3}, 0\right)$

Co-ordinates of R is (h, k)

$$\therefore \frac{1 \times h + \frac{2 \times 3\sqrt{3}}{2}}{2+1} = \sqrt{3}$$

$$h + 3\sqrt{3} = 3\sqrt{3} \Rightarrow h = 0$$

and $\frac{1 + k \times 2 \times \frac{3}{2}}{1+2} = 1$

$k + 3 = 3 \Rightarrow k = 0$

$\therefore$ R = (0, 0)

Coordinates of E and F are

$$\left(\frac{\sqrt{3}+0}{2}, \frac{3+0}{2}\right) \text{and} \left(\frac{2\sqrt{3}+0}{2}, \frac{0+0}{2}\right)$$

i.e. $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\sqrt{3}, 0\right)$

23. Equations of the sides QR, RP are

(A) $y = \frac{2}{\sqrt{3}}x + 1, \ y = -\frac{2}{\sqrt{3}}x - 1$

(B) $y = -\frac{1}{\sqrt{3}}x, \ y = 0$

(C) $y = \frac{\sqrt{3}}{2}x + 1, \ y = -\frac{\sqrt{3}}{2}x - 1$

(D) $y = \sqrt{3}x, \ y = 0$

Sol. (D)

Equation of QR and PR are

$y = \sqrt{3}x$ and $y = 0$

PHYSICS

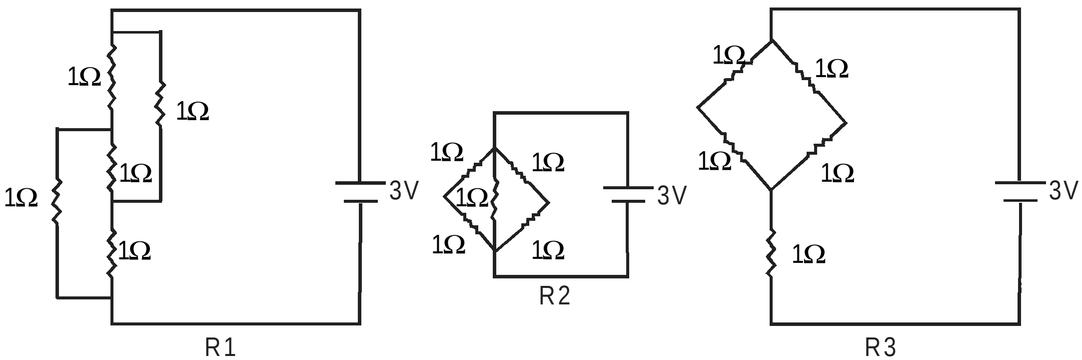
PART II

SECTION – I

Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

24. Figure shows three resistor configurations R1, R2 and R3 connected to 3V battery. If the power dissipated by the configurations R1, R2 and R3 is P1, P2 and P3, respectively, then
Figure :



(A) P1 > P2 > P3 (B) P1 > P3 > P2 (C) P2 > P1 > P3 (C) P3 > P2 > P1

Sol. (C)

After solving $R_1 = 1\Omega$

$$R_2 = \frac{1}{2}\Omega$$

$$\& \ R_3 = 2\Omega$$

& we know $P = \frac{V^2}{R}$

$P_2 > P_1 > P_3$

25. Which one of the following statements is **WRONG** in the context of X-rays generated from a X-ray tube?

(A) Wavelengh of characteristic X-rays decreases when the atomic number of the target increases

(B) Cut-off wavelength of the continuous X-rays depends on the atomic number of the target

(C) Intensity of the characteristic X-rays depends on the electrical power given to the X-rays tube

(D) Cut-off wavelength of the continuous X-rays depends on he energy of the electrons in the X-rays tube

Sol. (B)

Theoritical

26. Student I, II and III perform an experiment for measuring the acceleration due to gravity (g) using a simple pendulum. They use different length of the pendulum and/or record time for different number of oscillations. The observations area shown in the table .

Least count for length = 0.1 cm

Least count for time = 0.1 s

Student	Length of the pendulum (cm)	Number of oscillations (n)	Total time for (n) oscillations (s)	Time period (s)
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If E_I, E_{II} and E_{III} are the percentage errors in g, i.e., $\left(\frac{\Delta g}{g}\times 100\right)$ for students, I, II and II, respectively,

(A) $E_I = 0$

(B) E_I is minimum

(C) $E_I = E_{II}$

(D) E_{II} is maximum

Sol. (B)

We know

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad \Rightarrow T^2 = 4\pi^2\frac{l}{g}$$

$$g = 4\pi^2\frac{l}{T^2} \qquad \Rightarrow \frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$$

Exp I $$\frac{\Delta g}{g}\times 100 = \left[\frac{.1}{64} + 2\times\frac{.1}{16}\right]\times 100$$

$$= \frac{[.1+.2]\times 100}{64} = \left[\frac{30}{64}\right]$$

because No. of oscillation taken by student I is 8 as compare to 4 of student II.

$\therefore$ Student I is more accurate as compare to II

So $E_I \neq E_2$

Exp II:

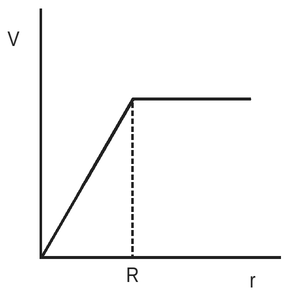
$$\frac{\Delta g}{g}\times 100 = \left[\frac{.1}{20} + 2\frac{.1}{9}\right]\times 100$$

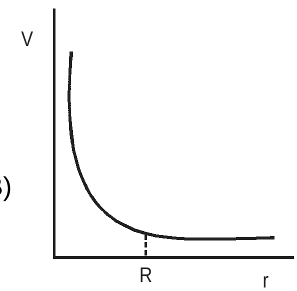
$$= \frac{.9+4}{1000} = \frac{4.9}{1000}\times 100 = \frac{490}{1000}$$

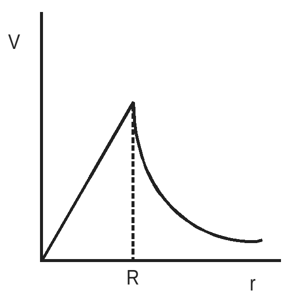
27. A spherically symmetric gravitational system of particles has a mass density

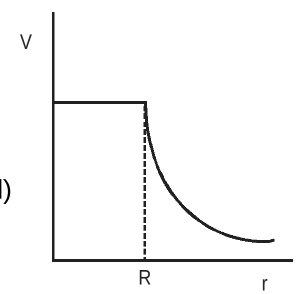
$$\rho = \begin{cases} \rho_o & \text{for } r \leq R \\ 0 & \text{for } r > R \end{cases}$$

where ρ_o is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed V as a function of distance r $(0 < r < \infty)$ from the centre of the system is represented by

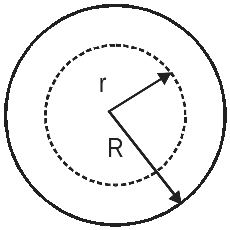
(A)


(B)


(c)


(d)


Sol. **(C)**



$$\rho = \begin{cases} \rho_o & r \le R \\ 0 & r > R \end{cases}$$

Gravitational field inside the sphere of distance r from the centre

$$g = \frac{GM}{R^3} r$$

where $M = \left[\frac{4}{3}\pi R^3 \rho_o\right]$

so $g = \dfrac{G\frac{4}{3}\pi R^3 \rho_o}{R^3} r$

$$g = \frac{4}{3} G\pi\rho_o r$$

$\therefore$ Inside the sphere $mg = \dfrac{mv^2}{r}$

$$\Rightarrow g = \frac{v^2}{r}$$

$$\Rightarrow \frac{4}{3}\pi G\rho_o r^2 = v^2$$

$$\Rightarrow v = \sqrt{\frac{4}{3}\pi G\rho_o}\; r$$

$\Rightarrow v \propto r$

& for r > R

$$g = \frac{GM}{r} = \frac{G \times \frac{4}{3}\pi R^3 \rho_o}{r^2} = \frac{4}{3}\frac{G\pi R^3 \rho_o}{r^2}$$

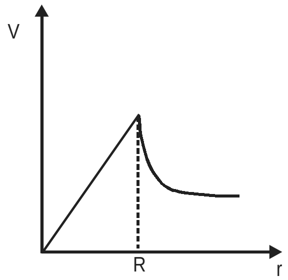
so out side the sphere

$$mg = \frac{mv^2}{r}, \quad g = \frac{v^2}{r}$$

$$\frac{4}{3}\frac{G\pi R^3\rho_o}{r^2} = \frac{v^2}{r}$$

$$\Rightarrow v \propto \frac{1}{\sqrt{r}}$$

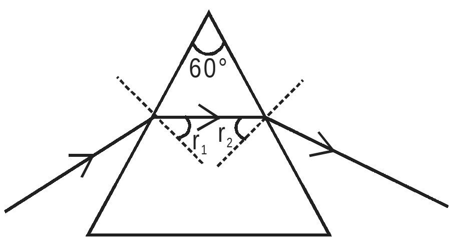
so graph will be like this



28. Two beams of red and violet colours are made to pass separately through a prism (angle of the prism is 60°). In the position of minimum deviation, the angle of refraction will be

(A) 30° for both the colours
(B) greater for the violet colour
(C) greater for the red colour
(D) equal but not 30° for both the colours

Sol. (A)



Min^m deviation $r_1 = r_2$

& we know $r_1 + r_2 = A$

$\therefore 2r_1 = 60°$

$r_1 = 30°$

It will be same for both colours.

29. An ideal gas is expanding such that PT^2 = constant. The coefficient of volume expansion of the gas is

(A) $\frac{1}{T}$ (B) $\frac{2}{T}$ (C) $\frac{3}{T}$ (D) $\frac{4}{T}$

Sol. (C)

$PT^2 = C$ ....(i)

by gas equation PV = nRT ....(ii)

from equation (i) put $p = \frac{C}{T^2}$

$$CV = nRT^3$$

$$C\frac{dV}{dT} = 3nRT^2$$

$$\frac{dV}{dT} = \frac{3nRT^2}{PT^2} \quad \because C = PT^2$$

$$= \frac{3nRT}{PT} = \frac{3PV}{PT} \quad \because nRT = PV$$

$$\frac{dV}{dT} = \frac{3V}{T}$$

$$\frac{1}{V}\frac{dV}{dT} = \frac{3}{T}$$

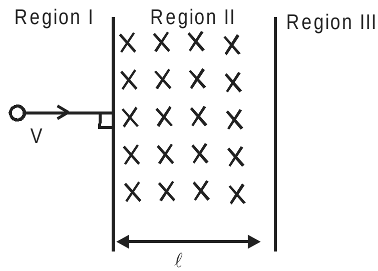
Cofficient of volume expression $\gamma = \frac{3}{T}$

Section II

Multiple Correct Answer Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices. (A), (B), (C) and (D) out of the which **ONE OR MORE** is/are correct.

30. A particle of mass m and charge q, moving with velocity V enters Region II normal to the boundary as shown in the figure. Region II has a uniform magnetic field B perpendicular to the plane of the paper. The length of the Region II is ℓ. Choose the correct choice(s).
Figure :



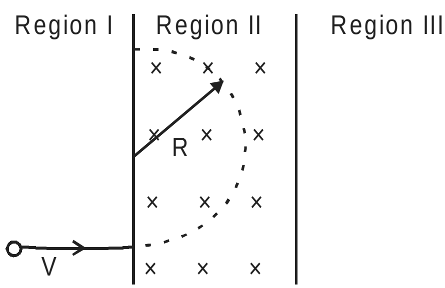
(A) The particle enters Region III only if its velocity $V > \frac{q\ell B}{m}$

(B) The particle enters Region III only if its velocity $V < \frac{q\ell B}{m}$

(C) Path length of the particle in Region II is maximum when velocity $V = \frac{q\ell B}{m}$

(D) Time spent in Region II s same for any velocity V as long as the particle returns to Region I

Sol. (A), (C), (D)



Radius of circular path inside region II is $R = \frac{mv}{qB}$

Time spent in region II as long as the particle returns to region I is

$$t = \frac{\pi R}{V} = \frac{\pi m}{qB}$$

particle enters region III if

$R > \ell$

$$\Rightarrow V > \frac{qB\ell}{m} \text{..............(i)}$$

Maximum path length is

$= \pi R = \pi \ell$

$\therefore R = \ell$

$$\Rightarrow V = \frac{qB\ell}{m} \text{...........(ii)}$$

Time spent $= \frac{\pi m}{qB}$, which is independent of V.

31. In a Young's double slit experiment, the separation between the two slits is d and the wavelength of the light is λ. The intensity of light falling on slit 1 is four times the intensity of light falling on slit 2. Choose the correct choice(s).

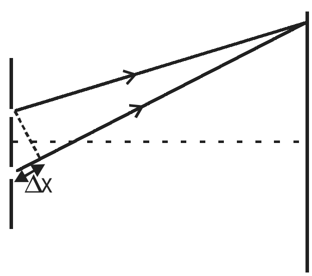
(A) If $d = \lambda$, the screen will contain only one maximum

(B) If $\lambda < d < 2\lambda$, at least one more maximum (besides the central maximum) will be observed on the screen

(C) If the intensity of light falling on slit 1 is reduced so that it becomes equal to that of slit 2, the intensities of the observed dark and bright fringes will increase

(D) If the intensity of light falling on slit 2 is increased so that if becomes equal to that of slit 1, the intensities of the observed dark and bright fringes will increase

Sol. (A), (B)



No matter what the relation between λ and d be, the central bright fringe will always be formed. Besides the central maximum, the first maximum will occur when the path difference $\Delta x = \lambda$.

If $d = \lambda, \Delta x = d \Rightarrow$ the maximum does not fall on the screen.

Hence, option (A) is correct.

If $\lambda < d < 2\lambda$, then $\frac{d}{2} < \lambda < d$

$\Rightarrow \frac{d}{2} < \Delta x < d$

$\Rightarrow$ at least one more maximum will be observed on the screen.

Initially, $I_{max} = I + 4I + 2\sqrt{I.4I} = 9I$

and $I_{min} = I + 4I - 2\sqrt{I.4I} = I$

For option (c),

$I'_{max} = I + I + 2\sqrt{I.I} = 4I$

$I'_{min} = I + I - 2\sqrt{I.I} = 0$

For option (d)

$I'_{max} = 4I + 4I + 2\sqrt{4I.4I} = 16I$

$I'_{min} = 4I + 4I - 2\sqrt{4I.4I} = 0$

Option (d) is incorrect because I_{max} has increased from 9I to 16I but I_{min} has decreased from I to 0.

32. Two balls, having linear momenta $\vec{P}_1 = P\hat{i}$ and $\vec{P}_2 = -P\hat{i}$, undergo a collision in free space. There is no external force acting on the balls. Let $\vec{P}'_1$ and $\vec{P}'_2$ be their final moment.a The following option (s) is (are) **NOT ALLOWED** for any non-zero value of p, a_1, a_2, b_1, b_2, c_1 and c_2.

(A) $\vec{P}'_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$
$\vec{P}_2 = a_2\hat{i} + b_2\hat{j}$

(B) $\vec{P}'_1 = c_1\hat{k}$
$\vec{P}_2 = c_2\hat{k}$

(C) $\vec{P}'_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$
$\vec{P}_2 = a_2\hat{i} + b_2\hat{j} - c_1\hat{k}$

(D) $\vec{P}'_1 = a_1\hat{i} + b_1\hat{j}$
$\vec{P}_2 = a_2\hat{i} + b_1\hat{j}$

Sol. (A), (D)

Since there is no external force on the balls, $\vec{p}_1 + \vec{p}_2 = \vec{p}'_1 + \vec{p}'_2 = 0$

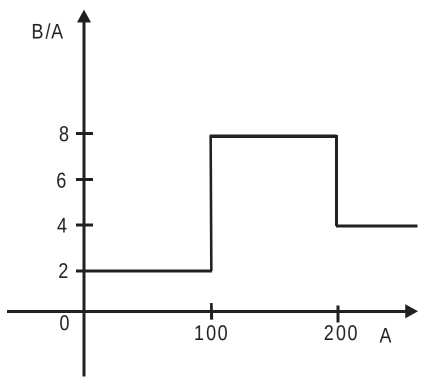
(A) $\rightarrow \vec{p}_1' + \vec{p}_2' = (a_1 + a_2)\hat{i} + (b_1 + b_2)\hat{j} + c_1\hat{k} \neq 0 \quad \because c_1 \neq 0$

(B) $\vec{p}_1' + \vec{p}_2' = (c_1 + c_2)\hat{k} = 0$ if $c_1 = -c_2$

(C) $\vec{p}_1' + \vec{p}_2' = (a_1 + a_2)\hat{i} + (b_1 + b_2)\hat{j} = 0$ if $a_1 = -a_2$ & $b_1 = -b_2$

(D) $\vec{p}_1' + \vec{p}_2' = (a_1 + a_2)\hat{i} + 2b_1\hat{j} \neq 0 \quad \because b_1 \neq 0$

33. Assume that the nuclear binding energy per nucleon (B/A) versus mass number (A) is as shown inthe figure. Use this plot to choose the correct choice(s) given below.
Figure:



(A) Fusion of nuclei with mass numbers lying in the range of 1 < A < 50 will release energy
(B) Fusion of two nuclei with mass numbers lying in the range of 51 < A < 100 will release energy
(C) Fission of a nucleus lying in the mass range of 100 < A < 200 will release energy when broken into two equal fragments
(D) Fission of a nucleus lying in the mass range of 200 < A < 260 will release energy when broken into two equal fragments

Sol. (B), (D)

(A) $\rightarrow$ Mass number of the resultant nucleus will be < 100. Thus, no change in B/A. Hence no energy released.

(B) $\rightarrow$ Mass number of the resultant nucleus will lie between 100 and 200. The B/A will increases $\Rightarrow$ resultant nuclues is more stable $\Rightarrow$ energy is released.

(C) $\rightarrow$ The daughter nuclei will have 50 < A < 100 $\Rightarrow$ B/A has decreased $\Rightarrow$ stability has decreased $\Rightarrow$ no release of energy.

(D) $\rightarrow$ The daughter nuclei will have 100 < A < 130 $\Rightarrow$ B/A has increased $\Rightarrow$ stability has increased $\Rightarrow$ energy is released.

SECTION - III

Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

34. STATEMENT-1

In a Meter Bridge experiment, null point for an unknown resistance is measured. Now, the unknown resistance is put inside an enclosure maintained at a higher temperature. The null point can be obtained at the same point as before by decreasing the value of the standard resistance.

and

STATEMENT-2

Resistance of a metal increases with increase in temperature.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol. (C)

Resistance of a metal increases with increase in temperature. **So statement 2 is true.** If unknown resistence increases, then the standard resistance must be **increased** to keep the ratio fixed (the null point remains the same).

SO STATEMENT 1 IS FALSE

35. STATEMENT-1

An astronaut in an orbiting space station above the Earth experiences weightlessness.

and

STATEMENT-2

An object moving around the Earth under the influence of Earth's gravitational force is in a state of 'free-fall'.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol. (A) In the frame of reference of the space station, the gravitational force balances the centrifugal force and hence the normal reaction is zero.
Hence statement 1 is true.
As far as weightlessness is concerned above situation is equivalent to a state of free fall.
Hence 2 is true and an explanation for 1.

36. STATEMENT-1
The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

and

STATEMENT-2
In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol. (A)

This is a common observation.

Statement I is true.
$A_1 V_1 = A_2 V_2$ speed speed decreases as the fluid moves upward, due to conservation of energy. Hence area of cross section increases.
Speed increases as the fluid moves down ward, due to conservation of energy y. Hence area of cross-section decreases.
Statement II is true and is a correct explanation of I

37. STATEMENT-1

Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.

and

STATEMENT-2

By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol. (D)

If linear acceleration along the incline is a and length of the incline is

l, $\ell, \ell = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2\ell}{a}}$

$$= \sqrt{\frac{2\ell}{\alpha r}}$$

$$= \sqrt{\frac{2\ell}{r}} \times \sqrt{\frac{I}{mg\sin\theta}}$$

Hence lower the moment of inertia, lower is the time taken. Hence solid cylinder will reach the bottom first.

Statement I is false

Using conservation of energy **statement II is true**

SECTION - IV

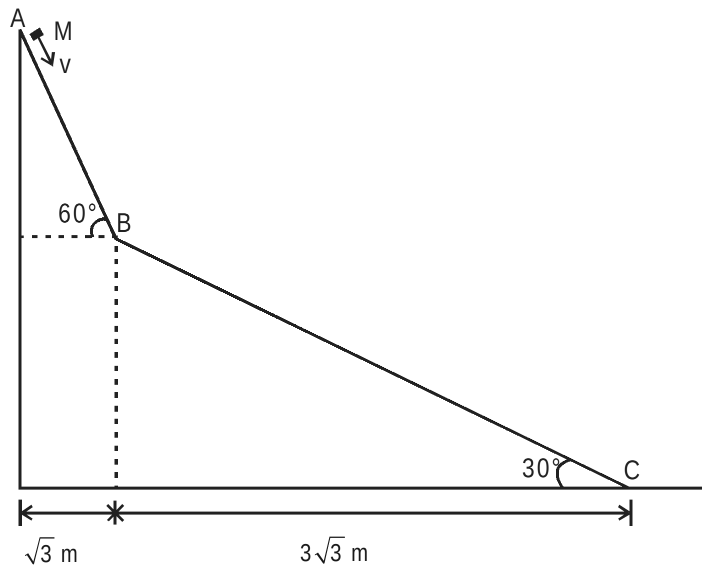
Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 38 to 40

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure. The angle of the incline suddenly changes from 60° to 30° at point B. The block is initially at rest at A. Assume that collisions between the block and the incline are totally inelastic (g = 10 m/s^2).

Figure :



38. The speed of the block at point B immediately after it strikes the second incline is

(A) $\sqrt{60}$ m/s (B) $\sqrt{45}$ m/s (C) $\sqrt{30}$ m/s (D) $\sqrt{15}$ m/s

Sol. (B)

Let 'V' be the speed at point B just before it strikes the second incline.

$$\therefore \frac{1}{2}MV^2 = Mg\left(\sqrt{3}\tan 60^\circ\right)$$

$$V = \sqrt{6g} \text{ m/s}$$

Let assume 'X' axis along the incline at 30° and 'Y' axis perpendicular to it.

$\therefore$ component of V along 'X'-direction $V_X = V\cos 30° = \frac{\sqrt{3}\ V}{2}$

Component of V along 'Y'-direction $V_Y = -V\sin 30° = -\frac{V}{2}$

After the inelastic collision with second incline, Y-component will be zero and 'X'-component will remain unchanged.

$\therefore$ After collision velocity of the block, $V' = \frac{\sqrt{3}V}{2}$ and is along 'X'-direction

or, $v' = \frac{\sqrt{3}V}{2} = \frac{\sqrt{3}\sqrt{6g}}{2} = \sqrt{45}$ m/s

39. The speed of the block at point C, immediately before it leaves the second incline is

(A) $\sqrt{120}$ m/s (B) $\sqrt{105}$ m/s (C) $\sqrt{90}$ m/s (D) $\sqrt{75}$ m/s

Sol. (B)

From conservation of energy

$$\frac{1}{2}MV_c^2 = Mg\,(3\sqrt{3}\tan 30°) + \frac{1}{2}MV'^2$$

$$\Rightarrow V_c^2 = 6g + V'^2$$

= 60 + 45 = 105

$$V_c = \sqrt{105}$$

40. If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point B, immediately after it strikes the second incline is

(A) $\sqrt{30}$ m/s (B) $\sqrt{15}$ m/s (C) 0 (D) $-\sqrt{15}$ m/s

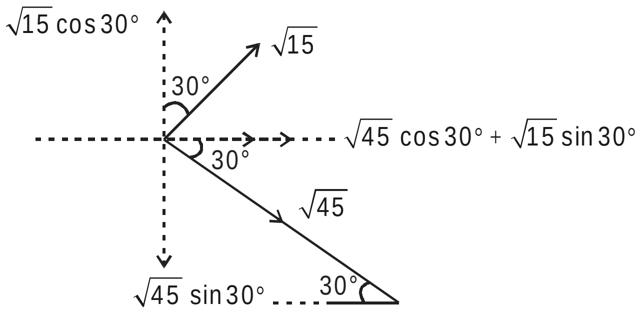
Sol. (C)

If collision is perfectly elastic, x-and y-components of velocity of the block after collision, at B

$$V_{B,X} = V_X = \frac{\sqrt{3}V}{2}V = \sqrt{45} \text{ m/s}$$

$$V_{B,\gamma} = -V_Y = \frac{V}{2} = \sqrt{15} \text{ m/s}$$

vertical component (upward) of the velocity of the block,



$$= \sqrt{15}\cos 30° - \sqrt{45}\sin 30°$$

$$= 0$$

Paragraph for Question Nos. 41 to 43

In a mixture of H – He^+ gas (He^+ is singly ionized He atom), H atoms and He^+ ions are excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to He^+ ions (by collisions). Assume that the Bohr model of atom is exactly valid.

41. The quantum number n of the state finally populated in He^+ ions is

(A) 2 (B) 3 (C) 4 (D) 5

Sol. (C)

Initially,

$$E_{n=2,H} = -\frac{13.6}{2^2}.1^2 = -\frac{13.6}{4}\text{eV}$$

$$E_{n=2,He^+} = -\frac{13.6}{2^2}.2^2 = -13.6\text{eV}$$

Excitation energy of H = $E_{n=2,H} - E_{n=1,H}$

$$= -\frac{13.6}{4} - \left(-\frac{13.6}{1}\right)$$

$$=\frac{3}{4}\times 13.6\text{eV}$$

Energy of H_e^+ after collisions,

$$E_{H_e^+}=\left(-13.6+\frac{3}{4}\times 13.6\right)\text{eV}$$

$$=-13.6\left(1-\frac{3}{4}\right)\text{eV}$$

$$=-\frac{13.6}{4}\text{eV} \quad =-\frac{13.6}{4^2}\times 4$$

$$=E_{n=4,H_e^+}$$

42. The wavelength of light emitted in the visible region by He^+ ions after collisions with H atoms is

(A) 6.5×10^{-7} m (B) 5.6×10^{-7} m (C) 4.8×10^{-7} m (D) 4.0×10^{-7} m

Sol. (C)

Due to transition from n = 4 to lower orbits, H_e^+ will emit radiations.

$$13.6\times Z^2_{H_e^+}\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]=\frac{hc}{\lambda}$$

$$\Rightarrow \lambda=\frac{hc}{13.6\times 4\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)};$$

$$=\frac{1242\times 10^{-9}}{13.6\times 4\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)}\text{m}$$

Putting transition from $n_2 = 4$ to $n_1 = 3$

$$\lambda=\frac{1242\times 10^{-9}}{13.6\times 4\left(\frac{1}{9}-\frac{1}{16}\right)}=\frac{22.8\times 144}{7}\times 10^{-9} \simeq 4.7\times 10^{-7}\text{m}$$

So transition will lie in visible region.

For other transitions, λ is less than 300 nm.

43. The ratio of the kinetic energy of the n = 2 electron for the H atom to that of He^+ ion is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2

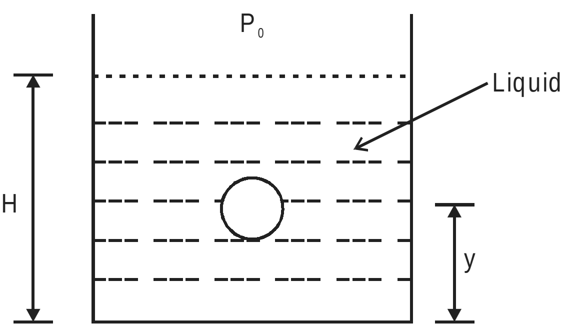
Sol. (C)

$$\frac{(K.E.)_{H,n=2}}{(K.E.)_{He^+,n=2}} = \frac{(T.E.)_{H,n=2}}{(T.E.)_{He^+,n=2}} = \frac{Z_H^2}{Z_{He^+}^2} = \frac{1}{4}$$

Paragraph for Question Nos. 44 to 46

A small spherical monoatomic ideal gas bubble $\left(\gamma = \frac{5}{3}\right)$ is trapped inside a liquid of density ρ_ℓ (see figure). Assume that the bubble does not exchange any heat with the liquid. The bubble contains n moles of gas. The temperature of the gas when the bubble is at the bottom is T_0, the height of the liquid is H and the atmospheric pressure is P_0 (Neglect surface tension).

Figure :



44. As the bubble moves upwards, besides the buoyancy force the following forces are acting on it

(A) Only the force of gravity

(B) The force due to gravity and the force due to the pressure of the liquid

(C) The force due to gravity, the force due to the pressure of the liquid and the force due to viscosity of the liquid

(D) The force due to gravity and the force due to viscosity of the liquid

Sol. (A)

Viscous force if present, will generate heat due to friction between bubble and liquid and hence heat exchange between the bubble & liquid contrary to what is given in the question. Hence no viscous forces are present.

45. When the gas bubble is at a height y from the bottom, its temperature is

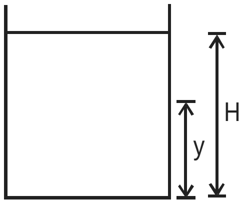
(A) $T_0\left(\frac{P_0+\rho_\ell gH}{P_0+\rho_\ell gy}\right)^{2/5}$

(B) $T_0\left(\frac{P_0+\rho_\ell g(H-y)}{P_0+\rho_\ell gH}\right)^{2/5}$

(C) $T_0\left(\frac{P_0+\rho_\ell gH}{P_0+\rho_\ell gy}\right)^{3/5}$

(D) $T_0\left(\frac{P_0+\rho_\ell g(H-y)}{P_0+\rho_\ell gH}\right)^{3/5}$

Sol. (B) $PV = nRT \rightarrow$ ideal gas equation.



$(P_0 + PgH)V_0 = nRT_0$

$PV = nRT\ [P = P_0 + P_\ell g(H-y)]$

also, $PV^\gamma = \text{const.}$

$\Rightarrow (P_0 + P_\ell gH)V_0^\gamma = [P_0 + P_\ell g(H-y)]V^\gamma$

$\Rightarrow V = \left[\frac{P_0 + P_\ell gH}{P_0 + P_\ell g(H-y)}\right]^{1/\gamma} V_0$

$\Rightarrow T = \frac{[P_0 + P_\ell g(H-y)]^{1-\frac{1}{\gamma}}}{[P_0 + P_\ell gH]^{1-\frac{1}{\gamma}}}\ T_0;\ \left(\gamma = \frac{5}{3}\right)$

46. The buoyancy force acting on the gas bubble is (Assume R is the universal gas constant)

(A) $\rho_\ell nRgT_0 \dfrac{(P_0+\rho_\ell gH)^{2/5}}{(P_0+\rho_\ell gy)^{7/5}}$

(B) $\dfrac{\rho_\ell nRgT_0}{(P_0+\rho_\ell gH)^{2/5}[P_0+\rho_\ell g(H-y)]^{3/5}}$

(C) $\rho_\ell nRgT_0 \dfrac{(P_0+\rho_\ell gH)^{3/5}}{(P_0+\rho_\ell gy)^{8/5}}$

(D) $\dfrac{\rho_\ell nRgT_0}{(P_0+\rho_\ell gH)^{3/5}(P_0+\rho_\ell g(H-y)^{2/5}}$

Sol. (B)

$$F_{Buoyancy} = \rho_l Vg = \rho g\left[\frac{(P_0+\rho gH)}{P_0+\rho g(H-y)}\right]^{1/\gamma} V_0$$

Using $V_0 = \dfrac{nRT_0}{(P_0+\rho gH)}$

$$F_B = \frac{\rho_l gnRT_0\left[P_0+\rho gH\right]^{-1+\frac{1}{\gamma}}}{\left[P_0+\rho g(H-y)\right]^{1/r}};\ \gamma\frac{5}{3}$$

CHEMISTRY

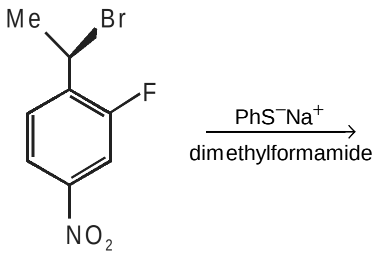
PART III

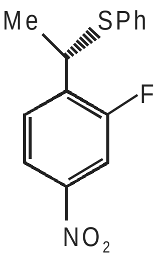
Section – I

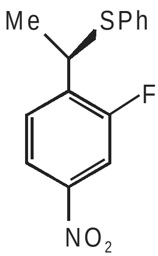
Straight Objective Type

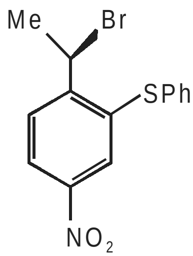
This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which only one is correct.

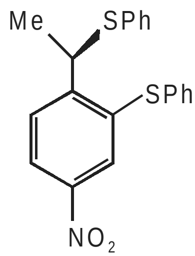
47. The major product of the following reaction is



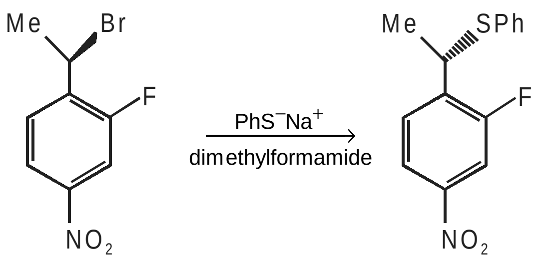
(A) 

(B) 

(C) 

(D) 

Sol. (A)



S_N2 reaction takes place in which nucleophile attacks from backside of leaving group due to which inversion takes place.

48. Aqueous solution of $Na_2S_2O_3$ on reaction with Cl_2 gives

(A) $Na_2S_4O_6$ (B) $NaHSO_4$ (C) NaCl (D) NaOH

Sol. (B)

$$Na_2S_2O_3 + 4Cl_2 + 5H_2O \longrightarrow 2NaHSO_4 + 8HCl$$

49. Hyperconjugation involves overlap of the following orbitals

(A) $\sigma - \sigma$ (B) $\sigma - p$ (C) $p - p$ (D) $\pi - \pi$

Sol. (B)

Hyperconjunction involves overlap of $\sigma - p$ orbitals.

50. 2.5 mL of $\frac{2}{5}$ weak monoacidic base $\left(K_b = 1\times10^{-12}\right)$ at 25°C is titrated with $\frac{2}{15}$ M HCl in water at 25°C. The concentration of H^+ at equivalences point is $\left(K_w = 1\times10^{-14} \text{ at } 25°C\right)$

(A) 3.7×10^{-13} M (B) 3.2×10^{-7} M (C) 3.2×10^{-2} M (D) 2.7×10^{-2} M

Sol. (D)

2.5 ml, $\frac{2}{5}$ M BOH + $\frac{2}{15}$ M HCl, Vml

At equivalence point.

$$M_1V_1 = M_2V_2$$

$$2.5\times\frac{2}{5} = V\times\frac{2}{15}$$

$$V = 2.5\times\frac{2}{5}\times\frac{15}{2} = 7.5 \text{ ml}$$

Number of moles of BCl formed = 1×10^{-3}

Total volume = $(2.5+7.5)\text{ml} = 10\text{ml} = 10\times10^{-3}\text{L}$

$$[BCl] = \frac{1}{10} = 0.1 \text{ M}$$

$BCl \rightleftharpoons B^+ + Cl^-$

$B^+ + H_2O \rightleftharpoons BOH + H^+$

C(1–h) Ch Ch

$$K_h = \frac{Ch^2}{(1-h)} = \frac{K_w}{K_b}$$

$$\frac{0.1h^2}{1-h} = \frac{10^{-14}}{10^{-12}} = 10^{-2}$$

$$\text{or } \frac{h^2}{1-h} = \frac{1}{10} \text{ or } h = 0.27$$

So $[H^+] = Ch = 0.1 \times 0.27 = 2.7 \times 10^{-2}$ M

51. Native silver metal forms a water soluble complex with a dilute aqueous solution of NaCN in the presence of

(A) nitrogen (B) oxygen (C) carbon dioxide (D) argon

Sol. (B)

Ag gets oxidised in presence of oxygen and dissolves forming the complex $Na\left[Ag(CN)_2\right]$.

52. Under the same reaction conditions, initial concentration of 1.386 mol dm^{-3} of a substance becomes half in 40 seconds and 20 seconds through first order and zero order kinetics, respectively. Ratio $\left(\frac{k_1}{k_0}\right)$ of the rate constants for first order (k_1) and zero order (k_0) of the reaction is

(A) $0.5\ mol^{-1}\ dm^3$ (B) $1.0\ mol\ dm^{-3}$ (C) $1.5\ mol\ dm^{-3}$ (D) $2.0\ mol^{-1}\ dm^3$

Sol. (A)

$$[A]_i = 1.38\ L\ mol/L \xrightarrow{\text{first order}} \frac{1.38}{2} \text{ in } 40 \text{ sec.}$$

$$k_1 = \frac{0.693}{t_{1/2}} = \frac{0.693}{40}$$

for zero order

$$t_{1/2} = 20 \text{ sec} = \frac{[A]_o}{2k_o}$$

$$k_0 = \frac{a}{2t_{1/2}} = \frac{1.38}{2\times 20}$$

$$\Rightarrow \frac{k_1}{k_0} = \frac{\frac{0.693}{40}}{\frac{1.386}{40}} = \frac{0.693}{1.38} = 0.502 \text{ mol}^{-1}\text{ dm}^3$$

Section – II

Multiple Correct Answers Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B) (C) and (D), out of which one or more is/are correct.

53. A solution of colourless salt H on boiling with excess NaOH produces a non-flammable gas. The gas evolution ceases after sometime. Upon addition of Zn dust to the same solution, the gas evolution restarts. The colourless salt(s) H is (are)

(A) NH_4NO_3 (B) NH_4NO_2 (C) NH_4Cl (D) $(NH_4)_2SO_4$

Sol. (A, B)

On addition of NaOH all ammonium salts give ammonia, but since the evolution of NH_3 resumes on addition of Zn, the salts can be NO_3^- or NO_2^- which on reduction give ammonia.

54. A gas described by van der Waals equation

(A) behaves similar to an ideal gas in the limit of large molar volumes

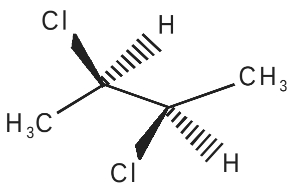
(B) behaves similar to an ideal gas in the limit of large pressures

(C) is characterised by van der Waals coefficients that are dependent on the identity of the gas but are independent of the temperature

(D) has the pressure that is lower than the pressure exerted by the same gas behaving ideally.

Sol. (A, C, D)

55. The correct statement(s) about the compound given below is (are)



(A) The compound is optically active

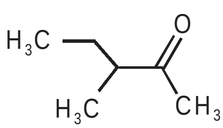
(B) The compound possesses centre of symmetry

(C) The compound possesses plane of symmetry

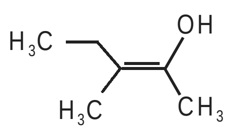
(D) The compound possesses axis of symmetry

Sol. (C, D)

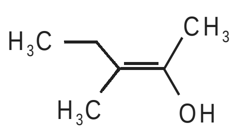
56. The correct statement(s) concerning the structures E, F and G is (are)



(E)



(F)



(G)

(A) E, F and G are resonance structure

(B) E, F and G are tautomers

(C) F and G are geometrical isomers

(D) F and G are diastereomers

Sol. (B, C, D)

Section – III

Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which only one is correct.

57. **Statement 1:** Bromobenzene upon reaction with Br_2/Fe gives 1,4-dibromobenzene as the major product

Statement 2: In bromobenzene, the inductive effect of the bromo group is more dominant than the mesomeric effect in directing the incoming electrophile.

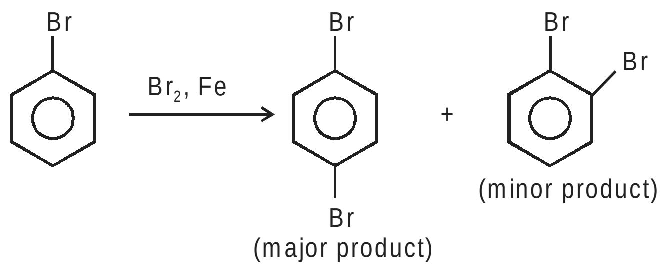
(A) **Statement 1** is true, **statement 2** is true, **statement 2** is a correct explanation for **statement 1.**

(B) **Statement 1** is true, **statement 2** is true, **statement 2** is **not** a correct explanation for **statement 1.**

(C) **Statement 1** is true, **statement 2** is false.

(D) **Statement 1** is false, **statement 2** is true.

Sol. (C)



The ortho, para - directive influence of Br is due to + mesomeric effect.

58. **Statement 1:** Pb^{4+} compounds are stronger oxidizing agents than Sn^{4+} compounds.

Statement 2: The higher oxidation states for the group 14 elements are more stable for the heavier members of the group due to 'inert pair effect'.

(A) **Statement 1** is true, **statement 2** is true, **statement 2** is a correct explanation for **statement 1.**

(B) **Statement 1** is true, **statement 2** is true, **statement 2** is **not** a correct explanation for **statement 1.**

(C) **Statement 1** is true, **statement 2** is false.

(D) **Statement 1** is false, **statement 2** is true.

Sol. (C)

Pb^{+4} is less stable than Pb^{2+} where Sn^{+4} is more stable than Sn^{+2}, Pb^{+4}, therefore is a stronger oxidizing agent.

The higher oxidation states of heavier elements are less stable due to inert pair effect.

59. **Statement 1:** For every chemical reaction at equilibrium, standard Gibbs energy of reaction is zero.

Statement 2: At constant temperature and pressure, chemical reactions are spontaneous in the direction of decreasing Gibbs energy.

(A) **Statement 1** is true, **statement 2** is true, **statement 2** is a correct explanation for **statement 1.**

(B) **Statement 1** is true, **statement 2** is true, **statement 2** is **not** a correct explanation for **statement 1.**

(C) **Statement 1** is true, **statement 2** is false.

(D) **Statement 1** is false, **statement 2** is true.

Sol. (D)

For a chemical reaction at equilibrium

$\Delta G = 0, \Delta G^{o} \neq 0$

For a spontaneous process

ΔG = –ve

60. **Statement 1:** The plot of atomic number (y-axis) versus number of neutrons (x-axis) for stable nuclei shows a curvature towards x-axis from the line of 45°C slope as the atomic number is increased.

Statement 2: Proton-proton electrostatic repulsions begin to overcome attractive forces involving protons and neutrons in heavier nuclides.

(A) **Statement 1** is true, **statement 2** is true, **statement 2** is a correct explanation for **statement 1.**

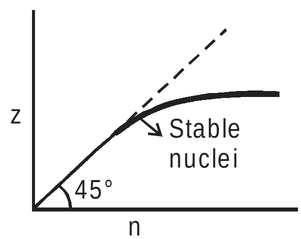
(B) **Statement 1** is true, **statement 2** is true, **statement 2** is **not** a correct explanation for **statement 1.**

(C) **Statement 1** is true, **statement 2** is false.

(D) **Statement 1** is false, **statement 2** is true.

Sol. (A)

Both statements are correct but statement 2 is not explanation of statement 1.



Section – IV

Linked Comprehension Type

This section contains 3 paragraph. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which only one is correct.

Paragraph for Question Nos. 61 to 63

Properties such as boiling point, freezing point and vapour pressure of a pure solvent change when solute molecules are added to get homogeneous solution. These are called colligative properties. Applications of colligative properties are very useful in day-to-day life. One of its examples is the use of ethylene glycol and water mixture as anti-freezing liquid in the radiator of automobiles

A solution **M** is prepared by mixing ethanol and water. The mole fraction of ethanol in the mixture is 0.9

Given : Freezing point depression constant of water $\left(K_f^{water}\right) = 1.86$ K kg mol^{-1}

Freezing point depression constant of ethanol $\left(K_f^{ethanol}\right) = 2.0$ K kg mol^{-1}

Boiling point elevation constant of water $\left(K_b^{water}\right) = 0.52$ K kg mol^{-1}

Boiling point elevation constant of ethanol $\left(K_b^{ethanol}\right) = 1.2$ K kg mol^{-1}

Standard freezing point of water = 273 K
Standard freezing point of ethanol = 155.7 K
Standard boiling point of water = 373 K
Standard boiling point of ethanol = 351.5 K
Vapour pressure of pure water = 32.8 mm Hg
Vapour pressure of pure ethanol = 40 mm Hg
Molecular weight of water = 18 g mol^{-1}
Molecular weight of ethanol = 46 g mol^{-1}

In answering the following questions, consider the solutions to be ideal dilute solutions and solutes to be non-volatile and non-dissociative.

61. The freezing point of the solution **M** is
(A) 268.7 K (B) 268.5 K (C) 234.2 K (D) 150.9 K

Sol. (D)
Let total moles be 1, then
$n_{ethanol} = 0.9$
mass of ethanol = 41.4 g = 41.4 × 10^{-3} kg
$n_{water} = 0.1$

molality, $m = \frac{0.1}{41.4} \times 10^3$ = 2.4 mol/kg.

$\Delta T_f = K_f m$ = 2 × 2.4 = 4.8

T_f (ethanol) = 155.7 – 4.8 = 150.9 K

62. The vapour pressure of the solution **M** is

(A) 39.3 mm Hg (B) 36.0 mm Hg (C) 29.5 mm Hg (D) 28.8 mm Hg

Sol. (B)

$$\frac{40 - P_S}{40} = 0.1$$

P_S = 36 mm of Hg.

63. Water is added to the solution **M** such that the mole fraction of water in the solution becomes 0.9. The boiling point of this solution is

(A) 380.4 K (B) 376.2 K (C) 375.5 K (D) 354.7 K

Sol. (B)

Let no of moles be 1

nH_2O = 0.9, $n_{ethanol}$ = 0.1

mass of H_2O = 0.9 ×18 g = 16.2 g = 0.0162 kg

$$molality = \frac{0.1}{0.0162} \text{ mol/kg} = 06.17 \text{ mol/kg}$$

$\Delta Tb = kb.m$

= 0.52 × 6.17 = 3.2

Boiling point of solution = 373 + 3.2 = 376.2 K

Paragraph for Question Nos. 64 to 66

There are some deposits of nitrates and phosphates in earth's crust. Nitrates are more soluble in water. Nitrates are difficult to reduce under the laboratory conditions but microbes do it easily. Ammonia forms large number of complexes with transition metal ions. Hybridization easily explains the ease of sigma donation capability of NH_3 and PH_3. Phosphine is a flammable gas and is prepared from white phosphorous.

64. Among the following, the correct statement is

(A) Phosphates have no biological significance in humans

(B) Between nitrates and phosphates, Phosphates are less abundant in earth's crust

(C) Between nitrates and phosphates, nitrates are less abundant in earth's crust

(D) Oxidation of nitrates is possible in soil

Sol. (C)

65. Among the following, the correct statement is

(A) Between NH_3 and PH_3, NH_3 is a better electron donor because the lone pair of electrons occupies spherical 's' orbital and is less directional

(B) Between NH_3 and PH_3, PH_3 is a better electron donor because the lone pair of electrons occupies sp_3 orbital and is more directional

(C) Between NH_3 and PH_3, NH_3 is a better electron donor because the lone pair of electrons occupies sp_3 orbital and is more directional

(D) Between NH_3 and PH_3, PH_3 is a better electron donor because the lone pair of electrons occupies spherical 's' orbital and is less directional

Sol. (C)

66. White phosphorus on reaction with NaOH gives PH_3 as one of the products. This is a

(A) dimerization reaction (B) disproportionation reaction

(C) condensation reaction (D) precipitation reaction

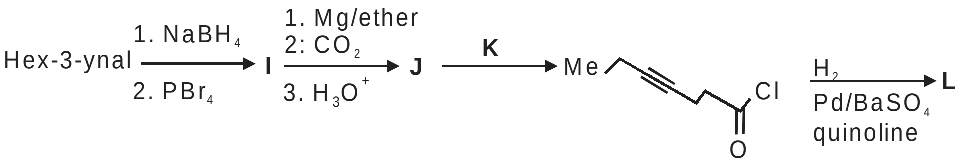
Sol. (B)

$$\overset{0}{P_4} + 3NaOH + 3H_2O \rightarrow \overset{-3}{P}H_3 + 3NaH_2\overset{+1}{P}O_2$$

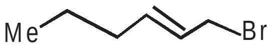
Disproportionation

Paragraph for Question Nos. 67 to 69

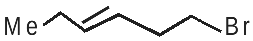
In the following reaction sequence, products **I, J** and **L** are formed. **K** represents a reagent.

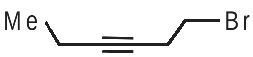


67. The structure of the product **I** is

(A) 

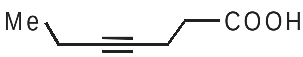
(B) 

(C) 

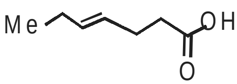
(D) 

Sol. (D)

68. The structures of compounds **J** and **K**, respectively, are

(A) 

and $SOCl_2$

(B) 

and SO_2Cl_2

(C) 

and $SOCl_2$

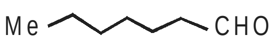
(D) 

and CH_3SO_2Sl

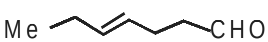
Sol. (A)

69. The structure of product **L** is

(A) 

(B) 

(C) 

(D) 

Sol. (C)

Explanation for questions 67 to 69:

$$CH_3-CH_2-C\equiv C-CH_2-CHO$$
$$\downarrow NaBH_4$$
$$CH-CH_2-C\equiv C-CH_2-CH_2OH$$
$$\downarrow PBr_3$$
$$CH_3-CH_2-C\equiv C-CH_2-CH_2-Br\ (I)$$
$$\downarrow Mg/ether$$
$$CH_3-CH_2-C\equiv C-CH_2-CH_2\,Mg.Br$$
$$\downarrow CO_2$$
$$CH_2-CH_2-C\equiv C-CH_2CH_2COOMgBr$$
$$\downarrow H_3O^+$$
$$CH_3-CH_2-C\equiv C-CH_2-CH_2COOH\ (J)$$
$$\downarrow SOCl_2\ (K)$$
$$CH_3CH_2-C\equiv C-CH_2-CH_2COCl$$
$$\downarrow H_2, Pd/BaSO_4\ quinoline$$
$$CH_3-CH_2-\overset{\overset{H}{|}}{C}=\overset{\overset{H}{|}}{C}-CH_2-CH_2-CHO\ (L)$$