MATHEMATICS

Directions : Questions number 1 to 5 are Assertion-Reason type questions. Each of these questions contains two statements : Statement-1 (Assertion) and Statement-2 (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

1. Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number", and r be the statement "x is a rational number iff y is a transcendental number".
Statement-1 :
r is equivalent to either q or p.

Statement-2 :
r is equivalent to $\sim(p \leftrightarrow \sim q)$.
(1) Statement-1 is true, Statement-2 is false
(2) Statement-1 is false, Statement-2 is true
(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1

Sol. (..)

				r	Statement – I	Statement – II
p	q	~p	~q	$\sim p \leftrightarrow q$	$q \vee p$	$\sim(p \leftrightarrow \sim q)$
F	F	T	T	F	F	T
F	T	T	F	T	T	F
T	F	F	T	T	T	F
T	T	F	F	F	T	T

r is not equivalent to either of the statements

2. In a shop there are five types of ice-creams available. A child buys six ice-creams available. A child buys six ice-creams
Statement-1 :
The number of different ways the child can buy the six ice-creams is ${}^{10}C_5$.
Statement-2 :
The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row.
(1) Statement-1 is true, Statement-2 is false
(2) Statement-1 is false, Statement-2 is true
(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1

Sol. (2)
$a + b + c + d + e = 6$
a ice creams of type I, b off type II, . . .

$\therefore$ total ways = ${}^{10}C_4$

$6A'S$ and $4B'S$ can be arranged in a row in $\dfrac{10!}{6!\,4!} = {}^{10}C_4$ ways

3. Statement-1 :

$$\sum_{r=0}^{n} (r+1)\ {}^{n}C_r = (n+2)2^{n-1}.$$

Statement-2 :

$$\sum_{r=0}^{n} (r+1)\ {}^{n}C_r x^r = (1+x)^n + nx\,(1+x)^{n-1}.$$

(1) Statement-1 is true, Statement-2 is false
(2) Statement-1 is false, Statement-2 is true
(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1

Sol. (3)

$$\sum_{r=0}^{n} (r+1)\ {}^{n}C_r = \sum_{r=0}^{n} r\ {}^{n}C_r + \sum_{r=0}^{n} {}^{n}C_r$$

$$= n \times 2^{n-1} + 2^n$$

$$= 2^{n-1}(n+2)$$

Statement-1 is true

$$\sum_{r=0}^{n} (r+1)\ {}^{n}C_r\ x^r = \sum_{r=0}^{n} r\ {}^{n}C_r\ x^r + \sum_{r=0}^{n} {}^{n}C_r\ x^r$$

$$= nx(1+x)^{n-1} + (1+x)^n$$

Statement-2 is true & Statement-2 explains Statement-1

4. Statement-1 :

For every natural number $n \geq 2$,

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}} > \sqrt{n}.$$

Statement-2 :

$$\sqrt{n(n+1)} < n+1.$$

(1) Statement-1 is true, Statement-2 is false
(2) Statement-1 is false, Statement-2 is true
(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1

Sol. (3) $n > 2$

$$1 > \frac{1}{\sqrt{n}}$$

$$\frac{1}{\sqrt{2}} > \frac{1}{\sqrt{n}}$$

$$\frac{1}{\sqrt{3}} > \frac{1}{\sqrt{n}}$$

$$\frac{1}{\sqrt{4}} > \frac{1}{\sqrt{n}}$$

$$\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{n}}$$

Adding them

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{n}} > \sqrt{n}$$

Statement-1 is True

$$\sqrt{n(n+1)} < n+1$$

$$\Rightarrow \sqrt{n} < \sqrt{n+1} \quad \text{(True)}$$

$$\frac{1}{\sqrt{n}} > \frac{1}{\sqrt{n+1}}$$

5. Let A be a 2 × 2 matrix with real entries. Let I be the 2 × 2 identity matrix. Denote by tr(A), the sum of diagonal entries of A. Assume that A = I.

Statement-1 :

If $A \neq I$ and $A \neq -I$, then det $A = -1$.

Statement-2 :

If $A \neq I$ and $A \neq -I$, then $\text{tr}(A) \neq 0$.

(1) Statement-1 is true, Statement-2 is false
(2) Statement-1 is false, Statement-2 is true
(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1

Sol. (1) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} a^2+bc & b(a+d) \\ (a+d)c & bc+d^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$a^2+bc=1, \quad b(a+d)=0$

$(a+d)c=0, \quad bc+d^2=1$

if b = 0 $\quad \Rightarrow a^2 = d^2 = 1$

now if a + d = 0

a = 1, d = –1 or a = –1, d = 1

$\begin{pmatrix} 1 & 0 \\ c & -1 \end{pmatrix}$ or $\begin{pmatrix} -1 & 0 \\ c & 1 \end{pmatrix}$

Then $A \neq I$ or $-I$

Then tr(A) = 0, $|A| = -1$, if $a+d \neq 0, c = 0$

then A = I or –I

6. The statement $p \to (q \to p)$ is equivalent to

(1) $p \to (q \leftrightarrow p)$
(2) $p \to (p \to q)$
(3) $p \to (p \vee q)$
(4) $p \to (p \wedge q)$

Sol (3)

p	q	$p \vee q$	$q \to q$	$p \to (q \to q)$	$p \to (p \vee q)$
F	F	F	T	T	T
F	T	T	F	T	T
T	F	T	T	T	T
T	T	T	T	T	T

7. The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3}+\tan^{-1}\frac{2}{3}\right)$ is

(1) $\frac{5}{17}$

(2) $\frac{6}{17}$

(3) $\frac{3}{17}$

(4) $\frac{4}{17}$

Sol (2) $\cot\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right)$

$$\cot\left(\tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}\right)$$

$$\cot\left(\tan^{-1}\frac{17}{6}\right)$$

$$=\frac{6}{17}$$

8. The differential equation of the family of circles with fixed radius 5 units and centre on the line y = 2 is

(1) $(x-2)^2\,y'^2=25-(y-2)^2$

(2) $(x-2)\,y'^2=25-(y-2)^2$

(3) $(y-2)\,y'^2=25-(y-2)^2$

(4) $(y-2)^2\,y'^2=25-(y-2)^2$

Sol (4) Let centre be (h, 2)
equation of circle becomes

$$(x-h)^2+(y-2)^2=25$$

$$2(x-h)+2(y-2)y'=0$$

$$x-h=(2-y)\,y'$$

$$(y-2)^2(y')^2+(y-2)^2=25$$

9. Let $I=\int_0^1 \frac{\sin x}{\sqrt{x}}dx$ and $J=\int_0^1 \frac{\cos}{\sqrt{x}}dx.$ Then which one of the following is true ?

(1) $I>\frac{2}{3}$ and $J<2$

(2) $I>\frac{2}{3}$ and $J>2$

(3) $I<\frac{2}{3}$ and $J<2$

(4) $I<\frac{2}{3}$ and $J>2$

Sol (3) $I=\int_0^1 \frac{\sin x}{\sqrt{x}}dx, J=\int_0^1 \frac{\cos x}{\sqrt{x}}dx$

for 0 < x < 1 sin x < x

$\Rightarrow \frac{\sin x}{\sqrt{x}}<\sqrt{x}$

$\Rightarrow \int_0^1 \frac{\sin x}{\sqrt{x}}dx<\int_0^1 \sqrt{x}\,dx$

$\Rightarrow I<\frac{2}{3}$

$\int_0^1 \frac{\cos x}{\sqrt{x}}dx<\int_0^1 \frac{1}{\sqrt{x}}dx$

J < 2

10. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to

(1) $\frac{4}{3}$

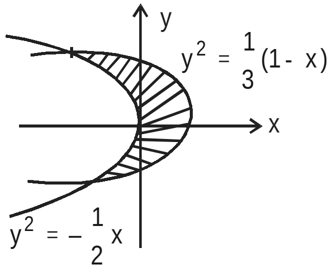
(2) $\frac{5}{3}$

(3) $\frac{1}{3}$

(4) $\frac{2}{3}$

Sol (1) $y^2=-\frac{1}{2}x$

$y^2=\frac{1}{3}(1-x)$



On solving

$$-\frac{1}{2}x = \frac{1}{3}(1-x)$$

$$-\frac{1}{2}x = \frac{1}{3}(1-x)$$

$$\Rightarrow \quad \frac{1}{3}x - \frac{1}{2}x = \frac{1}{3}$$

$$x = \frac{-1}{3}\times 6$$

$$= -2$$

$$\therefore \quad y^2 = -\frac{1}{2}x(-2)$$

$$y^2 = 1$$

$$y = \pm 1$$

Required Area

$$= \int_0^1 (x_1 - x_2)\, dy$$

$$= 2\int_0^1 [(1-3y^2) - (-2y^2)]\, dy$$

$$= 2\int_0^1 (1-y^2)\, dy$$

$$= 2\left[y - \frac{y^3}{3}\right]_0^1$$

$$= 2\left[1 - \frac{1}{3}\right]$$

$$= \frac{4}{3}$$

11. The value of $\sqrt{2}\int \frac{\sin x\, dx}{\sin\left(x - \frac{\pi}{4}\right)}$ is

(1) $x - \log|\cos(x - \frac{\pi}{4})| + c$

(2) $x + \log|\cos(x - \frac{\pi}{4})| + c$

(3) $x - \log|\sin(x - \frac{\pi}{4})| + c$

(4) $x + \log|\sin(x - \frac{\pi}{4})| + c$

Sol (4) $\sqrt{2}\int \frac{\sin x}{\sin\left(x - \frac{\pi}{4}\right)} dx$

$$= \sqrt{2}\int \frac{\sin\left(\left(x - \frac{\pi}{4}\right) + \frac{\pi}{4}\right)}{\sin\left(x - \frac{\pi}{4}\right)} dx$$

$$= \sqrt{2}\int \frac{\sin\left(x - \frac{\pi}{4}\right)\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\left(x - \frac{\pi}{4}\right)}{\sin\left(x - \frac{\pi}{4}\right)} dx$$

$$= \sqrt{2}\,\cos\frac{\pi}{4}\int dx + \sqrt{2}\sin\frac{\pi}{4}\int \cot\left(x - \frac{\pi}{4}\right) dx$$

$$= x + \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c$$

12. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60°. He moves away from the pole along the line BC to a point D such that CD = 7 m. From D the angle of elevation of the point A is 45°. Then the height of the pole is

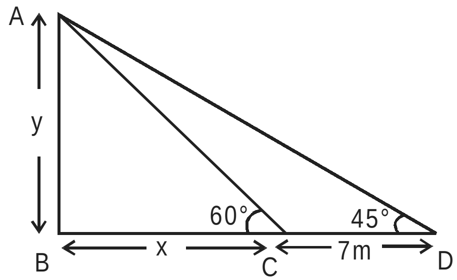
(1) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}+1}$m

(2) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}-1}$m

(3) $\frac{7\sqrt{3}}{2}(\sqrt{3}+1)$ m

(4) $\frac{7\sqrt{3}}{2}(\sqrt{3}-1)$ m

Sol (3) Let height of pole is y m



$\therefore \quad \tan 60° = \frac{y}{x}$

$\Rightarrow \quad x = \frac{y}{\sqrt{3}}$

& $\tan 45° = \frac{y}{x+7}$

$\Rightarrow \quad x + 7 = y$

$\Rightarrow \quad \frac{y}{\sqrt{3}} + 7 = y$

$\Rightarrow \quad y\left(1 - \frac{1}{\sqrt{3}}\right) = 7$

$\Rightarrow \quad y = \frac{7\sqrt{3}}{\sqrt{3}-1}$

$y = \frac{7\sqrt{3}}{2}(\sqrt{3}+1)$ m

13. How many real solutions does the equation $x^7 + 15x^5 + 16x^3 + 30x - 560 = 0$ have ?

(1) 5 (2) 7

(3) 1 (4) 3

Sol (3) $f(x) = x^7 + 14\,x^5 + 16\,x^3 + 30x - 560$

$\therefore$ $f'(x) > 0$

$\Rightarrow$ f(x) is increasing

$\Rightarrow$ f(x) = 0 has only one solution

14. Let $f(x) = \begin{cases} (x-1)\sin\dfrac{1}{x-1} & \text{if } \neq 1 \\ 0 & \text{if } x = 1 \end{cases}$ Then which one of the following is true ?

(1) f is differentiable at x = 1 but not at x = 0

(2) f is neither differentiable at x = nor at x = 1

(3) f is differentiable at x = 0 and at x = 1

(4) f is differentiable at x = 0 but not at x = 1

Sol (4) $f'(x) = \sin\dfrac{1}{x-1} - \dfrac{1}{x-1}\cos\dfrac{1}{x-1}$

f(x) is differentiable at x = 0

but not differentiable at x = 1

15. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is

(1) 4 (2) –4

(3) –12 (4) 12

Sol (4) $a + ar = 12$

$a(1 + r) = 12$

$ar^2 + ar^3 = 48$

$ar^2\,(1 + r) = 48$

$r^2 = 4$

$r = \pm\,2$

$\Rightarrow$ $r = -2$

$\therefore$ $a(1 - 2) = 12$

$a = -12$

16. It is given that the events A and B are such that $P(A)=\frac{1}{4}, P(A\,|\,B)=\frac{1}{2}$ and $P(B\,|\,A)=\frac{2}{3}$. Then $P(B)$ is

(1) $\frac{1}{2}$

(2) $\frac{1}{6}$

(3) $\frac{1}{3}$

(4) $\frac{2}{3}$

Sol. (3)

$$P(A\cap B)=P(A)P(B\,|\,A)=P(B).P(A\,|\,B)$$

$$\Rightarrow \frac{1}{4}\times\frac{2}{3}=P(B)\times\frac{1}{2}$$

$$\Rightarrow P(B)=\frac{1}{3}$$

17. A Die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A\cup B)$ is

(1) $\frac{2}{5}$

(2) $\frac{3}{5}$

(3) 0

(4) 1s

Sol. (4)

A = {4, 5, 6}, B = {1, 2, 3, 4}

$A\cup B$ = {1, 2, 3, 4, 5, 6}

$$P(A\cup B)=1$$

18. Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$. Then which one of the following holds ?

(1) The cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

(2) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$

(3) The cubic has minima at $-\sqrt{\frac{p}{3}}$ and maxima $\sqrt{\frac{p}{3}}$

(4) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

Sol. (2)

Let $f(x) = x^3 - px + q$

$f'(x) = 3x^2 - p = 0$

$\Rightarrow x = \pm\sqrt{\frac{p}{3}}$

$f''(x) = 6x$

$f''(x)$ at $x = \sqrt{\frac{p}{3}}$ is positive

$\therefore$ Minima at $x = \sqrt{\frac{p}{3}}$

and $f''(x)$ at $x = -\sqrt{\frac{p}{3}}$ is negative

So, maxima at $x = -\sqrt{\frac{p}{3}}$

19. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent ?

(1) $7.{}^6C_4.{}^8C_4$ (2) $8.{}^6C_4.{}^7C_4$

(3) $6.7.{}^8C_4$ (4) $6.8.{}^7C_4$

Sol. (1)

Total words which have no two s are adjacent

$= {}^8C_4 \times \frac{7!}{4! \times 2!} = {}^8C_4 \times 7 \times \frac{6!}{4! \times 2!}$

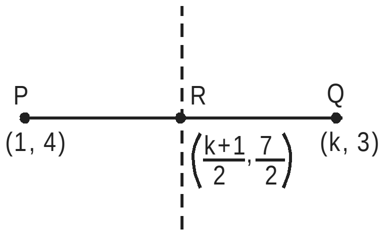
$= 7 \times {}^6C_4 \times {}^8C_4$

20. The perpendicular bisector of the line segment joining P(1, 4) and Q(k, 3) has y-intercept –4. Then a possible value of k is

(1) – 4 (2) 1

(3) 2 (4) – 2

Sol. (1)



Let the equation of perpendicular bisector is y = mx $-$ 4

Now, this line is passing through $R\left(\frac{k+1}{2}, \frac{7}{2}\right)$

So, $\frac{7}{2} = m\left(\frac{k+1}{2}\right) - 4$

$\Rightarrow 7 = mk + m - 8$ ... (i)

Now, $m = -\left(\frac{1}{\text{slope of PQ}}\right)$

$\Rightarrow m = -\frac{(1-k)}{1} = (k-1)$ ... (ii)

Putting value of m in equation (i)

$(k-1)k + (k-1) - 8 = 7$

$\Rightarrow k^2 - 1 = 15$

$\Rightarrow k^2 = 16$

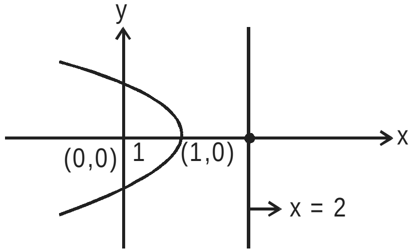
$\Rightarrow k = \pm 4$

21. A parabola has the origin as its focus and the line x = 2 as the directrix. Then the vertex of the parabola is at

(1) (2, 0) (2) (0, 2)

(3) (1, 0) (4) (0, 1)

Sol. (3)



22. The point diametrically opposite to the point P(1, 0) on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is

(1) (3, 4) (2) (3, – 4)

(3) (– 3, 4) (4) (– 3, – 4)

Sol. (4)

Let the other point is (h, k).

$$\frac{h+1}{2} = -1$$

$$\Rightarrow h = -3$$

$$\frac{k}{2} = -2$$

$$\Rightarrow k = -4$$

23. A focus of an ellipse is at the origin. The directrix is the line x = 4 and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is

(1) $\frac{5}{3}$ (2) $\frac{8}{3}$

(3) $\frac{2}{3}$ (4) $\frac{4}{3}$

Sol. (2)

Distance between focus and directrix = $\left(\frac{a}{e} - ae\right)$

$$\Rightarrow a\left(\frac{1-e^2}{e}\right) = 4 \quad \text{(given)}$$

$$\Rightarrow a\left(\frac{1-\frac{1}{4}}{\frac{1}{2}}\right)=4$$

$$\Rightarrow a=\frac{8}{3}$$

24. The solution of the differential equation $\frac{dy}{dx}=\frac{x+y}{x}$ satisfying the condition y(1) = 1 is

(1) $y = x \ln x + x$ (2) $y = \ln x + x$

(3) $y = x \ln x + x^2$ (4) $y = x e^{(x-1)}$

Sol. (1)

$$\frac{dy}{dx}=1+\frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx}-\frac{y}{x}=1$$

$$\Rightarrow y.\frac{1}{x}=\int\frac{1}{x}\,dx$$

$$\Rightarrow \frac{y}{x}=\ln x+c$$

$$y=x\ln x+cx$$

Now, $1=1.\ln 1+c \quad \Rightarrow c=1$

$$\therefore y=x\ln x+x$$

25. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that x = cy + bz, y = az + cx, and z = bx + ay. Then $a^2 + b^2 + c^2 + 2abc$ is equal to

(1) 1 (2) 2

(3) –1 (4) 0

Sol. (1)

$$\because \Delta=0$$

$$\Rightarrow \begin{vmatrix}1 & -c & -b\\ c & -1 & a\\ b & a & -1\end{vmatrix}=0$$

$$\Rightarrow 1\left(1-a^2\right)+c(-c-ab)-b(ac+b)=0$$

$$\Rightarrow 1-a^2-c^2-abc-abc-b^2=0$$

$$\Rightarrow a^2+b^2+c^2+2abc=1$$

26. Let A be a square matrix all of whose entries are integers. Then which one of the following is true ?

(1) If det $A=\pm 1$, then A^{-1} need not exist

(2) If det $A=\pm 1$, then A^{-1} exists but all its entries are not necessarily integers

(3) If det $A\neq \pm 1$, then A^{-1} exists and all its entries are non-integers

(4) If det $A=\pm 1$, then A^{-1} exists and all its entries are integers

Sol. (4)

$\because$ det $A=\pm 1$ $\Rightarrow$ inverse of matrix A exists and since all entries of matrix A are integers and Adj A is matrix of transpose of co-factors of matrix A.

$\therefore$ Entries of Adj A is also integers.

27. The quadratic equations $x^2-6x+a=0$ and $x^2-cx+6=0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common roots is

(1) 2 (2) 1

(3) 4 (4) 3

Sol. (1)

Let α, β are roots of $x^2-6x+a=0$ ... (i)

and α, γ are roots of $x^2-cx+6=0$ ... (ii)

$\therefore \alpha+\beta=6, \alpha\beta=a$

and $\alpha+\gamma=c, \alpha\gamma=6$

$$\because \frac{\alpha\beta}{\alpha\gamma}=\frac{a}{6} \Rightarrow \frac{\beta}{\gamma}=\frac{a}{6} \Rightarrow \frac{4}{3}=\frac{a}{6} \Rightarrow a=8$$

$\therefore$ From (i) $x^2-6x+8=0 \Rightarrow x=2, 4$; $\because \alpha\gamma=6 \Rightarrow \alpha$ cannot be 4 ($\because \gamma$ is integer)

$\Rightarrow \alpha=2$

28. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is $6\cdot 80$. Then which one the following gives possible values of a and b ?

(1) a = 3, b = 4 (2) a = 0, b = 7

(3) a = 5, b = 2 (4) a = 1, b = 6

Sol. (1)

$$\frac{a+b+8+5+10}{5}=6$$

$\Rightarrow a+b=7$ ... (i)

$$\frac{(a-6)^2+(b-6)^2+4+1+16}{5}=6.8$$

$\Rightarrow a^2+b^2+93-12(a+b)=34$

$\Rightarrow a^2+b^2=25$ ... (ii)

By equation (i) and (ii), we get
a = 3, b = 4

29. The vector $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$ lies in the plane of the vectors $\vec{b}=\hat{i}+\hat{j}$ and $\vec{c}=\hat{j}+\hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β?

(1) $\alpha=1,\ \beta=1$ (2) $\alpha=2,\ \beta=2$

(3) $\alpha=1,\ \beta=2$ (4) $\alpha=2,\ \beta=1$

Sol. (1)

$\because \vec{a}, \vec{b}, \vec{c}$ are co-planar

$$\therefore \begin{vmatrix}\alpha & 2 & \beta\\ 1 & 1 & 0\\ 0 & 1 & 1\end{vmatrix}=0$$

$\Rightarrow \alpha+\beta=2$ ... (i)

and angle between $\vec{a}$ and $\vec{b}$ is same as $\vec{a}$ and $\vec{c}$

$$\therefore \frac{\alpha+2}{\sqrt{\alpha^2+4+\beta^2}\sqrt{2}}=\frac{2+\beta}{\sqrt{\alpha^2+4+\beta^2}\sqrt{2}} \Rightarrow \alpha=\beta \quad \text{... (ii)}$$

From (i) and (ii), we get
α = 1, β = 1

30. The non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{a} = 8\vec{b}$ and $\vec{c} = -7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$

(1) π (2) 0

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

Sol. (1)

$\because \vec{a}$ and $\vec{b}$ are parallel with same direction and $\vec{b}$ and $\vec{c}$ are parallel with opposite direction.

$\therefore$ Angle between $\vec{a}$ and $\vec{c}$ is π.

31. The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then

(1) a = 8, b = 2 (2) a = 2, b = 8

(3) a = 4, b = 6 (4) a = 6, b = 4

Sol (4) Equation of line is

$$\frac{x-5}{2} = \frac{y-1}{1-b} = \frac{z-a}{a-1}$$

$\therefore$ It passes through $\left(0, \frac{17}{2}, -\frac{13}{2}\right)$

$$\therefore \quad -\frac{5}{2} = \frac{\frac{17}{2}-1}{1-b} = \frac{-\frac{13}{2}-1}{a-1}$$

From Ist and 2nd ratio

$$-5+5b = 17-2$$

$$\Rightarrow b = 4$$

and from Ist and IIrd ratio

$$-5a+5 = -13-2a$$

$$\Rightarrow \quad 3a = 18 \Rightarrow a = 6.$$

32. If the straight lines $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$ intersect at a point, then the integer k is equal to

(1) –2 (2) –5

(3) 5 (4) 2

Sol (2) $\therefore$ Lines are intersecting

$$\therefore \begin{vmatrix} 2-1 & 3-2 & 1-3 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 1 & -2 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix} = 0$$

$\Rightarrow \quad 1(4-3k)-1(2k-9)-2(k^2-6)=0$

$\Rightarrow \quad 4-3k-2k+9-2k^2+12=0$

$\Rightarrow \quad 2k^2+5k-25=0$

$\Rightarrow \quad k=-5, 5/2$

33. The conjugate of a complex number is $\frac{1}{i-1}$. Then that complex number is

(1) $\frac{1}{i-1}$ (2) $\frac{-1}{i-1}$

(3) $\frac{1}{i+1}$ (4) $\frac{-1}{i+1}$

Sol (4) $\therefore \quad z\bar{z} = |z|^2$

$\Rightarrow \quad z \times \frac{1}{-1+i} = \frac{1}{2}$

$\Rightarrow \quad z = \frac{-1+i}{2} \times \frac{-1-i}{-1-i}$

$= \frac{1+1}{2(-1-i)} = \frac{-1}{1+i}$

34. Let R be the real line. Consider the following subsets of the plane R × R :
S = {(x, y) : y = x + 1 and 0 < x < 2}
T = {(x, y) : x – y is an integer}.
Which one of the following is true ?
(1) T is an equivalence relation on R out S is not
(2) Neither S nor T is an equivalence relation on R
(3) Both S and T are equivalence relations on R
(4) S is an equivalence relation on R but T is not

Sol (1) $\because$ S is not reflexive $\Rightarrow$ S in not equivalance relation.
T is reflexive, Symmetric, and transitive
$\Rightarrow$ T is equivalance relation.

35. Let $f : N \to Y$ be a function defined as
f(x) = 4x + 3 where
$Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$.
Show that f is invertible and its inverse is

(1) $g(y) = \frac{y-3}{4}$ (2) $g(y) = \frac{3y+4}{3}$

(3) $g(y) = 4 + \frac{y+3}{4}$ (4) $g(y) = \frac{y+3}{4}$

Sol (1) y = 4x + 3

$\Rightarrow \quad x = \frac{y-3}{4}$

$\therefore \quad$ g(y) $g(y) = \frac{y-3}{4}$