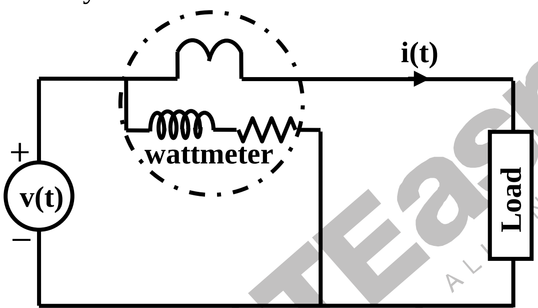
GATE 2012

Electrical Engineering

Set - B

Q.1 – Q.25 carry one mark each.

1. For the circuit shown in the figure, the voltage and current expressions are $v(t) = E_1 \sin(\omega t) + E_3 \sin(3\omega t)$ and $i(t) = I_1 \sin(\omega t - \phi_1) + I_3 \sin(3\omega t - \phi_3) + I_5 \sin(5\omega t)$. The average power measured by the Wattmeter is



(A) $\frac{1}{2}E_1I_1\cos\phi_1$

(B) $\frac{1}{2}[E_1I_1\cos\phi_1 + E_1I_3\cos\phi_3 + E_1I_5]$

(C) $\frac{1}{2}[E_1I_1\cos\phi_1 + E_3I_3\cos\phi_3]$

(D) $\frac{1}{2}[E_1I_1\cos\phi_1 + E_3I_1\cos\phi_1]$

[Ans. C]

Approach-1

$V_i(t) = E_1\sin(\omega t) + E_3\sin(3\omega t)$

$i_i(t) = I_1\sin(\omega t + \phi_1) + I_3\sin(3\omega t - \phi_3) + I_5\sin(5\omega t)$

$P_{avg} = \frac{1}{2\pi}\int_o^{2\pi} V_i(t)\, i_i(t)$

On solving, we get,

$P_{avg} = \frac{1}{2}E_1I_1\cos(\phi_1) + \frac{1}{2}E_3I_3\cos(\phi_3)$

Approach-2

$V(t) = E_1 \sin \omega t + E_3 \sin 3\omega t$

$I(t) = I_1 \sin(\omega t - \phi_1) + I_3 \sin(\omega t - \phi_3) + I_5 \sin(5\omega t)$

We know that Power transfer always happen between same harmonics of voltage and current

Power due to fundamental component $=E_{1\,rms} \times I_{1\,rms} \cos\phi_1 = \frac{E_1}{\sqrt{2}} \times \frac{I_1}{\sqrt{2}} \cos\phi_1 = \frac{E_1 I_1}{2} \cos\phi_1$

Power due to 3rd harmonic $= \frac{E_3}{\sqrt{2}} \times \frac{I_3}{\sqrt{2}} \times \cos\phi_3 = \frac{E_3 I_3}{2} \cos\phi_3$

Power due 5th harmonic = 0 (as there is no 5th harmonic voltage)

Total power $= \frac{E_1 I_1}{2} \cos\phi_1 + \frac{E_3 I_3}{2} \cos\phi_3$

2. The typical ratio of latching to holding current in a 20A thyristor is

(A) 5.0

(B) 2.0

(C) 1.0

(D) 0.5

[Ans. Marks to All*] (*Ambiguous options as per IIT Delhi, although typical ratio is 5)

3. A half-controlled single-phase bridge rectifier is supplying an R-L load. It is operated at a firing angle α and the load current is continuous. The fraction of cycle that the freewheeling diode conduct is

(A) $^1/_2$

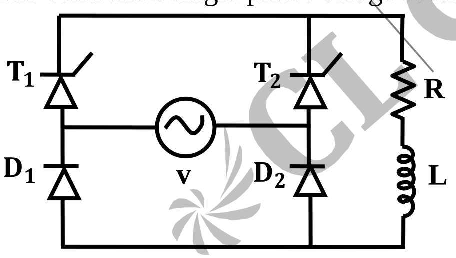
(B) $(1 - ^{\alpha}/_{\pi})$

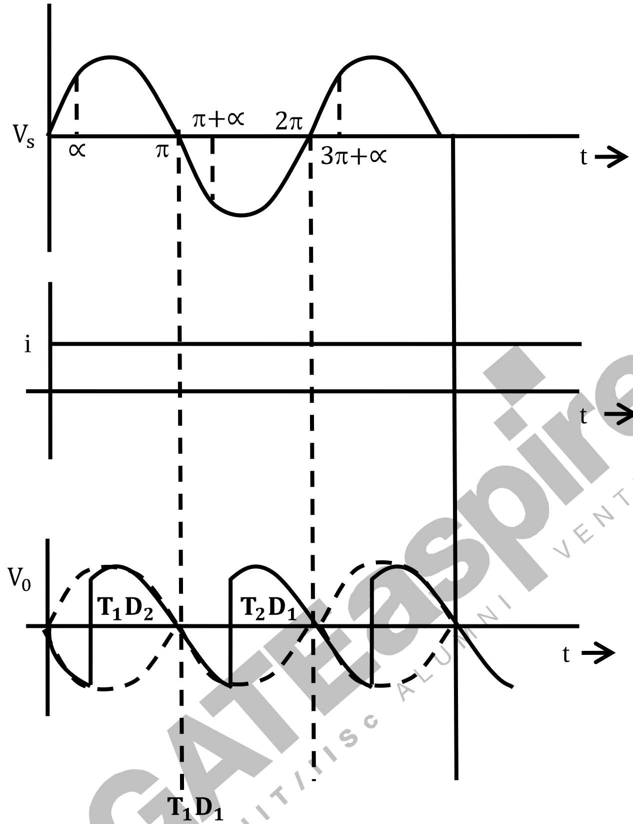
(C) $^{\alpha}/_{2\pi}$

(D) $^{\alpha}/_{\pi}$

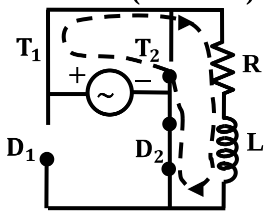
[Ans. D]

Half controlled single phase bridge rectifier Ckt

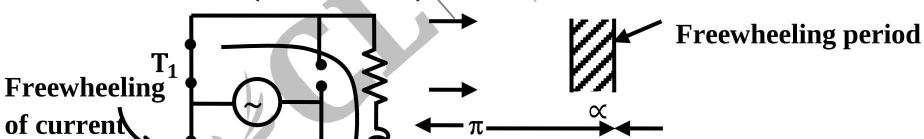




+ve half ($\propto < t < \pi$)



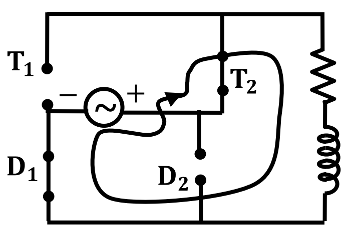
−ve half ($\pi < t < \pi+\propto$)



Freewheeling period = $\frac{2}{\pi}$

(fraction of cycle)

−ve half ($\pi+\propto < t < 2\pi$)



4. The sequence components of the fault current are as follows: $I_{positive} = j1.5$ pu, $I_{negative} = -j0.5$ pu, $I_{zero} = -j1$ pu. The type of fault in the system is

(A) LG
(B) LL
(C) LLG
(D) LLLG

[Ans. C]

$I_1 = I_2 + I_0$

So, it is LLG fault

5. The figure shows a two-generator system supplying a load of $P_D = 40$MW, connected at bus 2.

The fuel cost of generators G_1 and G_2 are:

$C_1(P_{G1}) = 10{,}000$ Rs/MWh and $C_2(P_{G2}) = 12{,}500$ Rs/MWh and the loss in the line is $P_{loss(pu)} = 0.5\ P^2_{G1(pu)}$, where the loss coefficient is specified in pu on a 100 MVA base. The most economic power generatration schedule in MW is

(A) $P_{G1} = 20, P_{G2} = 22$
(B) $P_{G1} = 22, P_{G2} = 20$
(C) $P_{G1} = 20, P_{G2} = 20$
(D) $P_{G1} = 0, P_{G2} = 40$

[Ans. A]

$C_1(P_{G1}) = 10{,}000$ Rs/MWh $C_2(P_{G2}) = 12{,}500$ Rs/MWh

$P_{loss}(pu) = 0.5\ P^2_{G1(pu)}$

$P_{G1} + P_{G2} - P_L = P_D$

$P_{G1} + P_{G2} - 0.5\ P^2_{G1(pu)} = 40 \qquad -(1)$

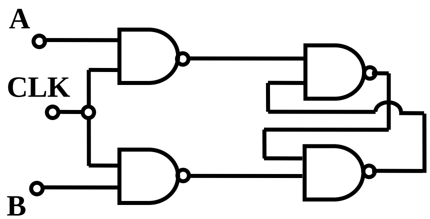
Going through given options, option 'A' will be satisfying equation (1)

$20 + 22 - 0.5 \times \left(\frac{20}{100}\right)^2 = 40$

If base power is taken as 100 MW

$P_{G1(pu)} = \frac{20}{100}$

6. Consider the given circuit.



In this circuit, race around

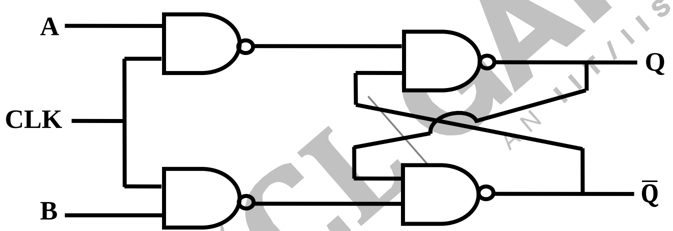
(A) does not occur

(B) occurs when CLK = 0

(C) occurs when CLK = 1 and A = B = 1

(D) occurs when CLK = 1 and A = B = 0

[Ans. A]



$Q_{next} = \overline{\overline{A.CLK}.\overline{Q}}$

$= A.CLK + Q$

$\overline{Q}_{next} = A.CLK + \overline{Q}$

If CLK = 1 and A and B = 1

then $\left.\begin{matrix} Q_{next} = 1 \\ \overline{Q}_{next} = 1 \end{matrix}\right\}$ No race around

If CLK = 1 and A = B = 0

$\left.\begin{matrix} Q_{next} = Q \\ \overline{Q}_{next} = \overline{Q} \end{matrix}\right\}$ No race around

Thus race around does not occur in the circuit.

7. The output Y of a 2-bit comparator is logic 1 whenever the 2-bit input A is greater than the 2-bit input B. The number of combinations for which the output is logic 1, is

(A) 4
(B) 6
(C) 8
(D) 10

[Ans. B]

Approach – 1

A > B   A & B are 2 bit

01   00   –1

10   00 01   –2

11   00   01   10 $\frac{-3}{6}$

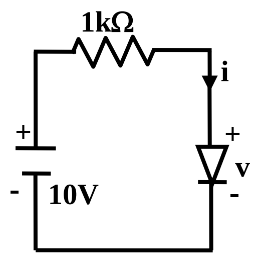
Approach – 2

Input A		Input B		Y
A_2	A_1	B_2	B_1	
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

Thus for 6 combinations output in logic 1.

8. The i-v characteristics of the diode in the circuit given below are

$$i = \begin{cases} \frac{v-0.7}{500} A, & v \geq 0.7V \\ 0 A, & v < 0.7 V \end{cases}$$



The current in the circuit is

(A) 10 mA

(B) 9.3 mA

(C) 6.67 mA

(D) 6.2 mA

[Ans. D]

Approach – 1

I_D(diode current) = 500 i + 0.7

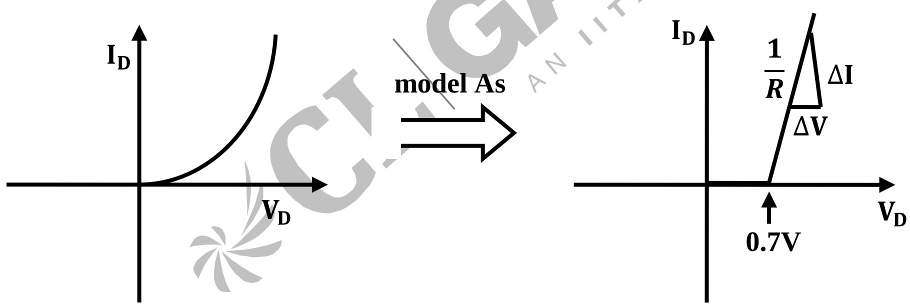
Applying KCL,

10 = 1000 i + 500 i + 0.7

$\Longrightarrow i = \frac{9.3}{1.5} = 6.2mA$

Approach – 2

Here diode equivalent circuit model is given which is graphically presented as →



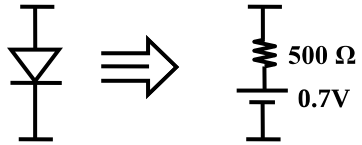
$V \geq 0.7V$

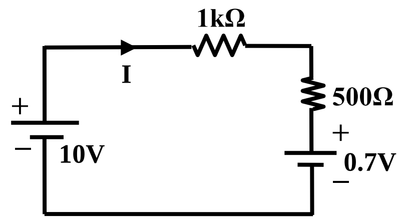
$$I = \frac{V - 0.7}{500}\ A$$

Slope $= \frac{1}{R} = \frac{1}{500}$

R = 500 Ω

Equivalent circuit Model:





Now applying KVL for Linear circuit

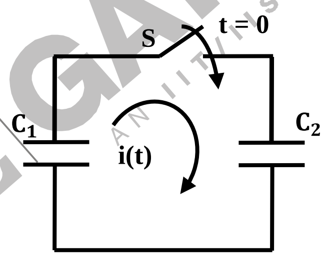
$-10V + 1K\Omega \times I + 500\,\Omega \times I + 0.7V = 0$

$\Rightarrow -9.3V + 1500\,\Omega\ \mathrm{I} = 0$

$\Rightarrow I = \dfrac{9.3}{1500}\dfrac{V}{\Omega} = 6.2 \times 10^{-3}\ A$

$I = 6.2$ mA

9. In the following figure, C_1 and C_2 are ideal capacitors. C_1 has been charged to 12 V before the ideal switch S is closed at t = 0. The current i(t) for all t is



(A) zero
(B) a step function
(C) an exponentially decaying function
(D) an impulse function

[Ans. D]

Initially when switch is closed, impulse current flows from C1 to C2 till voltages of C1 and C2 becomes equal. This happens due to the fact that there is a potential difference initially between C_1 and C_2, but resistance in the circuit is zero leading to an infinite current. Once charge is equal in C1 and C2, current i(t) will be zero.

10. The average power delivered to an impedance (4 – j3) Ω by a current 5 cos(100 π t + 100) A is

(A) 44.2 W
(B) 50 W
(C) 62.5 W
(D) 125 W

[Ans. B]

$P_{avg} = \frac{1}{2}\text{Re}\{VI^*\}$

$= \frac{1}{2}\text{Re}\{IZI^*\}$

$= \frac{1}{2}\text{Re}\{|I|^2 Z\}$

$= \frac{1}{2}|I|^2\text{Re}\{Z\}$

$= \frac{1}{2} \times 5^2 \times 4 \qquad (\because \text{Re}\{Z\} = 4)$

= 50 W

11. The unilateral Laplace transform of f(t) is $\frac{1}{s^2+s+1}$. The unilateral Laplace transform of tf(t) is

(A) $-\frac{s}{(s^2+s+1)^2}$
(B) $-\frac{2s+1}{(s^2+s+1)^2}$
(C) $\frac{s}{(s^2+s+1)^2}$
(D) $\frac{2s+1}{(s^2+s+1)^2}$

[Ans. D]

$L\{t.f(t)\} = (-1)^1.\frac{d}{ds}F(s)$

$= -\frac{d}{ds}\left(\frac{1}{s^2+s+1}\right)$

= 1

12. With initial condition x(1) = 0.5, the solution of the differential equation.

$t\frac{dx}{dt} + x = t$ is

(A) $x = t - \frac{1}{2}$
(B) $x = t^2 - \frac{1}{2}$
(C) $x = \frac{t^2}{2}$
(D) $x = \frac{t}{2}$

[Ans. D]

<u>Approach – 1</u>

Just substitute, $x = \frac{t}{2}$, or divide by t, and take integrating fact.

Approach – 2

Given DE is $t\frac{dx}{dt} + x = t \Rightarrow \frac{dx}{dt} + \frac{x}{t} = 1$

IF $= e^{\int \frac{1}{t}dt} = e^{\log t} = t$; solution is x (IF) = $\int (IF)$ tdt

$xt = \int t.tdt \Rightarrow xt = \frac{t^2}{2} + c$; Given that $x(1) = 0.5 \Rightarrow 0.5 = \frac{1}{2} + c \Rightarrow c = 0$

∴ the required solution is $xt = \frac{t^2}{2} \Rightarrow x = \frac{t}{2}$

Approach – 3

Given: $t\frac{dx}{dt} + x = t, x(1) = 0.5$

$t\frac{dx}{dt} + x = t$

$t\,dx + xdt = tdt$

$d(xt) = t\,dt$

$xt = \frac{t^2}{2} + C$

Using initial condition, at t = 1, x = 0.5

$0.5 \times 1 = \frac{1}{2} + C$

$C = 0$

$\therefore xt = \frac{t^2}{2}$

$x = \frac{t}{2}$

13. A two-phase load draws the following phase currents: $i_1(t) = I_m \sin(\omega t - \phi_1)$, $i_2(t) = I_m \cos(\omega t - \phi_2)$. These currents are balanced if ϕ_1 is equal to

(A) $-\phi_2$

(B) ϕ_2

(C) $\left(\frac{\pi}{2} - \phi_2\right)$

(D) $\left(\frac{\pi}{2} + \phi_2\right)$

[Ans. B]

$i_1(t) = I_m \sin(\omega t - \phi_1)$

$i_2(t) = I_m \cos(\omega t - \phi_2)$

$i_1(t) = I_m \cos\left(\frac{\pi}{2} - \omega t + \phi_1\right)$

$i_1(t) = I_m \cos\left(\omega t - \frac{\pi}{2} - \phi_1\right)$

Equality the phasors

$\frac{\pi}{2} + \phi_1;\ \phi_2$

Since ϕ_1 and ϕ_2 are 90° apart

if $\phi_1 = \phi_2$

Phasors are

$\frac{\pi}{2} + \phi_2, \phi_2$ i.e. 90 apart

So $\boxed{\phi_1 = \phi_2}$

14. The slip of an induction motor normally does not depend on

(A) rotor speed

(B) synchronous speed

(C) shaft torque

(D) core-loss component

[Ans. D]

Slip = $\frac{N_S - N_r}{N_S}$

→ So depends on N_s (synchronous speed)

→ So depends on N_r (rotor speed)

→ If torque increases N_r decrease

→ It will not dependent on core loss

15. The bus admittance matrix of a three-bus three-line system is

$$Y = j\begin{bmatrix} -13 & 10 & 5 \\ 10 & -18 & 10 \\ 5 & 10 & -13 \end{bmatrix}$$

If each transmission line between the two buses is represented by an equivalent π - network, the magnitude of the shunt susceptance of the line connecting bus 1 and 2 is

(A) 4

(B) 2

(C) 1

(D) 0

[Ans. B]

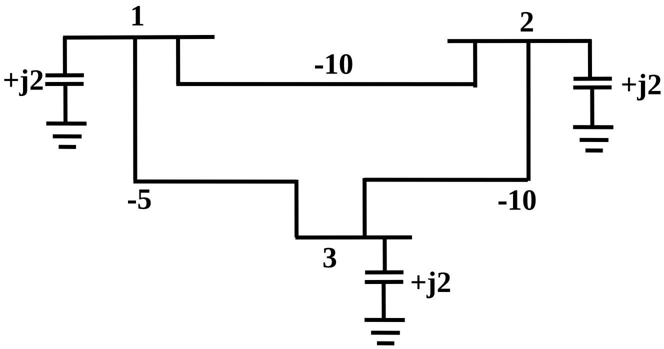
$$Y = j\begin{bmatrix} -13 & 10 & 5 \\ 10 & -18 & 10 \\ 5 & 10 & -13 \end{bmatrix} = j\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

For Bus admittance Matrix.

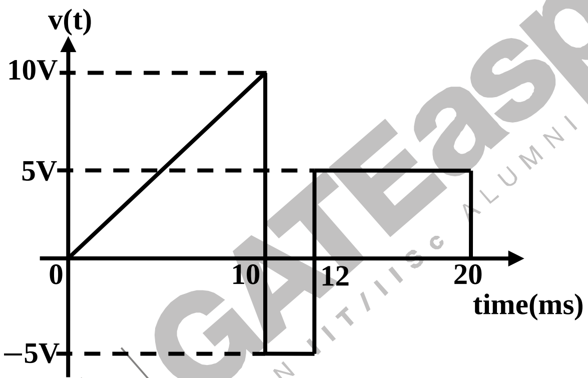
$Y_{11} + Y_{12} + Y_{13} = 0$

$-13 + 10 + 5 \neq 0$

So, shunt susceptance of the line connecting bus 1 and 2 is + j2



16. A periodic voltage waveform observed on an oscilloscope across a load is shown. A permanent magnet moving coil (PMMC) meter connected across the same load reads



(A) 4 V
(B) 5 V
(C) 8 V
(D) 10 V

[Ans. A]

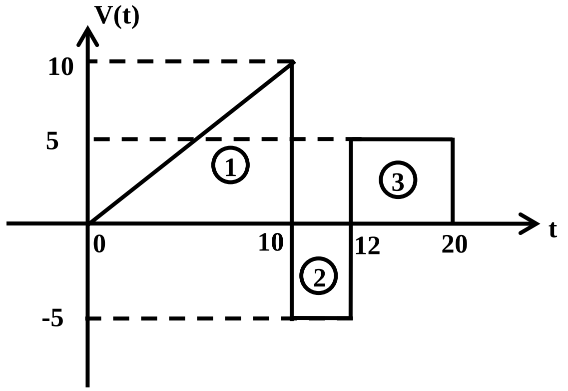
Approach-1

PMMC meter will read the average value of the applied waveform.

Vavg = (1/time period) $\int V(t)\,dt$ = (1/20)[$_0\int^{10}$ t.dt + $_{10}\int^{12}$ (-5).dt + $_{12}\int^{20}$ (+5)dt]

Solving and substituting the limits, we get the value of Vavg as 4V

Approach-2



PMMC types of instrument measures average value.

$V_{avg} = \frac{\text{area of graph}}{\text{total time}} \quad = \frac{1}{T}\int_o^T f(t)dt$

$= \frac{\text{area of (1)} - \text{area of (2)} + \text{area of (3)}}{20}$

$= \frac{\frac{1}{2} \times 10 \times 10 - 5 \times 2 + 8 \times 5}{20} = \frac{80}{20} = 4V.$

17. If $x[n] = (1/3)^{|n|} - (1/2)^n u[n]$, then the region of convergence (ROC) of its Z-transform in the Z-plane will be

(A) $\frac{1}{3} < |z| < 3$

(B) $\frac{1}{3} < |z| < \frac{1}{2}$

(C) $\frac{1}{2} < |z| < 3$

(D) $\frac{1}{3} < |z|$

[Ans. C]

Given: $x[n] = (1/3)^{|n|} - (1/2)^n u[n]$

$x[n] \rightleftharpoons x(z)$

$x(z) = \sum_{n=-\infty}^{\infty} x[n]\ z^{-n}$

First consider $\left(\frac{1}{3}\right)^{|n|}$

$\sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^{|n|}\ z^{-n}$

$= \sum_{n=-\infty}^{-1} \left(\frac{1}{3}\right)^{-n}\ z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^{n}\ z^{-n}$

$= \sum_{n=-\infty}^{-1} \left({}^{3}/_{z}\right)^{n} + \sum_{n=0}^{\infty} \left(\frac{1}{3z}\right)^{n}$

$= \left(\frac{3}{z}\right)^{-1} + \left({}^{3}/_{z}\right)^{-2} + ------+1+\frac{1}{3z}+\left(\frac{1}{3z}\right)^{2}+------$

$\left|\frac{3}{z}\right| > 1$ $\left|\frac{1}{3z}\right| < 1$

$\left|\frac{z}{3}\right| < 1$ $|z| > {}^{1}/_{3}$

$|z| < 3$

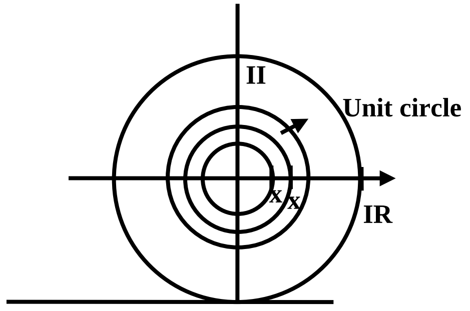
Consider $\left(\frac{1}{2}\right)^{n}$ u(n)

$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n} z^{-n}$

$= 1 + \frac{1}{2z} + \left(\frac{1}{2z}\right)^{2} +$ -------------

$\left|\frac{1}{2z}\right| < 1 \quad |z| > {}^{1}/_{2}$

ROC is $\frac{1}{2} < |z| < 3$



18. A system with transfer function

$$G(s) = \frac{(s^2+9)(s+2)}{(s+1)(s+3)(s+4)}$$

is excited by sin (ωt). The steady-state output of the system is zero at

(A) ω = 1 rad/s
(B) ω = 2 rad/s
(C) ω = 3 rad/s
(D) ω = 4 rad/s

[Ans. C]

$G(s) = \frac{(S^2+9)\,(S+2)}{(S+1)(S+3)(S+4)}$

By substituting s = jw and equating |G(s)|=0 we get w=3

19. The bridge method commonly used for finding mutual inductance is

(A) Heaviside Campbell bridge
(B) Schering bridge
(C) De Sauty bridge
(D) Wien bridge

[Ans. A]

Mutual inductance measurement is normally done by Heaviside Campbell bridge.

20. In the sum of products function $f(X, Y, Z) = \sum(2, 3, 4, 5)$, the prime implicants are

(A) $\overline{X}\,Y, X\,\overline{Y}$

(B) $\overline{X}\,Y, X\,\overline{Y}\,\overline{Z}, X\,\overline{Y}\,Z$

(C) $\overline{X}\,Y\,\overline{Z}, \overline{X}Y\,Z, X\,\overline{Y}$

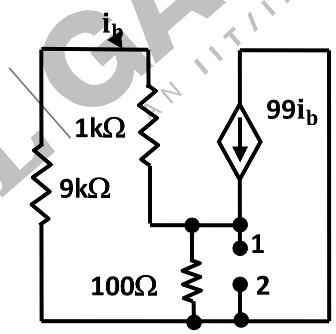
(D) $\overline{X}\,Y\,\overline{Z}, \overline{X}\,Y\,Z, X\,\overline{Y}\,\overline{Z}, X\,\overline{Y}\,Z$

[Ans. A]

x \ yz	00	01	11	10
0	0	0	1	1
1	1	1	0	0

$f(x, y, z) = \bar{x}y + x\bar{y}$

21. The impedance looking into nodes 1 and 2 in the given circuit is



(A) 50 Ω

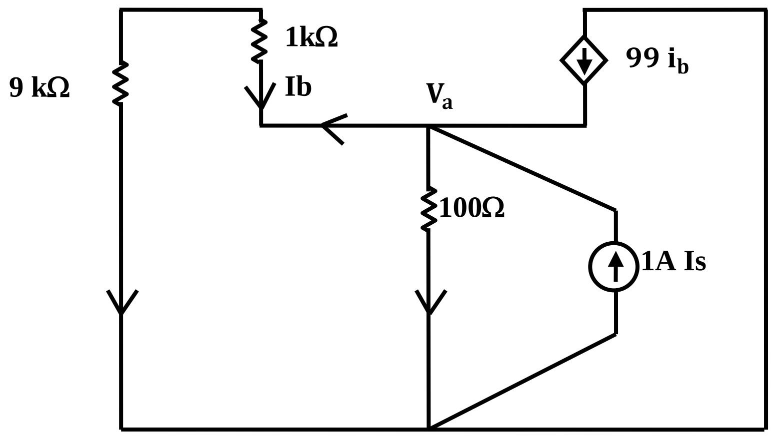
(B) 100 Ω

(C) 5 kΩ

(D) 10.1kΩ

[Ans. A]

<u>Approach – 1</u>



By nodal analysis at node a,

$$\frac{V_a - 0}{10k} + \frac{V_a}{100} - 99\, i_b = 1$$

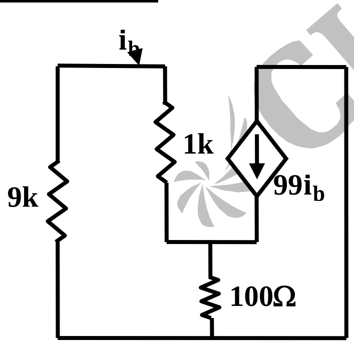
$$\frac{V_a - 0}{10k} + \frac{V_a}{100} - 1 + \frac{99\, V_a}{10k} = 0$$

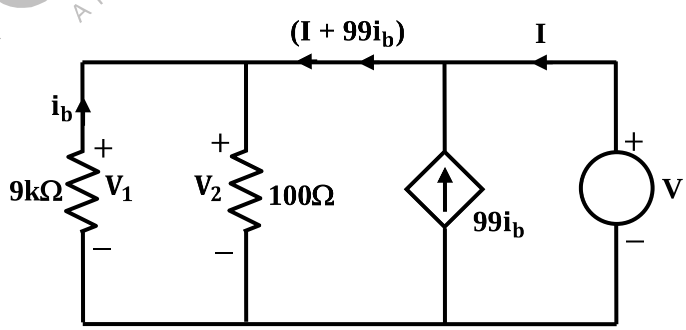
$$\Rightarrow V_a\left[\frac{100}{10k} + \frac{100}{10k}\right] = 1.$$

$$\Rightarrow V_a = 50V$$

$$\therefore R\,(\text{thevenin}) = \frac{V_a}{I_s} = 50\Omega$$

Approach – 2





After connecting a voltage source of V

$$V_1 = V_2 \Rightarrow (10k)(-i_b) = 100(I + 99i_b + i_b);$$

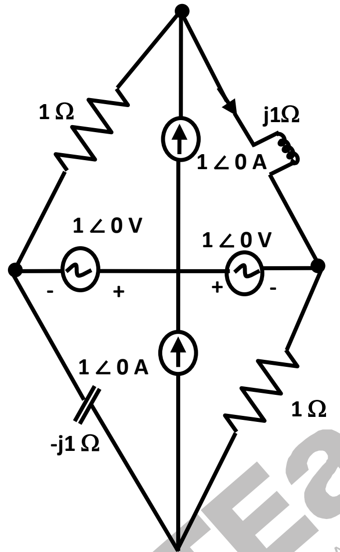
$$-10000i_b = 100I + 100 \times 100i_b = 100I + 10000i_b$$

$$-20000i_b = 100I \Rightarrow i_b = -\left(\frac{100}{20000}\right)I = \left[-\frac{I}{200}\right]$$

$$V = 100[I + 99i_b + i_b] = 100\left[I + 100\left(\frac{-I}{200}\right)\right] = 50I$$

$$R_{th} = \frac{V}{I} = \frac{50I}{I} = 50\Omega$$

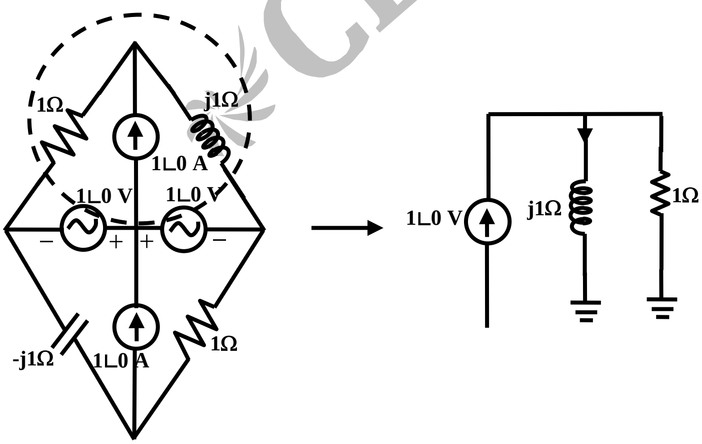
22. In the circuit shown below, the current through the inductor is



(A) $\frac{2}{1+j}$ A
(B) $\frac{-1}{1+j}$ A
(C) $\frac{1}{1+j}$ A
(D) 0 A

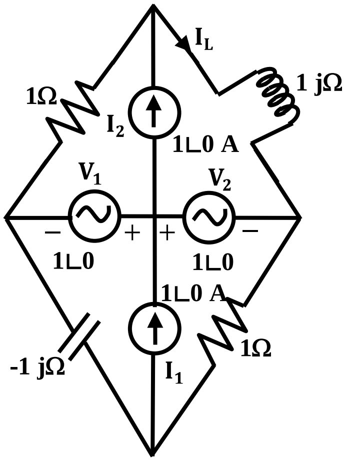
[Ans. C]

Approach – 1

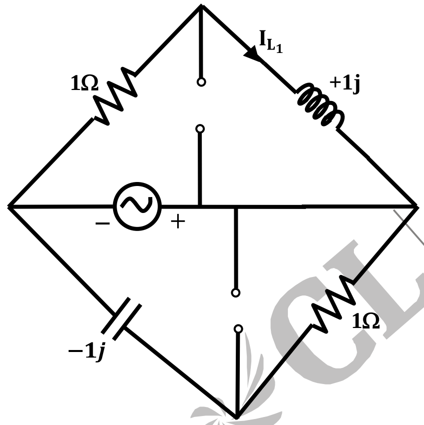


$$I_L = 1\angle 0 \times \frac{1}{1+j1} = \frac{1}{1+j1}\ A$$

Approach – 2



Apply Superposition theorem,
V_1 only: Short circuit
V_2 open circuit I_1 and I_2



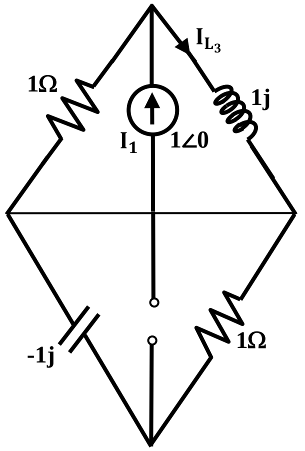
$I_{L_1} = \frac{-1}{1+1j}$ --------------(1)

$\left[I_{L_1} = \frac{-V_1}{Total\ Impedance}\right]$

Similarly for V_2 only

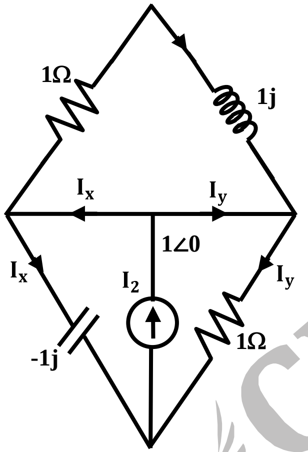
$I_{L_2} = \frac{1}{1+1j}$ --------------(2)

For I_1 only
Using current divider rule



$I_{L_3} = \frac{1}{1+1j} \times 1 < 0 = \frac{1}{1+1j}$ -------------- (3)

For I_2 only



$I_2 = I_x + I_y$

Current in Inductor = 0

So $I_{L_4} = 0$ ------------ (4)

From (1), (2), (3), (4), Total current,

$$I_L = \frac{-1}{1+1j} + \frac{1}{1+1j} + \frac{1}{1+1j} + 0$$

$\Rightarrow I_L = \frac{1}{1+1j}$

23. Given

$f(z) = \frac{1}{z+1} - \frac{2}{z+3}$. If C is a counterclockwise path in the z-plane such that $|z+1| = 1$, the value of $\frac{1}{2\pi j}\oint_C f(z)dz$ is

(A) – 2
(B) – 1
(C) 1
(D) 2

[Ans. C]

Z = –3 is outside circle

Z = 1 is inside circle

$\lim_{z\to -1}(z+1).\frac{1}{(z+1)} = 1$

24. Two independent random variables X and Y are uniformly distributed in the interval [−1, 1]. The probability that max[X, Y] is less than 1/2 is

(A) 3/4
(B) 9/16
(C) 1/4
(D) 2/3

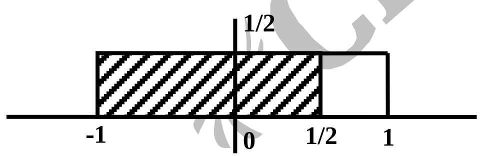
[Ans. B]

$P\left[\max[X,Y] < \frac{1}{2}\right] = P\left[X < \frac{1}{2}, Y < \frac{1}{2}\right]$ (If maximum is $< \frac{1}{2}$ then both are less than $\frac{1}{2}$)

$P\left[x < \frac{1}{2}, y < \frac{1}{2}\right]$

$P\left[x < \frac{1}{2}\right].P\left[y < \frac{1}{2}\right]$ (since independent events)

Probability density function of X and Y



$= \frac{3}{4} \times \frac{3}{4}$

$= \frac{9}{16}$

25. If $x = \sqrt{-1}$, then the value of x^x is

(A) $e^{-\pi/2}$
(B) $e^{\pi/2}$
(C) x
(D) 1

[Ans. A]

Approach – 1

Given, $x = \sqrt{-1}$; $x^x = \left(\sqrt{-1}\right)^{\sqrt{-1}} = i^i$

We know that $e^{i\theta} = \cos\theta + i\sin\theta \Rightarrow e^{i\frac{\pi}{2}} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$

$\therefore\ (i)^i = \left(e^{i\pi/2}\right)^i = e^{-\pi/2}$

Approach – 2

$x = \sqrt{-1} = i = e^{i\pi/2}$

Let $y = x^x$

Taking Natural logarithm on both sides

$\log y = \log x^x$

$\log y = x \log x$

$= e^{i\pi/2} \log e^{i\pi/2}$

$= i\left(i\pi/2\right)$

$\log y = -\pi/2$

$y = e^{-\pi/2}$

$\therefore\ x^x = e^{-\pi/2}$

Q.26 to Q.55 carry two marks each.

26. A single phase 10 kVA, 50 Hz transformer with 1 kV primary winding draws 0.5 A and 55 W, at rated voltage and frequency, on no load. A second transformer has a core with all its linear dimensions $\sqrt{2}$ times the corresponding dimensions of the first transformer. The core material and lamination thickness are the same in both transformers. The primary windings of both the transformers have the same number of turns. If a rated voltage of 2 kV at 50 Hz is applied to the primary of the second transformer, then the no load current and power, respectively, are

(A) 0.7 A, 77.8 W

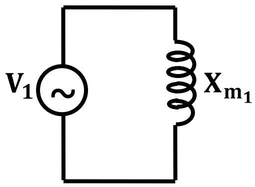
(B) 0.7 A, 155.6 W

(C) 1 A, 110 W

(D) 1 A, 220 W

[Ans. B]

Equivalent circuit of 1st transformer we neglect coreless component resistance as that value is very large.



For 2nd transformer linear dimension is $\sqrt{2}$ times 1st one.

Now, $L = \frac{N^2}{Re} = \frac{N^2 \mu a}{l}$

L = inductance

N = no. of the turns

a = c/s area of core

l = length of flux path

$\frac{L_{m_1}}{L_{m_2}} = \frac{N_1^2 \mu a_1}{l_1} \times \frac{l_2}{N_2^2 \mu a_2}$

$= \frac{a_1 l_2}{a_2 l_1}$ $(N_1 = N_2)$

$= \frac{a_1 \times \sqrt{2} l_1}{2a_1 \times l_1} = \frac{1}{\sqrt{2}}$

or, $L_{m_2} = \sqrt{2} L_{m_1}$

$\therefore X_{m_2} = \sqrt{2} X_{m_1}$

$\therefore I_{m_2}$ = Magnetising current of 2nd transformer

$= \frac{V_2}{X_{m_2}} = \frac{2V_1}{\sqrt{2} X_{m_1}} = \sqrt{2} I_{m_1} = \sqrt{2} \times 0.5 = 0.7$ A

Now specific corelss (per unit volume) of both transformers are san

$\therefore \frac{P_{C_2}}{P_{C_1}} = \frac{\text{Vol.of 2nd transformer}}{\text{Vol.of 1st transformer}} = 2\sqrt{2}$

Or, $P_{C_2} = 2\sqrt{2}\, P_{C_1} = 155.5$W

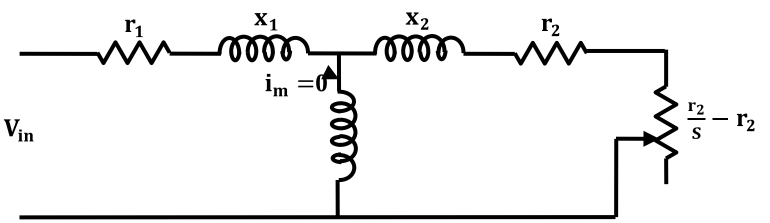
27. The locked rotor current in a 3-phase, star connected 15 kW, 4-pole, 230 V, 50 Hz induction motor at rated conditions is 50 A. Neglecting losses and magnetizing current, the approximate locked rotor line current drawn when the motor is connected to a 236 V, 57 Hz supply is

(A) 58.5 A
(B) 45.0 A
(C) 42.7 A
(D) 55.6 A

[Ans. Marks to All*] (*Ambiguous options)

One of the solution is as follows:

Neglecting stator resistance and magnetizing current



Since losses are neglected

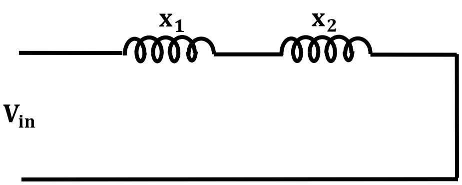
$r_1 = r_2 = 0$

Magnetizing current is neglected. So that branch is open circuited.

∴ rotor is blocked S = 1

$\therefore \frac{r_2}{s} - r_2 = 0$

∴ ckt is reduced to



$I = \frac{V_{in}}{x_1 + x_2}$

$I_1 = \frac{230}{x_1 + x_2}$

$I_1 = 50A$

When frequency in increased

$x_1 \to x_1 \times \frac{57}{50}$

$x_2 \to x_2 \times \frac{57}{50}$

$I_2 = \frac{236}{\frac{57}{50}x_1 + \frac{57}{50}x_2}$

$= \frac{50}{57} \times \frac{236}{x_1 + x_2}$

$\therefore \frac{I_2}{I_1} = \frac{236}{230} \times \frac{50}{57}$

$= 45.0$ A

28. An analog voltmeter uses external multiplier settings. With a multiplier setting of 20 kΩ, it reads 440 V and with a multiplier setting of 80 kΩ, it reads 352 V. For a multiplier setting of 40 kΩ, the voltmeter reads

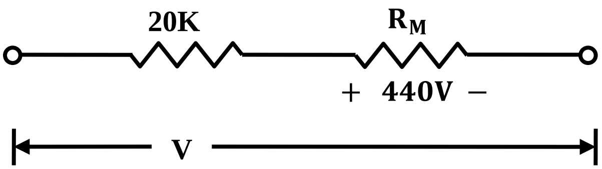
(A) 371 V
(B) 383 V
(C) 394 V
(D) 406 V

[Ans. Marks to All*] (*Ambiguous options)

One of the solution is as follows:

Here the problem is solved by assuming the terminal voltage across the meter + the multiplier resistor remains same.

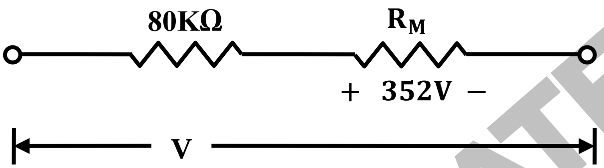
1) Given voltmeter reading as 440V when a multiplier resistance of 20 kΩ is used.



Let V be the terminal voltage and R_M be the meter resistance

$\therefore 440 = V\left[\frac{R_M}{R_M+20k}\right] \quad \rightarrow (1)$

2) Voltmeter reading was 352V for a multiplier resistance of 80 kΩ.



$\therefore 352 = V\left[\frac{R_M}{R_M+80k}\right] \quad \rightarrow (2)$

Solving (1) & (2), we get $R_M = 220k\Omega$

$\therefore$Terminal voltage = $440 + 20k\left[\frac{440}{220k}\right]$

$= 480$ V

$\therefore$ For a multiplier resistance of 40 kΩ, the voltmeter reading is $480\left[\frac{220k}{220k+40k}\right] = 406.15V$

≈ 406 V

29. The input x(t) and output y(t) of a system are related as $y(t) = \int_{-\infty}^{t} x(\tau)\cos(3\tau)\, d\tau$. The system is

(A) time-invariant and stable

(B) stable and not time-invariant

(C) time-invariant and not stable

(D) not time-invariant and not stable

[Ans. D]

- Assume a bounded input x(t) = cos (3t)

$$y(t) = \int_{-\infty}^{t} cos^2\ (3\tau)\ d\tau$$

Thus, y(t) is unbounded, hence, system is not stable.

- Assume $x(t) = \delta(t)$

$$y(t) = \int_{-\infty}^{t} \delta\,(\tau)\cos(3\tau)\ d\tau$$

$= u(t)\cos(0) = u(t)$

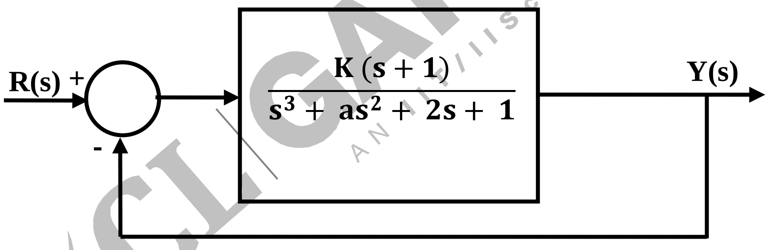
Time shifted input $x\left(t - \frac{\pi}{6}\right) = \delta\left(t - \frac{\pi}{6}\right)$

$$y'(t) = \int_{-\infty}^{t} \delta\left(\tau - \frac{\pi}{6}\right)\cos(3\tau)\ d\tau$$

$= u(t)\cos\left(3 \times \frac{\pi}{6}\right) = 0$

$y'(t) \neq y\left(t - \frac{\pi}{6}\right) \Longrightarrow$ System is not time – invariant

30. The feedback system shown below oscillates at 2 rad/s when



(A) K = 2 and a = 0.75
(B) K = 3 and a = 0.75
(C) K = 4 and a = 0.5
(D) K = 2 and a = 0.5

[Ans. A]

$1 + G(S)H(S) = S^3 + as^2 + (2+k)s + 1 + k$

s^3	$1\ (2+k)$
s^2	$a\ (2+k)$
s	$a(2+k) - (2+k)0$
s^0	$(1+k)^a$

For system to oscillate,

$$a(2+k)-(1+k)=0$$

$$a=\left(\frac{1+k}{2+k}\right)$$

$$A.E \Rightarrow as^2+(1+k)=0 \Rightarrow s=\sqrt{\frac{1+k}{a}}=2 \Rightarrow \left(\frac{1+k}{a}\right)=a \Rightarrow 2+k=4 \Rightarrow k=2$$

Thus a = 0.75

31. The Fourier transform of a signal h(t) is H(jω) = (2 cos ω) (sin 2ω)/ ω. The value of h(0) is

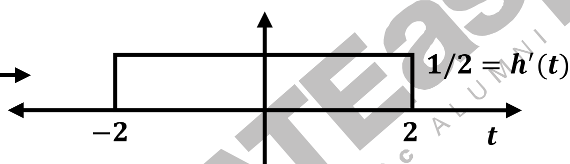
(A) 1/4 (C) 1

(B) 1/2 (D) 2

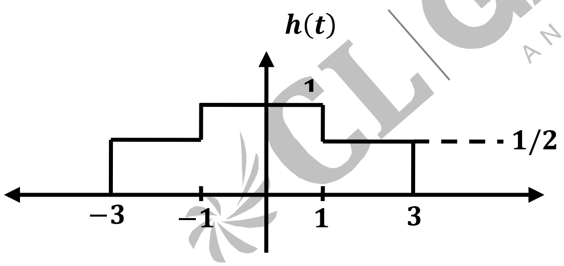
[Ans. C]

Approach – 1

$$\frac{\sin 2\omega}{\omega} \longleftrightarrow$$



$$2\cos\omega\left(\frac{\sin 2\omega}{\omega}\right)=\left[e^{j\omega}+e^{-j\omega}\right]\left(\frac{\sin 2\omega}{\omega}\right) \leftrightarrow h(t)=h'(t-1)+h'(t+1)$$

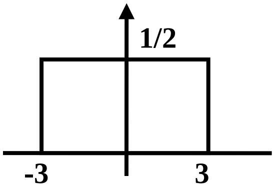


Approach – 2

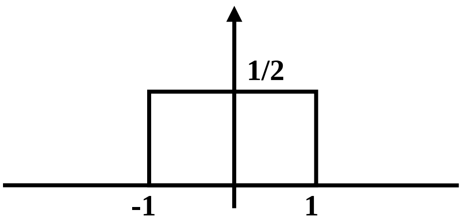
Given: $H(j\omega) = (2\cos\omega)\ (\sin 2\omega)/\omega$

Solution: $H(j\omega)=\frac{2\cos\omega\sin 2\omega}{\omega}$

$$H(j\omega)=\frac{\sin 3\omega}{\omega}+\frac{\sin\omega}{\omega}$$

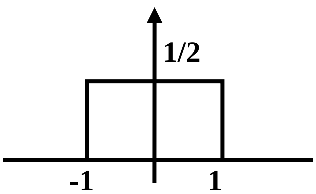


$\rightleftharpoons \quad \frac{\sin 3\omega}{\omega}$



$\rightleftharpoons \quad \frac{\sin \omega}{\omega}$

$h(t) =$



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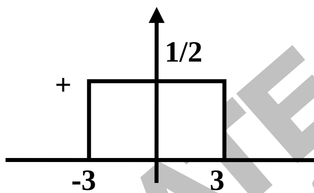


$h(0) = \frac{1}{2} + \frac{1}{2}$

$h(0) = 1$

32. The state variable description of an LTI system is given by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u$$

$$y = (1 \quad 0 \quad 0) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

where y is the output and u is the input. The system is controllable for

(A) $a_1 \neq 0, a_2 = 0, a_3 \neq 0$

(B) $a_1 = 0, a_2 \neq 0, a_3 \neq 0$

(C) $a_1 = 0, a_2 \neq 0, a_3 = 0$

(D) $a_1 \neq 0, a_2 \neq 0, a_3 = 0$

[Ans. D]

The controllability matrix

$= [B \quad AB \quad A^2B]$

$$A = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\Rightarrow$ Controllability matrix $= \begin{bmatrix} 0 & 0 & a_1 a_2 \\ 0 & a_2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

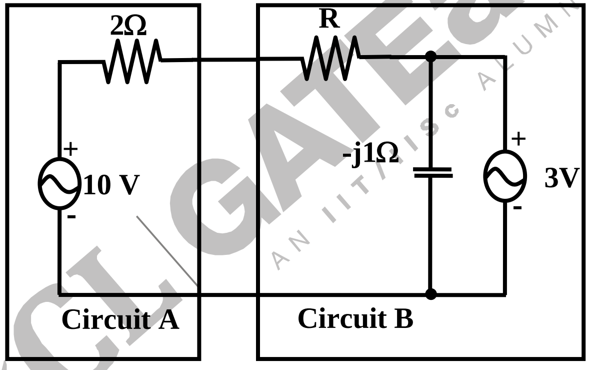
$\Rightarrow a_1 \neq 0$

$a_2 \neq 0$

a_3 can be zero

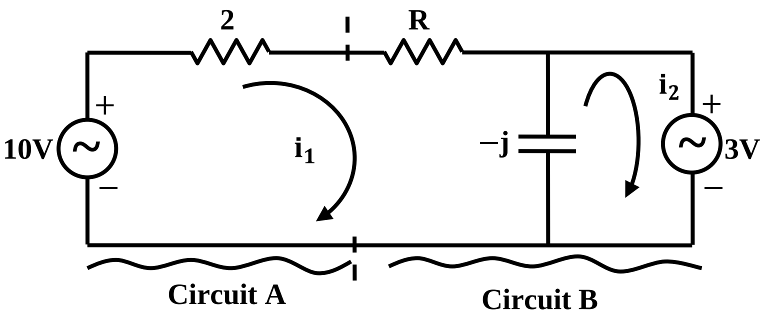
For system to be controllable, determinant of control ability matrix should not be zero.

33. Assuming both the voltage sources are in phase, the value of R for which maximum power is transferred from circuit A to circuit B is



(A) 0.8 Ω
(B) 1.4 Ω
(C) 2 Ω
(D) 2.8 Ω

[Ans. A]



From KVL: $10 = (2+R)i_1 + (i_1 - i_2)(-j)$

$10 = (2 + R - j)i_1 + ji_2$ -------------- (1)

$3 = -j(i_1 - i_2) \Rightarrow 3 = -ji_1 + ji_2$ ------------- (2)

From (1) & (2): $i_1 = \frac{7}{2+R}$; $i_2 = \frac{7}{2+R} - 3j$

From (2) $i_1 - i_2 = 3j$

Power transfer from circuit A to circuit B

$P = i_1^2 R + (i_1 - i_2)^2(-j) + 3i_2$

$= \frac{49R}{(2+R)^2} + 9j + 3\left(\frac{7}{2+R} - 3j\right) = \frac{7(10R+6)}{(2+R)^2}$

$\frac{dP}{dR} = 0$ for max power $\Rightarrow R = 0.8\ \Omega$

34. Consider the differential equation

$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \delta(t)$ with $y(t)|_{t=0^-} = -2$ and $\left.\frac{dy}{dt}\right|_{t=0^-} = 0$

The numerical value of $\left.\frac{dy}{dt}\right|_{t=0^+}$ is

(A) −2

(B) −1

(C) 0

(D) 1

[Ans. D]

Approach – 1

$$\frac{d^2y\,(t)}{dt^2} + \frac{2\,dy\,(t)}{dt} + y\,(t) = \delta(t)$$

Converting to s – domain,

$s^2y(s) - sy(0) - y'(0) + 2[sy(s) - y(0)] + y(s) = 1$

$[s^2 + 2s + 1]y(s) + 2s + 4 = 1$

$$y(s) = \frac{-3 - 2s}{(s^2 + 2s + 1)}$$

Find inverse Laplace transform

$y(t) = [-2e^{-t} - te^{-t}]\, u(t)$

$\frac{dy(t)}{dt} = 2e^{-t} + te^{-t} - e^{-t}$

$\left.\frac{dy(t)}{dt}\right|_{t=0^+} = 2 - 1 = 1$

Approach – 2

$\frac{d^2y(t)}{dt^2} + \frac{2dy(t)}{dt} + y(t) = \delta(t)$

Applying Laplace Transform on both sides

$s^2y(s) - sy(0^-) - \left.\frac{dy}{dt}\right|_{t=0^-} + 2\big(sy(s) - y(0^-)\big) + y(s) = 1$

$s^2y(s) + 2s + 2sy(s) + y(s) = 1 - 4$

$y(s) = \frac{-3 - 2s}{s^2 + 2s + 1} = \frac{-3}{(s+1)^2} - \frac{2s}{(s+1)^2}$

$y(t) = -3te^{-t} - 2\frac{d}{dt}\,(te^{t})$

$= -3te^{-t} - 2(-te^{-t} + e^{-t})$

$y(t) = -te^{-t} - 2e^{-t}$

$\frac{dy(t)}{dt} = +te^{-t} + (e^{-t}) + 2e^{-t}$

$= te^{-t} + e^{-t}$

$\left.\frac{dy(t)}{dt}\right|_{t=0^+} = 1$

35. The direction of vector A is radially outward from the origin, with $|A| = kr^n$ where $r^2 = x^2 + y^2 + z^2$ and k is a constant. The value of n for which ∇·A = 0 is

(A) −2 (C) 1

(B) 2 (D) 0

[Ans. A]

We know that, $\nabla.\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\,(r^2\,A_r)$

Now, $\nabla.\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\,(r^2A_r)$

$= \frac{1}{r^2}\frac{\partial}{\partial r}\,(kr^{rH2}) = \frac{k}{r^2}(n+2)\,r^{n+1}$

$= k(n+2)\,r^{n+1}$

$\therefore$ For, $\nabla.\vec{A} = 0, \Rightarrow (n+2) = 0 \Rightarrow n = -2$

36. A fair coin is tossed till a head appears for the first time. The probability that the number of required tosses is odd, is

(A) 1/3
(B) 1/2
(C) 2/3
(D) 3/4

[Ans. C]

If required tosses is odd the possible sequence of heads and tails will be:

H, TTH, TTTTH, TTTTTT H, ……………

Since, these events are mutually exclusive, we can add the prob. of each event.

Thus, the required prob. is given by

$P = \frac{1}{2} + \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} + \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} + \ldots\ldots\ldots\ldots\ldots..$

This is a geometric series with $a = \frac{1}{2}, r = \frac{1}{4}$

$$P = \frac{\frac{1}{2}}{1-\frac{1}{4}} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$

37. A 220 V, 15 kW, 1000 rpm shunt motor with armature resistance of 0.25 Ω, has a rated line current of 68 A and a rated field current of 2.2 A. The change in field flux required to obtain a speed of 1600 rpm while drawing a line current of 52.8 A and a field current of 1.8 A is

(A) 18.18% increase
(B) 18.18% decrease
(C) 36.36% increase
(D) 36.36% decrease

[Ans. D]

V = 220 V $N_1 =$ 100 rpm

$R_a = 0.25\,\Omega$

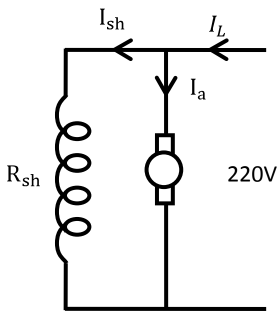
$I_{L_1} = 68$ A

$Ish_1 = 2.2$ A

$N_2 = 1600$ rpm

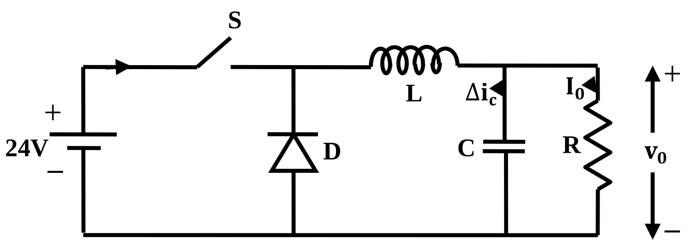
$I_{L_2} = 52.8$ A

$Ish_2 = 1.8$ A



$I_{a_1} = I_{L_1} = - Ish_1 = 68 - 22$
$= 65.8$ A
$Rsh_1 = \frac{220}{2.2} = 100\,\Omega$
$Rsh_2 = \frac{220}{1.8} = 122.22\,\Omega$
$I_{a_2} = I_{L_2} - Ish_2 = 52.8 - 1.8$
$= 51$A
$E_b = \frac{QPN_2}{60A} = kPN$
$\frac{E_b}{E_{b_2}} = \frac{Q_1 N_1}{d_2 n_2}$
$\frac{220 - I_{a_1}\ R_a}{220 - I_{a_2}\ Ra} = \frac{Q_1}{Q_2} \times \frac{100}{1600}$
$Q_1 = 1.57\, Q_2$
$\frac{Q_1 - Q_2}{Q_1} = \frac{1 - 0.637}{1} = 36.3\%.$

38. In the circuit shown, an ideal switch S is operated at 100 kHz with a duty ratio of 50%. Given that Δi_c is 1.6 A peak-to-peak and I_0 is 5 A dc, the peak current in S is



(A) 6.6 A
(B) 5.0 A
(C) 5.8 A
(D) 4.2 A

[Ans. C]

Peak current = $I_o + \frac{\Delta i_c}{2} = 5 + \frac{1.6}{2} = 5.8A$

39. A cylindrical rotor generator delivers 0.5 pu power in the steady-state to an infinite bus through a transmission line of reactance 0.5 pu. The generator no-load voltage is 1.5 pu and the infinite bus voltage is 1 pu. The inertia constant of the generator is 5 MW-s/MVA and the generator reactance is 1 pu. The critical clearing angle, in degrees, for a three-phase dead short circuit fault at the generator terminal is

(A) 53.5
(B) 60.2
(C) 70.8
(D) 79.6

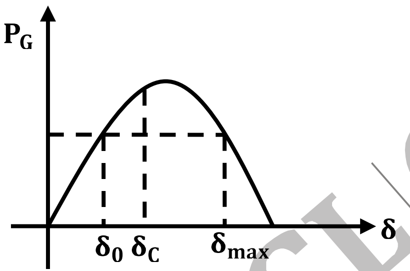
[Ans. D]

The critical clearing angle formula for 3 – ϕ dead short ckt fault is

$\delta_C = \cos^{-1}\left[\frac{P_S(\delta_{max} - \delta_0) + P_{m3}\cos\delta_{max}}{P_{m3}}\right]$

$P_S \rightarrow$ Steady state power = 0.5 p.u

$\delta_{max} = 180 - \delta_0$



$P_{m_2} \rightarrow$ Max power output during fault = 0

$P_{m_3} \rightarrow$ Max power output After fault

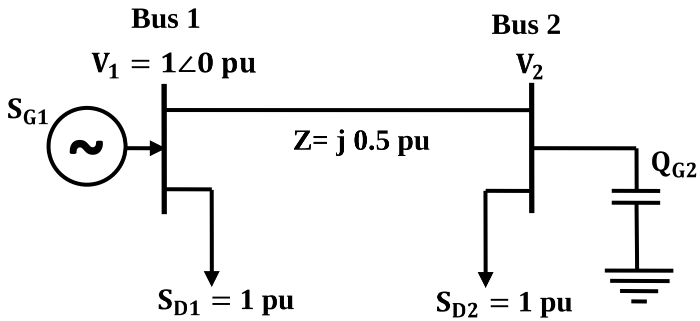
$P_S = P_{max\,1}\sin\delta_0 = \frac{V_1V_2}{X}\sin\delta_0 \quad V_1 = 1.5 \;\; V_2 = 0.5$

$X = X_l + X_g = 0.5 + 1 = 1.5$

$0.5 = \frac{1.5}{1.5}\sin\delta_0 \implies \delta_0 = 30°$

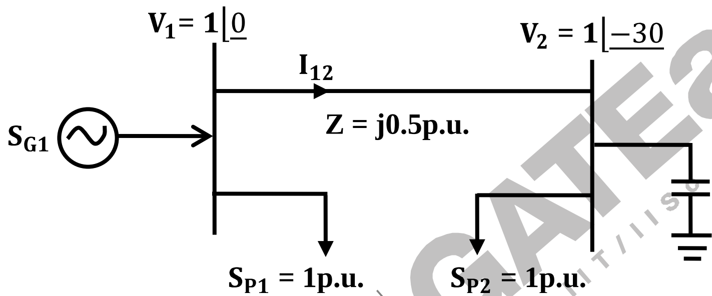
$\delta_{max} = 180 - 30 = 150°$

40. For the system shown below, S_{D1} and S_{D2} are complex power demands at bus 1 and bus 2 respectively. If $|V_2| = 1$ pu, the VAR rating of the capacitor (Q_{G2}) connected at bus 2 is



(A) 0.2 pu
(B) 0.268 pu
(C) 0.312 pu
(D) 0.4 pu

[Ans. B]



Line lossless $S_{G1} = S_{D1} + S_{D2} = 1 + 1 = 2$p.u.
Power transfer from bus-1 to bus-2 is 1p.u.

$\therefore 1 = \frac{|V_1|\,|V_2|}{X_{12}} \sin(\theta_1 - \theta_2) = \frac{1\times 1}{0.5} \sin(\theta_1 - \theta_2)$; $\sin(\theta_1 - \theta_2) = 0.5$

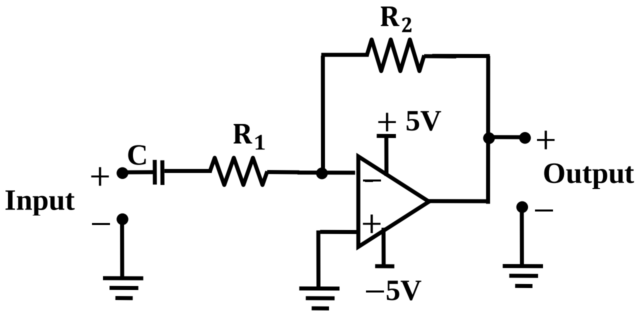
$\theta_1 - \theta_2 = \sin^{-1} 0.5 = 30$; $\theta_1 = 0$ $\{V_1 = 1\angle 0\}$; $\therefore \theta_2 = -30^o$; $V_2 = 1\angle -30$

$I_{12} = \frac{V_1 - V_2}{2} = \frac{1\angle 0 - 1\angle -30}{j0.5} = 1 - j0.288$

Current $S_{D2} = 1\angle -30$; Current in $Q_{G2} = 1\angle -30 - [1 - j0.268] = 0.268\angle -120$

VAR rating of capacitor = $|V_2|\,|I_Q| \sin(|V_2||I_2|) = 1 \times 0.268 \times \sin(+90) = 0.268$

41. The circuit shown is a



(A) low pass filter with $f_{3dB} = \frac{1}{(R_1+R_2)C}$ rad/s

(B) high pass filter with $f_{3dB} = \frac{1}{R_1C}$ rad/s

(C) low pass filter with $f_{3dB} = \frac{1}{R_1C}$ rad/s

(D) high pass filter with $f_{3dB} = \frac{1}{(R_1+R_2)C}$ rad/s

[Ans. B]

The transfer function of the N/W is $\frac{V_0(s)}{V_i(s)} = \frac{-R_2}{R_1+\frac{1}{CS}} = -\frac{R_2CS}{R_1CS+1}$

This represents H.P filter with cutoff frequency at $\frac{1}{R_1C}$

42. Let y[n] denote the convolution of h[n] and g[n], where h[n] – $(1/2)^n$ u[n] and g[n] is a causal sequence. If y[0] = 1 and y[1] = 1/2, then g[1] equals

(A) 0
(B) 1/2
(C) 1
(D) 3/2

[Ans. A]

$$y[n] = \sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^k g(n-k)$$

$$y[0] = \sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^k g(-k) = 1$$

$$\Rightarrow \left(\frac{1}{2}\right)^O g(0) = 1$$

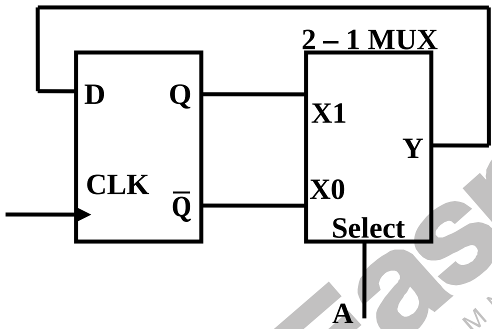
$\Rightarrow g(0) = 1$ since g(n) is Causal sequence $g(-1), g-2),$………….. = 0

$$y[1] = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k g[1-k]$$

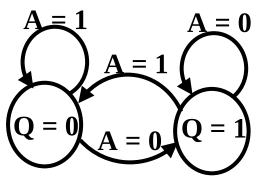
$$\Rightarrow \left(\frac{1}{2}\right)^0 g[1] + \left(\frac{1}{2}\right)^1 g(0) = {}^1/_2$$

$$g[1] = 0$$

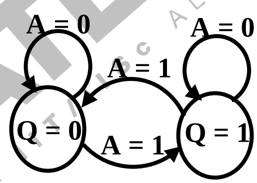
43. The state transition diagram for the logic circuit shown is



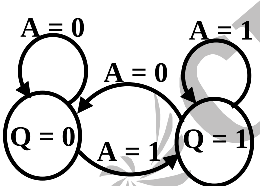
(A)



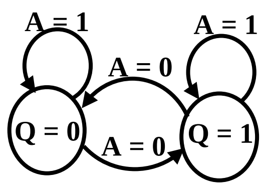
(B)



(C)

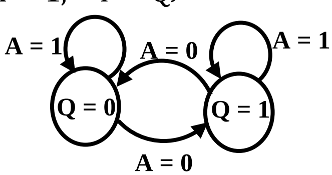


(D)



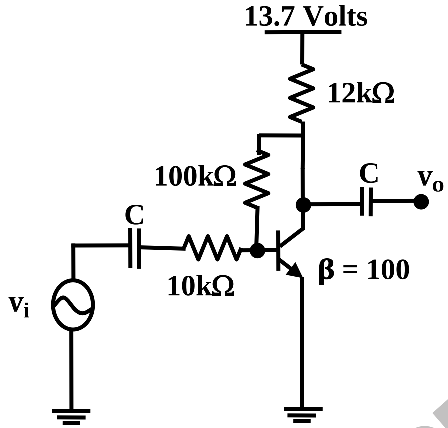
[Ans. D]

$$\left.\begin{array}{ll} A = 0, & Y = Q \\ A = 1, & Y = \overline{Q} \end{array}\right\}$$ whenever A = 1, output gets into same state



Whenever A = 0, output gets toggled

44. The voltage gain A_v of the circuit shown below is



(A) $|A_v| \approx 200$

(B) $|A_v| \approx 100$

(C) $|A_v| \approx 20$

(D) $|A_v| \approx 10$

[Ans. D]

Approach – 1

This is voltage shunt feedback if we neglect base to emitter voltage. Feedback factor $P_f = \frac{1}{100 \times 103}$

$= \frac{1}{105}\Omega - 1$

Now with F/B,

$$A_2 = \frac{V_0}{I_i} = 12 \times 10^5 \Omega$$

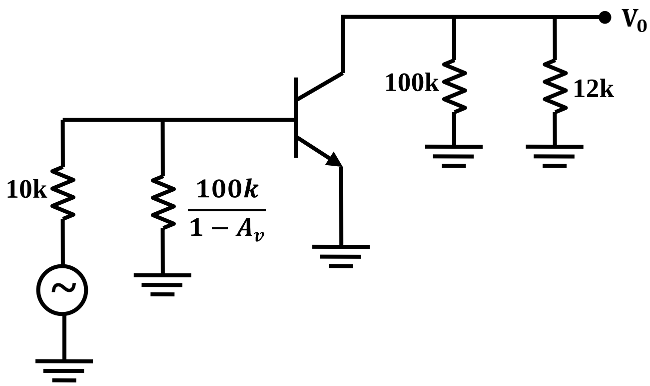
So with feedback

$$\frac{V_0}{I_f} = A_{2_f} = \frac{A_2}{1+\beta\ A_2} = \frac{12\times105}{\beta} \simeq 10^5$$

But $I_i = \frac{V_L}{10\times10^3} = \frac{V_L}{10^4}$

$$\frac{V_0}{V_L} \simeq \frac{10^5}{10^4} \simeq 10$$

Approach – 2



KVL in input loop, $13.7 - (I_C + I_B)12k - 100k\ (I_B) - 0.7 = 0$

$\Rightarrow\ I_B - 9.9\mu A$; $I_C = \beta I_B = 0.99mA$; $I_E = 1mA$

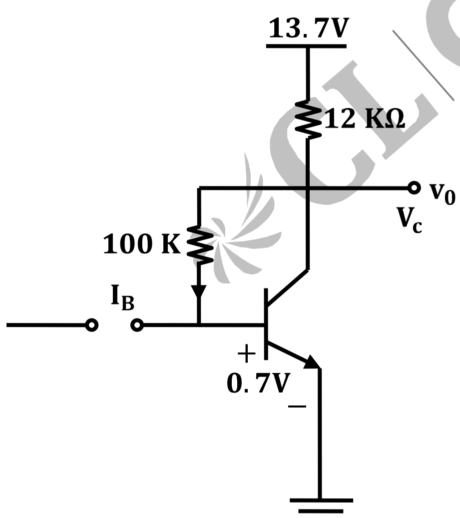
$\therefore\ r_e = \frac{26mA}{I_E} = 26\,\Omega$; $z_i = \beta r_e = 2.6k\Omega$; $\therefore A_v = \frac{(100k||12k)}{26} = 412$

$z_i' = z_i||\left(\frac{100k}{1+412}\right) = 221\,\Omega$; $A_{vs} = A_v \frac{z_i'}{z_i'+R_s} = (412)\left(\frac{221}{221+10k}\right)$

$|A_{vs}| \approx 10$

Approach – 3

This is a shunt – shunt feedback amplifier output voltage is sampled and current is feedback , , , DC circuit reduces to



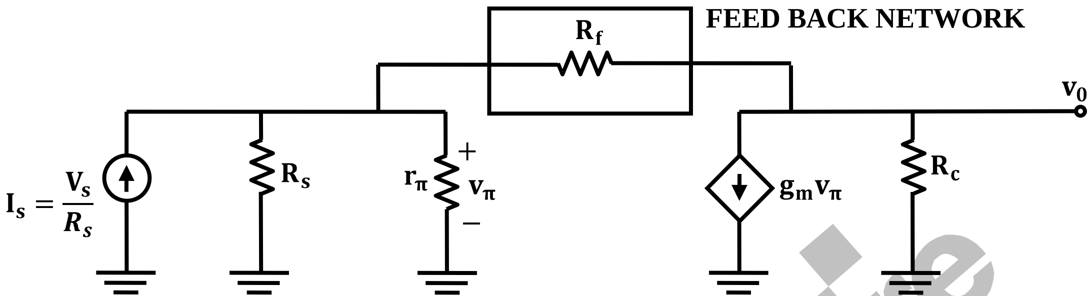
$V_C = 100\ I_B + 0.7$ ---------------- (1)

$\frac{13.7 - V_C}{12} = (\beta + 1)\ I_B$ -------------- (2)

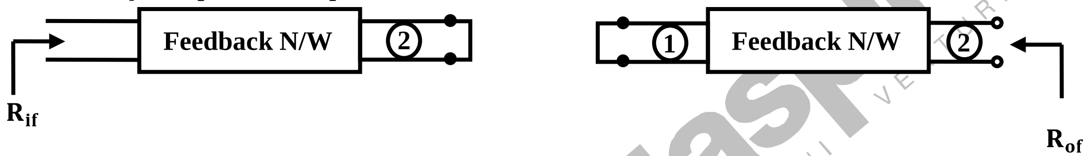
From (1) and (2)

$$\frac{13.7 - 100\,I_B - 0.7}{12} \simeq \beta I_B = 100\,I_B$$

$\Rightarrow I_B = 0.01\,mA$ and $I_C = \beta I_B = 1\text{ mA}$

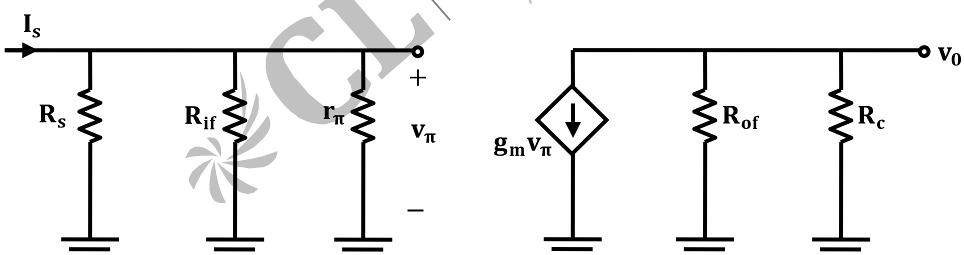


To divide R_f to input and output side



So

So, our circuit redeces to,



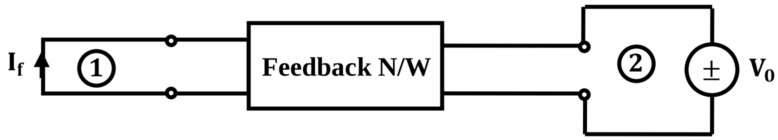
$$A = \frac{V_0}{I_S} = -\frac{g_m(R_f||R_c)v_\pi}{\dfrac{v_\pi}{(R_s||R_{if}||r_\pi)}} = -g_m\,(R_{of}||R_c)(R_s||R_{if}||r_\pi)$$

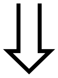
Now $R_{of} = R_{if} = 100k\Omega$, $R_c = 12k\Omega$, $r_\pi = \frac{V_T}{I_B} = 2.5k\,\Omega$, and $R_s = 10\text{ k}$

Also, $g_m = \frac{I_C}{V_T} = \frac{40\,mA}{V}$

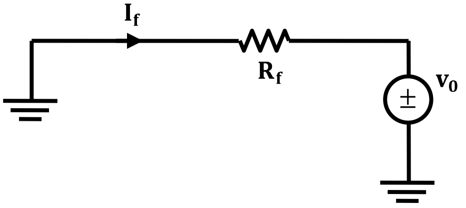
$A = -840.336\,k\Omega$ ---------------- (1)

Note: This is a trans-resistance amplifier. Now feedback factor, for, shunt – shunt configuration is





$$\beta \equiv \left.\frac{I_f}{V_0}\right|_{V_I=0}$$



$$\frac{v_0}{I_f} = \frac{1}{\beta} \Rightarrow \beta = \frac{I_f}{v_0} = -\frac{1}{R_f}$$

So here $\beta = \frac{-1}{100k\Omega}$ ---------- (2)

So gain after feedback

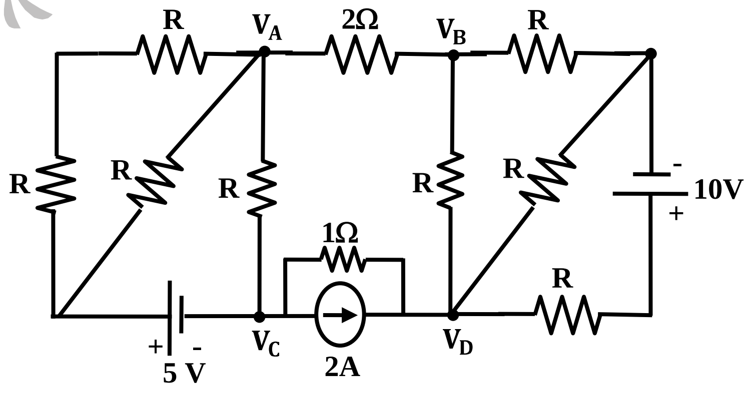
$$\frac{V_0}{I_s} = \frac{A}{1+A\beta} \Rightarrow \frac{v_0}{I_s} = \frac{-840.336k}{1+8.40336} \approx 10^5 \text{ (Approx.)}$$

So,

$$\frac{V_0}{V_I} = \frac{V_0}{I_s\, R_s} = \frac{1}{R_s} \times \frac{v_0}{I_s} = \frac{1}{10k} \times 10^5 = 10$$

So $|A_v| \approx 10$

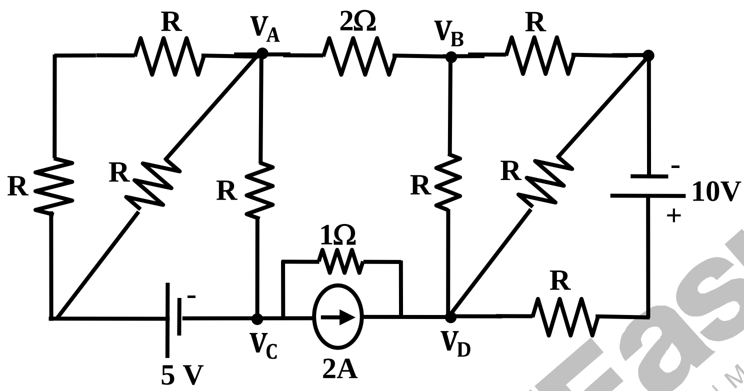
45. If $V_A - V_B = 6$ V, then $V_C - V_D$ is

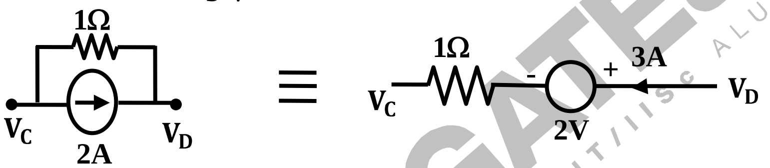


(A) $-5V$
(B) 2V
(C) 3V
(D) 6V

[Ans. A]

$I = \frac{V_A - V_B}{2} = \frac{6}{2} = 3A$; Since current entering any network is same as leaving in $V_C - V_D$ branch also it is I = 3A





$V_D = 2 + 3 + V_C = 5 + V_C; V_C - V_D = -5V$

46. The maximum value of $f(x) = x^3 - 9x^2 + 24x + 5$ in the interval [1, 6] is
(A) 21
(B) 25
(C) 41
(D) 46

[Ans. C]

$f(x) = x^3 - 9x^2 + 24x + 5$

$f(x) = x^3 - 9x^2 + 24x + 5$

$\frac{df(x)}{dx} = 3x^2 - 18x + 24 = 0$

$x^2 - 6x + 8 = 0$

$(x - 2)(x - 4) = 0$

x = 2, x = 4 (critical points)

$\frac{d^2f(x)}{dx^2} = 6x - 18$

$= 6(2) - 18 < 0$ (for x=2)

$\frac{d^2f(x)}{dx^2} =$ 6(4) – 18 > 0 (for x=4)

∴ Maximum at x = 2

f(2) = $2^3 - 9(2)^2 + 24(2) + 5$

$= 8 - 36 + 48 + 5$

= 25

We have to find the maximum in the close interval [1, 6]

Hence, we have to check at end points also (as extremum exists at the critical points or end points)

f(6) = $(6)^3 - 9(6)^2 + 24(6) + 5$

= 41

∴ Maximum value = 41

47. Given that

$A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the value of A^3 is

(A) 15 A + 12 I
(B) 19 A + 30 I
(C) 17 A + 15 I
(D) 17 A + 21 I

[Ans. B]

Given: $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$

We know that Every characteristic equation satisfies its own matrix

$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} -5-\lambda & -3 \\ 2 & -\lambda \end{vmatrix} = 0$

$+5\lambda + \lambda^2 + 6 = 0$

$\Rightarrow \lambda^2 + 5\lambda + 6 = 0$

We know by Cayley Hamilton Theorem that every characteristic equation satisfies its own matrix.

$\therefore A^2 + 5A + 6I = 0$

$\Rightarrow A^3 + 5A^2 + 6A = 0$

$A^3 + 5(-5A - 6I) + 6A = 0$

$\therefore A^3 = 19A + 30I$

Common Data Questions

Common Data for Question 48 and 49:

With 10 V dc connected at port A in the linear nonreciprocal two-port network shown below, the following were observed:

(i) 1 Ω connected at port B draws a current of 3 A

(ii) 2.5 Ω connected at port B draws a current of 2 A

48. With 10 V dc connected at port A, the current drawn by 7 Ω connected at port B is

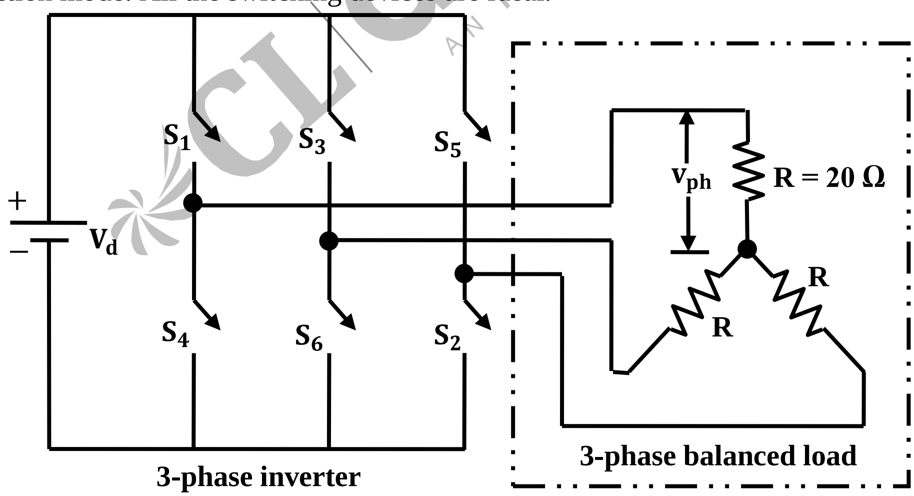
(A) 3/7 A
(B) 5/7 A
(C) 1 A
(D) 9/7 A

49. For the same network, with 6 V dc connected at port A, 1 Ω connected at port B draws 7/3 A. If 8 V dc is connected to port A, the open circuit voltage at port B is

(A) 6 V
(B) 7 V
(C) 8 V
(D) 9 V

Common Data for Questions 50 and 51:

In the 3-phase inverter circuit shown, the load is balanced and the gating scheme is 180°-conduction mode. All the switching devices are ideal.



50. If the dc bus voltage $V_d = 300$ V, the power consumed by 3-phase load is

(A) 1.5 kW
(B) 2.0 kW

(C) 2.5 kW (D) 3.0 kW

[Ans. D]

Power consumed by 3 phase load

$P = 3 \times \frac{(V_{ph}rms)^2}{R} = 3 \times \frac{(141.4)^2}{20} = 3\ kW$

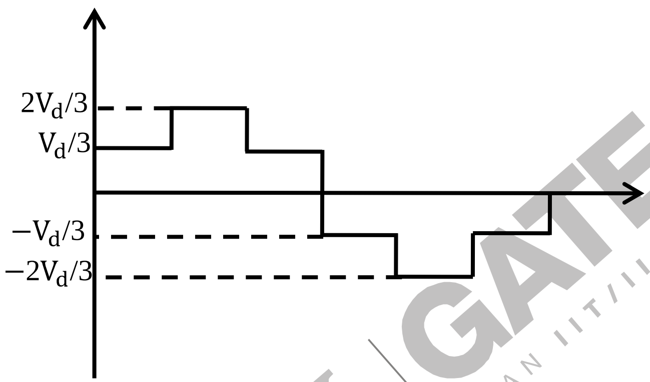
51. The rms value of load phase voltage is

(A) 106.1 V (C) 212.2 V

(B) 141.4 V (D) 282.8 V

[Ans. B]

Load phase voltage waveform



RMS value of phase voltage = $V_{rms} = \sqrt{\frac{2}{9}}\ V_d = \sqrt{\frac{2}{9}} \times 300 = 141.4V$

Linked Answer Questions

Statement for Linked Answer Question 52 and 53:

Transfer function of a compensator is given as

$G_c(s) = \frac{s+a}{s+b}$

52. $G_c(s)$ is a lead compensator if

(A) $a = 1, b = 2$ (C) $a = -3, b = -1$

(B) $a = 3, b = 2$ (D) $a = 3, b = 1$

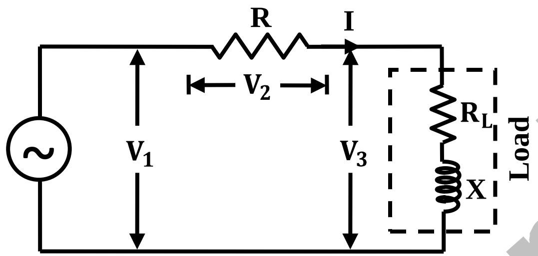
53. The phase of the above lead compensator is maximum at

(A) $\sqrt{2}$ rad/s (B) $\sqrt{3}$ rad/s

(C) $\sqrt{6}$ rad/s
(D) $1/\sqrt{3}$ rad/s

Statement for Linked Answer Questions 54 and 55:

In the circuit shown, the three voltmeter readings are $V_1 = 220$ V, $V_2 = 122$ V, $V_3 = 136$ V.



54. The power factor of the load is
(A) 0.45
(B) 0.50
(C) 0.55
(D) 0.60

[Ans. A]

$\cos\theta = \frac{V_1^2 - V_2^2 - V_3^2}{2V_1V_2} = \frac{220^2 - 122^2 - 136^2}{2\times122\times136} = 0.45$

55. If $R_L = 5\Omega$, the approximate power consumption in the load is
(A) 700 W
(B) 750 W
(C) 800 W
(D) 850 W

[Ans. B]

$\cos\theta = \frac{R_L}{z}; 0.45 = \frac{5}{z} \Rightarrow z = 11.11$

$I = \frac{V_3}{z} = \frac{136}{11.11} = 12.24A; P_L = I^2R_L = 12.24^2 \times 5 = 750W$

General Aptitude (GA) Questions (Compulsory)

Q. 56 – Q. 60 carry one mark each.

56. Choose the most appropriate word from the options given below to complete the following sentence:

Given the seriousness of the situation that he had to face, his _____ was impressive

(A) beggary
(B) nomenclature
(C) jealousy
(D) nonchalance

[Ans. D]
Nonchalance means behaving in a calm and relaxed way; giving the impression that you are not feeling any anxiety.

57. If $(1.001)^{1259} = 3.52$ and $(1.001)^{2062} = 7.85$, then $(1.001)^{3321} =$
(A) 2.23
(B) 4.33
(C) 11.37
(D) 27.64

[Ans. D]
$(1.001)^{1259} = 3.52$ and $(1.001)^{2062} = 7.85$, then $(1.001)^{1259} \times (1.001)^{2062} = 3.52 \times 7.85$ or $(1.001)^{(1259+2062)} = 3.52 \times 7.85$ or $(1.001)^{3321} = 27.64$

58. Which one of the following options is the closest in meaning to the word given below?
Latitude
(A) Eligibility
(B) Freedom
(C) Coercion
(D) Meticulousness

[Ans. B]
Latitude means freedom to choose what you do or the way that you do it.

59. Choose the most appropriate alternative from the options given below to complete the following sentence:
If the tired soldier wanted to lie down, he _______ the mattress out on the balcony.
(A) should take
(B) shall take
(C) should have taken
(D) will have taken

[Ans. A]

60. One of the parts (A, B, C, D) in the sentence given below contains an ERROR. Which one of the following is INCORRECT?
I requested that he should be given the driving test today instead of tomorrow.
(A) requested that
(B) should be given
(C) the driving test
(D) instead of tomorrow

[Ans. B]

The correct statement should be -" i requested that he be given the driving test today instead of tomorrow."

Q. 61 – Q. 65 carry two marks each.

61. There are eight bags of rice looking alike, seven of which have equal weight and one is slightly heavier. The weighing balance is of unlimited capacity. Using the balance, the minimum number of weighing required to identify the heavier bag is

(A) 2
(B) 3
(C) 4
(D) 8

[Ans. A]

62. **One of the legacies of the Roman legions was discipline. In the legions, military law prevailed and discipline was brutal. Discipline on the battlefield kept units obedient, intact and fighting, even when the odds and conditions were against them.**

Which one of the following statements best sums up the meaning of the above passage?

(A) Through regimentation was the main reason for the efficiency of the Roman legions even in adverse circumstances.
(B) The legions were treated inhumanly as if the men were animals.
(C) Discipline was the armies' inheritance from their seniors.
(D) The harsh discipline to which the legions were subjected to led to the odds and conditions being against them.

[Ans. A]

The passage states that the strict discipline kept the armies intact even when condition were against them.

63. The data given in the following table summarizes the monthly budget of an average household.

Category	**Amount (Rs.)**
Food	4000
Clothing	1200
Rent	2000
Savings	1500
Other expenses	1800

The approximate percentage of the monthly budget NOT spent on savings is

(A) 10%
(B) 14%
(C) 81%
(D) 86%

[Ans. D]

Total Income = 10500

Savings =1500

Percentage of budget spent on savings = $\frac{1500}{10500} \times 100 = 14.28\%$

Percentage of budget not spent on savings = 86%

64. Raju has 14 currency notes in his pocket consisting of only Rs. 20 notes and Rs. 10 notes. The total money value of the notes is Rs. 230. The number of Rs. 10 notes that Raju has is

(A) 5
(B) 6
(C) 9
(D) 10

[Ans. A]

Let the number of Rs. 10 notes = x

And the number of Rs. 20 notes = y

Now,

x + y = 14 and 10x + 20y =230.

Solving them, we get x = 5 and y = 9.

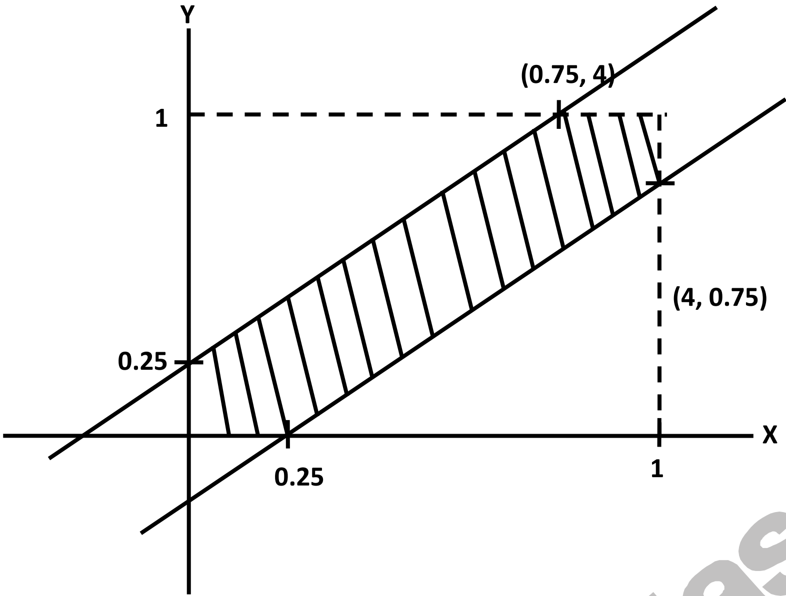
65. A and B are friends. They decide to meet between 1 PM and 2 PM on a given day. There is a condition that whoever arrives first will not wait for the other for more than 15 minutes. The probability that they will meet on that day is

(A) 1/4
(B) 1/16
(C) 7/16
(D) 9/16

[Ans. C]

We can solve this question graphically.

Let x axis represents the time when A reaches the meeting place and y axis represents the time when B reaches the same place.



Given conditions are

Total area is represented by

$0 \le x \le 1$ and $0 \le y \le 1$.

Total area = 1 × 1 = 1 unit

Desired area is represented by

$0 \le x \le 1, 0 \le y \le 1, x - y \le 1/4$ and $y - x \le 1/4$

$$\text{Desired area} = 1 - \left(\frac{1}{2}\times\frac{3}{4}\times\frac{3}{4}\right) - \left(\frac{1}{2}\times\frac{3}{4}\times\frac{3}{4}\right) = \frac{7}{16}$$

$$\text{Required probability} = \frac{desired\ area}{total\ area} = \frac{7}{16\times1} = \frac{7}{16}$$