GATE 2012

Instrumentation Engineering

Q. 1 – Q. 25 carry one mark each.

1. If x = $\sqrt{-1}$, then the value of x^x is

(A) $e^{-\pi/2}$

(B) $e^{\pi/2}$

(C) x

(D) 1

[Ans. A]

Approach – 1

Given, $x = \sqrt{-1}$; $x^x = \left(\sqrt{-1}\right)^{\sqrt{-1}} = i^i$

We know that $e^{i\theta} = \cos\theta + i\sin\theta \Rightarrow e^{i\frac{\pi}{2}} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$

$\therefore\ (i)^i = \left(e^{i\pi/2}\right)^i = e^{-\pi/2}$

Approach – 2

$x = \sqrt{-1} = i = e^{i\pi/2}$

Let y = x^x

Taking Natural logarithm on both sides

$\log y = \log x^x$

$\log y = x\log x$

$= e^{i\pi/2}\log e^{i\pi/2}$

$= i\left(i\pi/2\right)$

$\log y = -\pi/2$

$y = e^{-\pi/2}$

$\therefore\ x^x = e^{-\pi/2}$

2. With initial condition x(1) = 0.5, the solution of the differential equation.

$t\frac{dx}{dt} + x = t$ is

(A) $x = t - \frac{1}{2}$

(B) $x = t^2 - \frac{1}{2}$

(C) $x = \frac{t^2}{2}$

(D) $x = \frac{t}{2}$

[Ans. D]

Approach – 1

Just substitute, $x = \frac{t}{2}$, or divide by t, and take integrating fact.

Approach – 2

Given DE is $t\frac{dx}{dt} + x = t \Rightarrow \frac{dx}{dt} + \frac{x}{t} = 1$

IF $= e^{\int\frac{1}{t}dt} = e^{\log t} = t$; solution is x (IF) = $\int(IF)$ tdt

$xt = \int t.tdt \Rightarrow xt = \frac{t^2}{2} + c$; Given that $x(1) = 0.5 \Rightarrow 0.5 = \frac{1}{2} + c \Rightarrow c = 0$

$\therefore$ the required solution is $xt = \frac{t^2}{2} \Rightarrow x = \frac{t}{2}$

Approach – 3

Given: $t\frac{dx}{dt} + x = t, x(1) = 0.5$

$t\frac{dx}{dt} + x = t$

$t\,dx + xdt = tdt$

$d(xt) = t\,dt$

$xt = \frac{t^2}{2} + C$

Using initial condition, at t = 1, x = 0.5

$0.5 \times 1 = \frac{1}{2} + C$

$C = 0$

$\therefore xt = \frac{t^2}{2}$

$x = \frac{t}{2}$

3. Two independent random variables X and Y are uniformly distributed in the interval [−1, 1]. The probability that max[X, Y] is less than 1/2 is

(A) 3/4
(B) 9/16
(C) 1/4
(D) 2/3

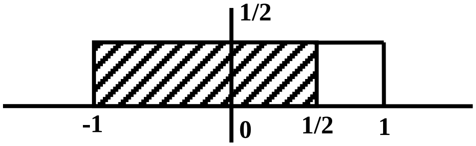
[Ans. B]

$P\left[\max[X, Y] < \frac{1}{2}\right] = P\left[X < \frac{1}{2}, Y < \frac{1}{2}\right]$ (If maximum is < $\frac{1}{2}$ then both are less than $\frac{1}{2}$)

$P\left[x < \frac{1}{2}, y < \frac{1}{2}\right]$

$P\left[x<\frac{1}{2}\right].P\left[y<\frac{1}{2}\right]$ (since independent events)

Probability density function of X and Y



$=\frac{3}{4}\times\frac{3}{4}$

$=\frac{9}{16}$

4. The unilateral Laplace transform of f(t) is $\frac{1}{s^2+s+1}$. The unilateral Laplace transform of t f(t) is

(A) $-\frac{s}{(s^2+s+1)^2}$

(B) $-\frac{2s+1}{(s^2+s+1)^2}$

(C) $\frac{s}{(s^2+s+1)^2}$

(D) $\frac{2s+1}{(s^2+s+1)^2}$

[Ans. D]

$L\{t.f(t)\}=(-1)^1.\frac{d}{ds}F(s)$

$=-\frac{d}{ds}\left(\frac{1}{s^2+s+1}\right)$

= 1

5. Given

$f(z)=\frac{1}{z+1}-\frac{2}{z+3}$. If C is a counterclockwise path in the z-plane such that $|z+1|=1$, the value of $\frac{1}{2\pi j}\oint_c f(z)dz$ is

(A) – 2

(B) – 1

(C) 1

(D) 2

[Ans. C]

Z = –3 is outside circle

Z = 1 is inside circle

$\lim_{z\to-1}(z+1).\frac{1}{(z+1)}=1$

6. The average power delivered to an impedance (4 – j3) Ω by a current 5 cos(100 π t + 100) A is

(A) 44.2 W

(B) 50 W

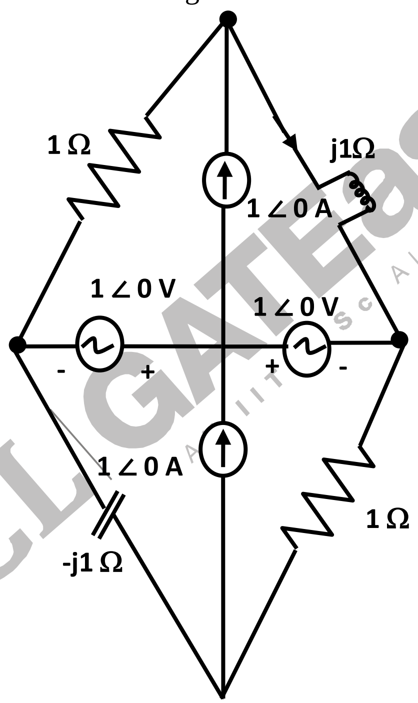
(C) 62.5 W

(D) 125 W

[Ans. B]

$$P_{avg} = \frac{1}{2}\text{Re}\{VI^*\}$$
$$= \frac{1}{2}\text{Re}\{IZI^*\}$$
$$= \frac{1}{2}\text{Re}\{|I|^2 Z\}$$
$$= \frac{1}{2}|I|^2\text{Re}\{Z\}$$
$$= \frac{1}{2} \times 5^2 \times 4 \qquad (\because \text{Re}\{Z\} = 4)$$
= 50 W

7. In the circuit shown below, the current through the inductor is



(A) $\frac{2}{1+j}$ A

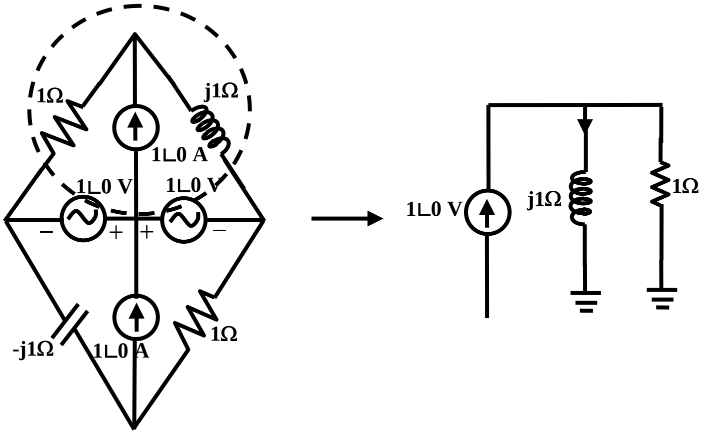
(B) $\frac{-1}{1+j}$ A

(C) $\frac{1}{1+j}$ A

(D) 0 A

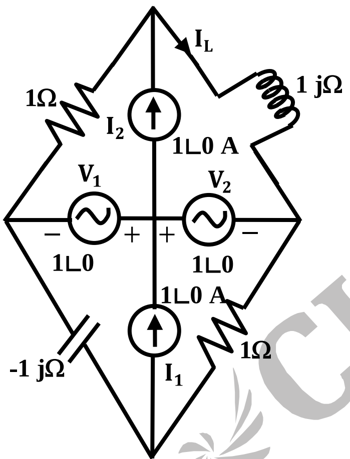
[Ans. C]

Approach – 1



$$I_L = 1\angle 0 \times \frac{1}{1+j1} = \frac{1}{1+j1}\ A$$

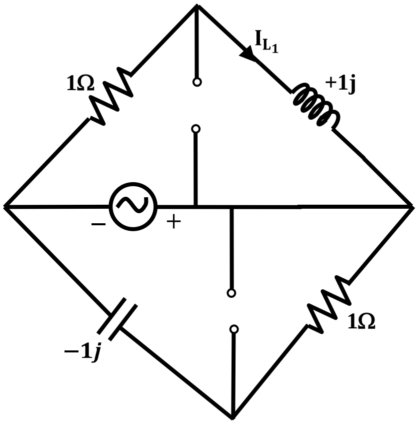
Approach – 2



Apply Superposition theorem,

V_1 only: Short circuit

V_2 open circuit I_1 and I_2



$I_{L_1} = \frac{-1}{1+1j}$ -------------- (1)

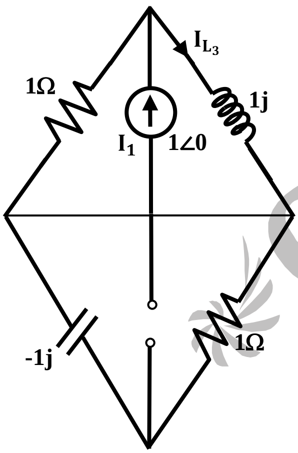
$\left[I_{L_1} = \frac{-V_1}{Total\ Impedance}\right]$

Similarly for V_2 only

$I_{L_2} = \frac{1}{1+1j}$ -------------- (2)

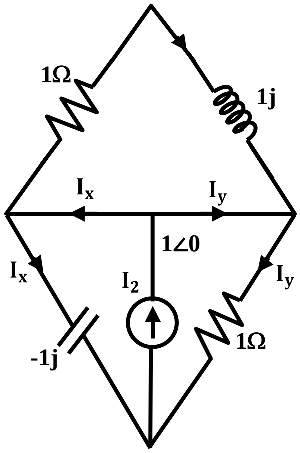
For I_1 only

Using current divider rule



$I_{L_3} = \frac{1}{1+1j} \times 1 < 0 = \frac{1}{1+1j}$ -------------- (3)

For I_2 only



$I_2 = I_x + I_y$

Current in Inductor = 0

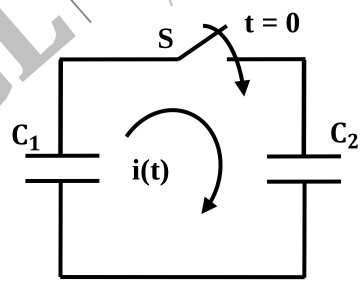
So $I_{L_4} = 0$ ------------ (4)

From (1), (2), (3), (4), Total current,

$$I_L = \frac{-1}{1+1j} + \frac{1}{1+1j} + \frac{1}{1+1j} + 0$$

$\Rightarrow I_L = \frac{1}{1+1j}$

8. In the following figure, C_1 and C_2 are ideal capacitors. C_1 has been charged to 12 V before the ideal switch S is closed at t = 0. The current i(t) for all t is

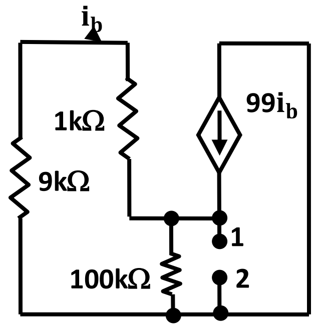


(A) zero
(B) a step function
(C) an exponentially decaying function
(D) an impulse function

[Ans. D]

Initially when switch is closed, impulse current flows from C1 to C2 till voltages of C1 and C2 becomes equal. This happens due to the fact that there is a potential difference initially between C_1 and C_2, but resistance in the circuit is zero leading to an infinite current. Once charge is equal in C1 and C2, current i(t) will be zero.

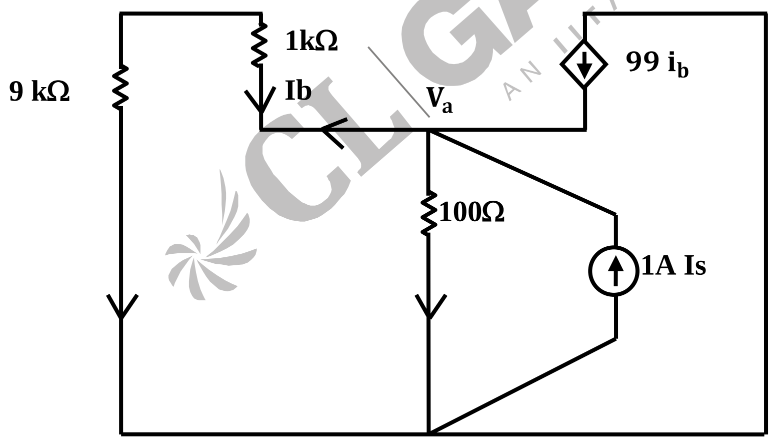
9. The impedance looking into nodes 1 and 2 in the given circuit is



(A) 50 Ω
(B) 100 Ω
(C) 5 kΩ
(D) 10.1kΩ

[Ans. A]

Approach – 1



By nodal analysis at node a,

$$\frac{V_a - 0}{10k} + \frac{V_a}{100} - 99\, i_b = 1$$

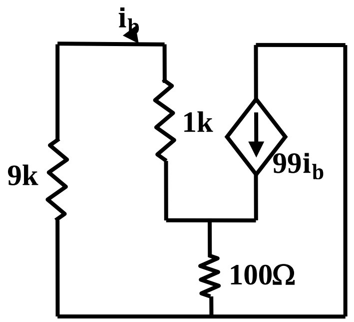
$$\frac{V_a - 0}{10k} + \frac{V_a}{100} - 1 + \frac{99\, V_a}{10k} = 0$$

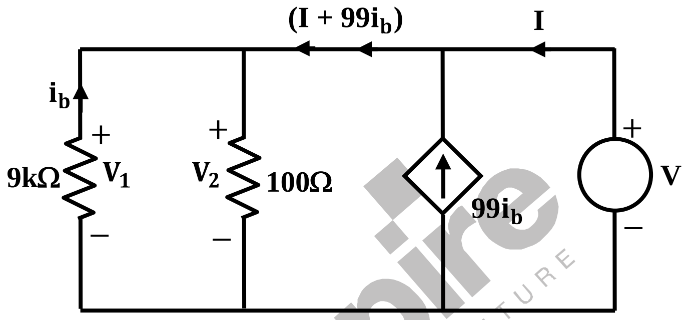
$$\Rightarrow V_a \left[\frac{100}{10k} + \frac{100}{10k}\right] = 1.$$

$\Rightarrow V_a = 50V$

$\therefore R\ (\text{thevenin}) = \frac{V_a}{I_s} = 50\Omega$

Approach – 2





After connecting a voltage source of V

$V_1 = V_2 \Rightarrow (10k)(-i_b) = 100(I + 99i_b + i_b);$

$-10000i_b = 100I + 100 \times 100i_b = 100I + 10000i_b$

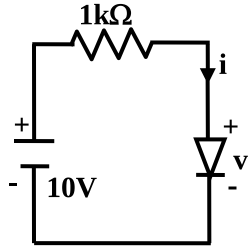
$$-20000i_b = 100I \Rightarrow i_b = -\left(\frac{100}{20000}\right)I = \left[-\frac{I}{200}\right]$$

$$V = 100[I + 99i_b + i_b] = 100\left[I + 100\left(\frac{-I}{200}\right)\right] = 50I$$

$$R_{th} = \frac{V}{I} = \frac{50I}{I} = 50\Omega$$

10. The i-v characteristics of the diode in the circuit given below are

$$i = \begin{cases} \frac{v-0.7}{500} A, & v \geq 0.7V \\ 0\ A, & v < 0.7\ V \end{cases}$$



The current in the circuit is

(A) 10 mA

(B) 9.3 mA

(C) 6.67 mA

(D) 6.2 mA

[Ans. D]

Approach – 1

I_D(diode current) = 500 i + 0.7

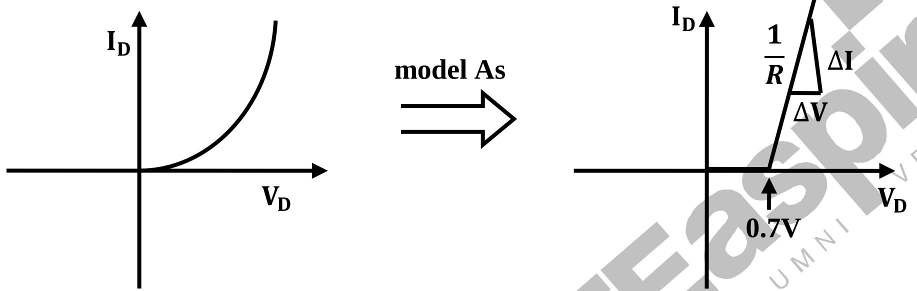
Applying KCL,

10 = 1000 i + 500 i + 0.7

$\Rightarrow i = \frac{9.3}{1.5} = 6.2mA$

Approach – 2

Here diode equivalent circuit model is given which is graphically presented as →



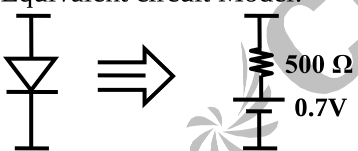
$V \geq 0.7V$

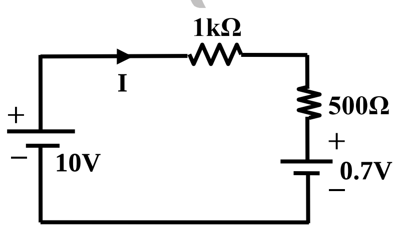
$$I = \frac{V - 0.7}{500}\ A$$

Slope $= \frac{1}{R} = \frac{1}{500}$

R = 500 Ω

Equivalent circuit Model:





Now applying KVL for Linear circuit

$-10V + 1K\Omega \times I + 500\Omega \times I + 0.7V = 0$

$\Rightarrow -9.3V + 1500\Omega\ \text{I} = 0$

$$\Rightarrow I = \frac{9.3}{1500}\frac{V}{\Omega} = 6.2 \times 10^{-3}\ A$$

$I = 6.2$ mA

11. A system with transfer function

$$G(s) = \frac{(s^2 + 9)(s + 2)}{(s + 1)\ (s + 3)\ (s + 4)}$$

is excited by sin (ωt). The steady-state output of the system is zero at

(A) ω = 1 rad/s
(B) ω = 2 rad/s
(C) ω = 3 rad/s
(D) ω = 4 rad/s

[Ans. C]

$$G(s) = \frac{(S^2+9)\ (S+2)}{(S+1)(S+3)(S+4)}$$

By substituting s = jw and equating |G(s)|=0 we get w=3

12. The output Y of a 2-bit comparator is logic 1 whenever the 2-bit input A is greater than the 2-bit input B. The number of combinations for which the output is logic 1, is

(A) 4
(B) 6
(C) 8
(D) 10

[Ans. B]

Approach – 1

A > B A & B are 2 bit

01 00 –1

10 00 01 –2

11 00 01 10 $\frac{-3}{6}$

Approach – 2

Input A		Input B		Y
A_2	A_1	B_2	B_1	
0	0	0	0……………………..	0
0	0	0	1……………………..	0
0	0	1	0……………………..	0
0	0	1	1……………………..	0
0	1	0	0……………………..	1
0	1	0	1……………………..	0
0	1	1	0……………………..	0

0	1	1	1…………………….. 0
1	0	0	0…………………….. 1
1	0	0	1…………………….. 1
1	0	1	0…………………….. 0
1	0	1	1…………………….. 0
1	1	0	0…………………….. 1
1	1	0	1…………………….. 1
1	1	1	0…………………….. 1
1	1	1	1…………………….. 0

Thus for 6 combinations output in logic 1.

13. In the sum of products function $f(X, Y, Z) = \sum(2, 3, 4, 5)$, the prime implicants are

(A) $\bar{X}Y, X\bar{Y}$

(B) $\bar{X}Y, X\bar{Y}\bar{Z}, X\bar{Y}Z$

(C) $\bar{X}Y\bar{Z}, \bar{X}YZ, X\bar{Y}$

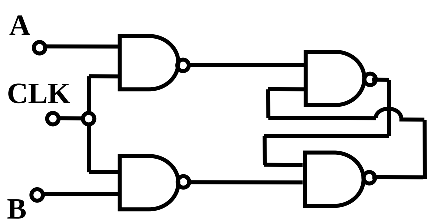
(D) $\bar{X}Y\bar{Z}, \bar{X}YZ, X\bar{Y}\bar{Z}, X\bar{Y}Z$

[Ans. A]

x \ yz	00	01	11	10
0	0	0	1	1
1	1	1	0	0

$f(x, y, z) = \bar{x}y + x\bar{y}$

14. Consider the given circuit.



In this circuit, race around

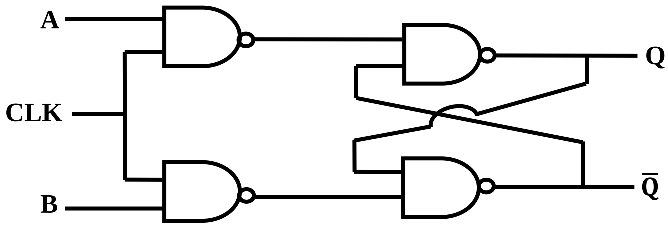
(A) does not occur

(B) occurs when CLK = 0

(C) occurs when CLK = 1 and A = B = 1

(D) occurs when CLK = 1 and A = B = 0

[Ans. A]



$Q_{next} = \overline{\overline{A.CLK.\overline{Q}}}$

$= A.CLK + Q$

$\overline{Q}_{next} = A.CLK + \overline{Q}$

If CLK = 1 and A and B = 1

then $\left.\begin{matrix} Q_{next} = 1 \\ \overline{Q}_{next} = 1 \end{matrix}\right\}$ No race around

If CLK = 1 and A = B = 0

$\left.\begin{matrix} Q_{next} = Q \\ \overline{Q}_{next} = \overline{Q} \end{matrix}\right\}$ No race around

Thus race around does not occur in the circuit.

15. If $x[n] = (1/3)^{|n|} - (1/2)^n u[n]$, then the region of convergence (ROC) of its Z-transform in the Z-plane will be

(A) $\frac{1}{3} < |z| < 3$

(B) $\frac{1}{3} < |z| < \frac{1}{2}$

(C) $\frac{1}{2} < |z| < 3$

(D) $\frac{1}{3} < |z|$

[Ans. C]

Given: $x[n] = (1/3)^{|n|} - (1/2)^n u[n]$

$x[n] \rightleftharpoons x(z)$

$x(z) = \sum_{n=-\infty}^{\infty} x[n]\ z^{-n}$

First consider $\left(\frac{1}{3}\right)^{|n|}$

$\sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^{|n|}\ z^{-n}$

$= \sum_{n=-\infty}^{-1} \left(\frac{1}{3}\right)^{-n} z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^{n}\ z^{-n}$

$= \sum_{n=-\infty}^{-1} \left(3/z\right)^{n} + \sum_{n=0}^{\infty} \left(\frac{1}{3z}\right)^{n}$

$= \left(\frac{3}{z}\right)^{-1} + \left(3/z\right)^{-2} + ------ + 1 + \frac{1}{3z} + \left(\frac{1}{3z}\right)^{2} + ------$

$\left|\frac{3}{z}\right| > 1$ $\left|\frac{1}{3z}\right| < 1$

$\left|\frac{z}{3}\right| < 1$ $|z| > {}^{1}/_{3}$

$|z| < 3$

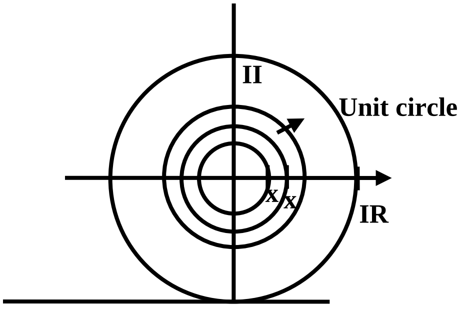
Consider $\left(\frac{1}{2}\right)^{n}$ u(n)

$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n} z^{-n}$

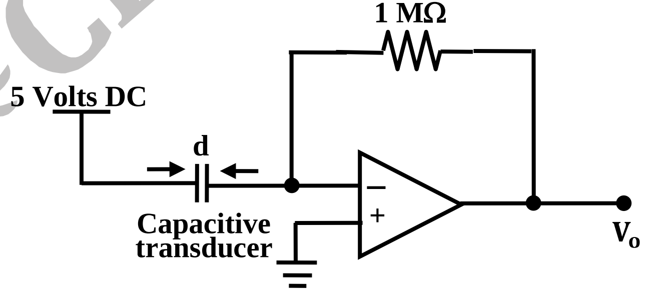
$= 1 + \frac{1}{2z} + \left(\frac{1}{2z}\right)^{2} +$ -------------

$\left|\frac{1}{2z}\right| < 1 \quad |z| > {}^{1}/_{2}$

ROC is $\frac{1}{2} < |z| < 3$



16. A capacitive motion transducer circuit is shown. The gap d between the parallel plates of the capacitor is varied as d (t) = 10^{-3} [1 + 0.1 sin (1000 π t)] m. If the value of the capacitance is 2pF at t = 0 ms, the output voltage V_o at t = 2 ms is



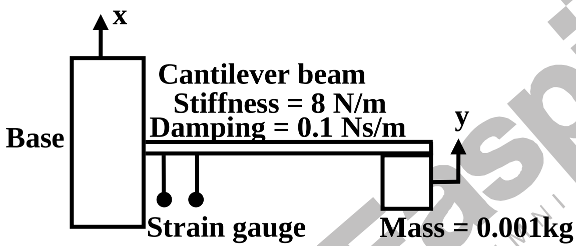
(A) $\frac{\pi}{2}$ mV
(B) π mV
(C) 2π mV
(D) 4π mV

[Ans. B]

17. A psychrometric chart is used to determine
(A) pH
(B) Sound velocity in glasses
(C) CO_2 concentration
(D) Relative humidity

[Ans. D]
Psychrometric chart is used to determine the relative humidity

18. A strain gauge is attached on a cantilever beam as shown. If the base of the cantilever vibrates according to the equation x(t) = sin ω_2t, where 2 rad/s < ω_1, ω_2 < 3 rad/s, then the output of the strain gauge is proportional to



(A) x
(B) $\frac{dx}{dt}$
(C) $\frac{d^2x}{dt^2}$
(D) $\frac{d(x-y)}{dt}$

[Ans. C]
As the base is moving, representing with a equation
$x(t) = \sin\omega_1 t + \sin\omega_2 t$ ------- (1)
Equation (1) is the equation of simple motion, it describes displacement
So, the attached strain gauge will be measuring the acceleration of the member as the given figure is a accelerometer with a mass of 0.001KG attached to it. So,
$V = \dot{x} = \frac{dx}{dt}$
$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \ddot{x}$
So, above accelerometer equation relates to,
$x(t)$ as $a = \frac{d^2x}{dt^2}$

19. The transfer of a Zero-Order-Hold system with sampling interval T is
(A) $\frac{1}{s}(1 - e^{-Ts})$
(B) $\frac{1}{s}(1 - e^{-Ts})^2$
(C) $\frac{1}{s}e^{-Ts}$
(D) $\frac{1}{s^2}e^{-Ts}$

[Ans. A]

20. An LED emitting at 1 μm with a spectral width of 50 mm is used in Michelson interferometer. To obtain a sustained interference, the maximum option path difference between the two arms of the interferometer is

(A) 200 μm
(B) 20 μm
(C) 1 μm
(D) 50 μm

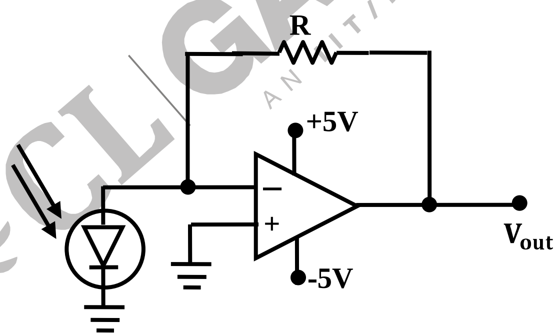
[Ans. B]

21. Light of wavelength 630 mm in vacuum, falling normally on a biological specimen of thickness 10 μm, splits into two beams that are polarized at right angles. The refractive index of the lissue for the two polarizations are 1.32 and 1.33. When the two beams emerge, they are out of phase by

(A) 0.13^{o}
(B) 74.3^{o}
(C) 99^{o}
(D) 128.6^{o}

[Ans. B]

22. The responsivity of the PIN photodiode shown is 0.9 A/W. To obtain V_{out} of –1V of an incident optical power of 1 mW, value of R to be used is



(A) 0.9 Ω
(B) 1.1 Ω
(C) 0.9 kΩ
(D) 1.1 kΩ

[Ans. D]

Given, responsivity = 0.9 A/w.

it can be considered as sensitivity because it's a ratio of o/p to i/p.

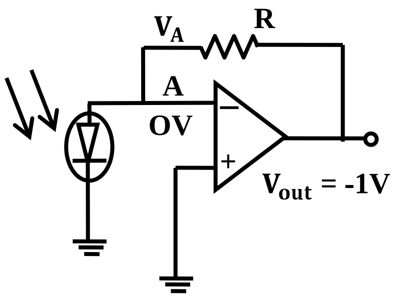
R= $^{o/p}/_{ip}$

$0.9 = \frac{o/p}{1 \times 10^{-3}}$ w

0.9×10^{-3} w = o/p.

$\frac{0.9\ A}{w} \times 1 \times 10^{-3}$ w = o/p

0.9 mA = o/p



By Applying concept of virtual ground at node A. we have V_A = OV

So, current will flow from V_A to V_{out} as V_{out} is at = -1V.

$\frac{V_A - V_{out}}{R} = I$

$\frac{0-(-1)}{R} = 0.9$ mA

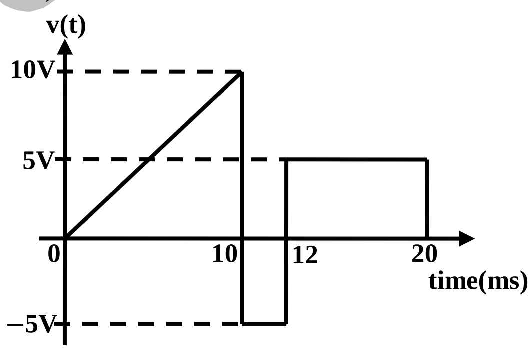
$\frac{1}{0.9 \times 10^{-3}} = R$

$\frac{1000}{0.9} = R$

⇒ 1111.11 Ω = R

or 1.1 KΩ = R

23. A periodic voltage waveform observed on an oscilloscope across a load is shown. A permanent magnet moving coil (PMMC) meter connected across the same load reads



(A) 4 V
(B) 5 V
(C) 8 V
(D) 10 V

[Ans. A]

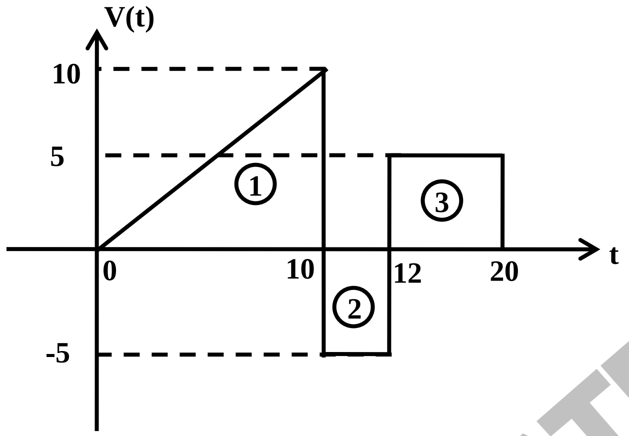
Approach-1

PMMC meter will read the average value of the applied waveform.

Vavg = (1/time period) $\int V(t)\,dt$ = (1/20)[$_0\int^{10}$ t.dt + $_{10}\int^{12}$ (-5).dt + $_{12}\int^{20}$ (+5)dt]

Solving and substituting the limits, we get the value of Vavg as 4V

Approach-2



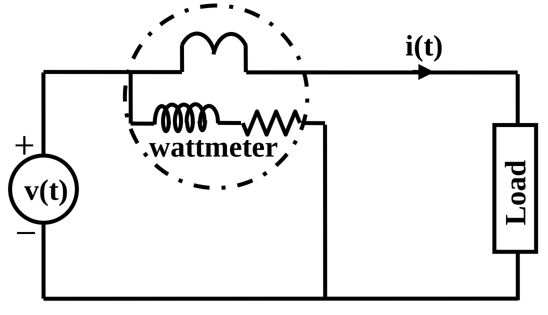
PMMC types of instrument measures average value.

$V_{avg} = \frac{\text{area of graph}}{\text{total time}} \quad = \frac{1}{T}\int_0^T f(t)dt$

$= \frac{\text{area of (1)} - \text{area of (2)} + \text{area of (3)}}{20}$

$= \frac{\frac{1}{2}\times 10\times 10 - 5\times 2 + 8\times 5}{20} = \frac{80}{20} = 4V.$

24. For the circuit shown in the figure, the voltage and current expressions are $v(t) = E_1\sin(\omega t) + E_3\sin(3\omega t)$ and $i(t) = I_1\sin(\omega t - \phi_1) + I_3\sin(3\omega t - \phi_3) + I_5\sin(5\omega t)$. The average power measured by the Wattmeter is



(A) $\frac{1}{2}E_1I_1\cos\phi_1$

(B) $\frac{1}{2}[E_1I_1\cos\phi_1 + E_1I_3\cos\phi_3 + E_1I_5]$

(C) $\frac{1}{2}[E_1I_1\cos\phi_1 + E_3I_3\cos\phi_3]$

(D) $\frac{1}{2}[E_1I_1\cos\phi_1 + E_3I_1\cos\phi_1]$

[Ans. C]

In V(t), only fundamental, 3rd harmonics are present and the 5th harmonics is zero.

$V_i(t) = E_1\sin(\omega t) + E_3\sin(3\omega t)$

$i_i(t) = I_1\sin(\omega t + \phi_1) + I_3\sin(3\omega t - \phi_3) + I_3\sin(5\omega t)$

We know $\int \pi\sin(n\theta \pm \propto).B\cos(m\theta \pm \beta) = 0$

$\therefore \frac{1}{2\pi}\int_0^{2\pi} E_1\sin(\omega t)I_1\sin(\omega t - \phi_1) + E_3I_3\sin(3\omega t)$

$\frac{1}{2}E_1I_1\cos(\phi_1) + \frac{1}{2}E_3I_3\cos(\phi_3)$

$= \frac{1}{2}\int E_1I_1\cos(\phi_1) + E_3I_3\cos(\phi_3)$

25. The bridge method commonly used for finding mutual inductance is

(A) Heaviside Campbell bridge

(B) Schering bridge

(C) De Sauty bridge

(D) Wien bridge

[Ans. A]

Mutual inductance measurement is normally done by Heaviside Campbell bridge.

Q. 26 To Q.55 carry two marks each.

26. A fair coin is tossed till a head appears for the first time. The probability that the number of required tosses is odd, is

(A) 1/3

(B) 1/2

(C) 2/3

(D) 3/4

[Ans. C]

If required tosses is odd the possible sequence of heads and tails will be:

H, TTH, TTTTH, TTTTTT H, ……………

Since, these events are mutually exclusive, we can add the prob. of each event.

Thus, the required prob. is given by

$P = \frac{1}{2} + \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} + \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} + \ldots\ldots\ldots\ldots\ldots..$

This is a geometric series with $a = \frac{1}{2}, r = \frac{1}{4}$

$$P = \frac{\frac{1}{2}}{1-\frac{1}{4}} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$

27. Given that
$A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the value of A^3 is
(A) 15 A + 12 I
(B) 19 A + 30 I
(C) 17 A + 15 I
(D) 17 A + 21 I

[Ans. B]
Given: $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$
We know that Every characteristic equation satisfies its own matrix
$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} -5-\lambda & -3 \\ 2 & -\lambda \end{vmatrix} = 0$
$+5\lambda + \lambda^2 + 6 = 0$
$\Rightarrow \lambda^2 + 5\lambda + 6 = 0$
We know by Cayley Hamilton Theorem that every characteristic equation satisfies its own matrix.
$\therefore A^2 + 5A + 6I = 0$
$\Rightarrow A^3 + 5A^2 + 6A = 0$
$A^3 + 5(-5A - 6I) + 6A = 0$
$\therefore A^3 = 19A + 30I$

28. The direction of vector A is radially outward from the origin, with $|A| = kr^n$ where $r^2 = x^2 + y^2 + z^2$ and *k* is a constant. The value of *n* for which ∇·A = 0 is
(A) −2
(B) 2
(C) 1
(D) 0

[Ans. A]
We know that, $\nabla.\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r)$
Now, $\nabla.\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r)$
$= \frac{1}{r^2}\frac{\partial}{\partial r}(kr^{rH2}) = \frac{k}{r^2}(n+2)\, r^{n+1}$
$= k(n+2)\, r^{n+1}$
$\therefore$ For, $\nabla.\vec{A} = 0, \Rightarrow (n+2) = 0 \Rightarrow n = -2$

29. The maximum value of $f(x) = x^3 - 9x^2 + 24x + 5$ in the interval [1, 6] is

(A) 21
(B) 25
(C) 41
(D) 46

[Ans. C]

$f(x) = x^3 - 9x^2 + 24x + 5$

$f(x) = x^3 - 9x^2 + 24x + 5$

$\frac{df(x)}{dx} = 3x^2 - 18x + 24 = 0$

$x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0$

x = 2, x = 4 (critical points)

$\frac{d^2f(x)}{dx^2} = 6x - 18$

$= 6(2) - 18 < 0$ (for x=2)

$\frac{d^2f(x)}{dx^2} = 6(4) - 18 > 0$ (for x=4)

∴ Maximum at x = 2

$f(2) = 2^3 - 9(2)^2 + 24(2) + 5$

$= 8 - 36 + 48 + 5$

= 25

We have to find the maximum in the close interval [1, 6]

Hence, we have to check at end points also (as extremum exists at the critical points or end points)

$f(6) = (6)^3 - 9(6)^2 + 24(6) + 5$

= 41

∴ Maximum value = 41

30. Consider the differential equation

$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \delta(t)$ with $y(t)|_{t=0^-} = -2$ and $\left.\frac{dy}{dt}\right|_{t=0^-} = 0$

The numerical value of $\left.\frac{dy}{dt}\right|_{t=0^+}$ is

(A) −2
(B) −1
(C) 0
(D) 1

[Ans. D]

<u>Approach – 1</u>

$$\frac{d^2y\,(t)}{dt^2}+\frac{2\,dy\,(t)}{dt}+y\,(t)=\delta(t)$$

Converting to s – domain,

$$s^2y(s)-sy(0)-y'(0)+2[sy(s)-y(0)]+y(s)=1$$

$$[s^2+2s+1]y(s)+2s+4=1$$

$$y(s)=\frac{-3-2s}{(s^2+2s+1)}$$

Find inverse Laplace transform

$$y(t)=[-2e^{-t}-te^{-t}]\,u(t)$$

$$\frac{dy(t)}{dt}=2e^{-t}+te^{-t}-e^{-t}$$

$$\left.\frac{dy(t)}{dt}\right|_{t=0^+}=2-1=1$$

Approach – 2

$$\frac{d^2y(t)}{dt^2}+\frac{2dy(t)}{dt}+y(t)=\delta(t)$$

Applying Laplace Transform on both sides

$$s^2y(s)-sy(0^-)-\left.\frac{dy}{dt}\right|_{t=0^-}+2\big(sy(s)-y(0^-)\big)+y(s)=1$$

$$s^2y(s)+2s+2sy(s)+y(s)=1-4$$

$$y(s)=\frac{-3-2s}{s^2+2s+1}=\frac{-3}{(s+1)^2}-\frac{2s}{(s+1)^2}$$

$$y(t)=-3te^{-t}-2\frac{d}{dt}(te^{t})$$

$$=-3te^{-t}-2(-te^{-t}+e^{-t})$$

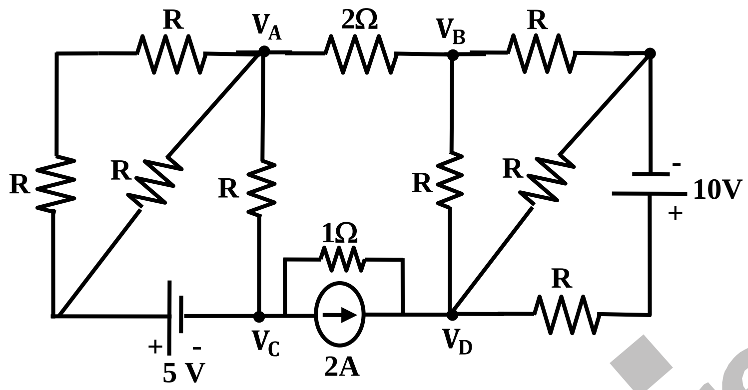
$$y(t)=-te^{-t}-2e^{-t}$$

$$\frac{dy(t)}{dt}=+te^{-t}+(e^{-t})+2e^{-t}$$

$$=te^{-t}+e^{-t}$$

$$\left.\frac{dy(t)}{dt}\right|_{t=0^+}=1$$

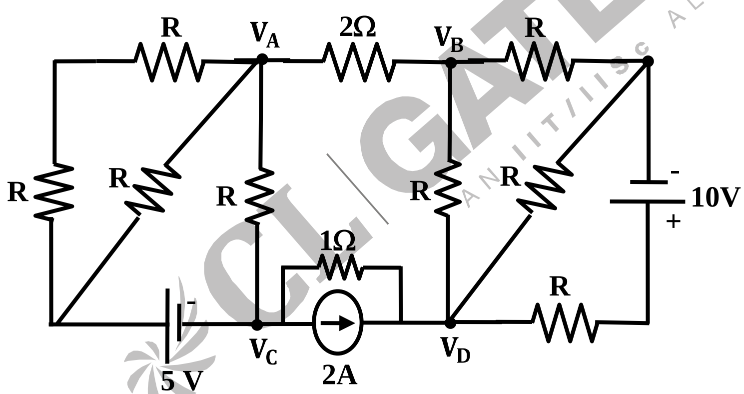
31. If $V_A-V_B=6$ V, then V_C-V_D is

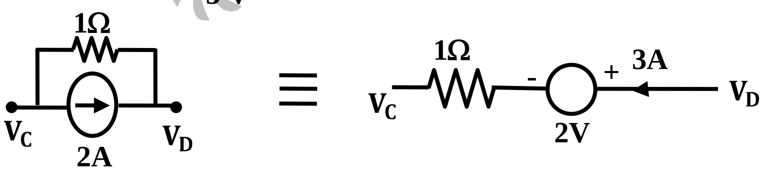


(A) −5V
(B) 2V
(C) 3V
(D) 6V

[Ans. A]

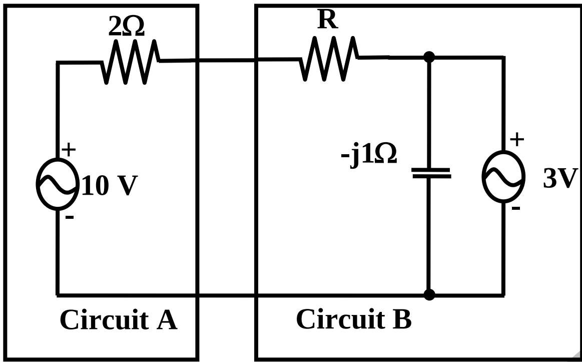
$I = \frac{V_A - V_B}{2} = \frac{6}{2} = 3A$; Since current entering any network is same as leaving in $V_C - V_D$ branch also it is I = 3A





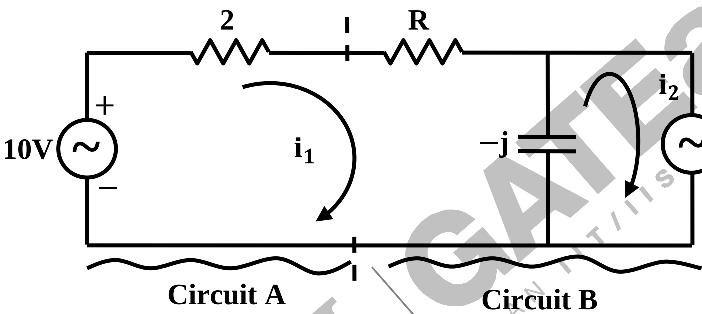
$V_D = 2 + 3 + V_C = 5 + V_C$; $V_C - V_D = -5V$

32. Assuming both the voltage sources are in phase, the value of R for which maximum power is transferred from circuit A to circuit B is



(A) 0.8 Ω
(B) 1.4 Ω
(C) 2 Ω
(D) 2.8 Ω

[Ans. A]



From KVL: $10 = (2 + R)i_1 + (i_1 - i_2)(-j)$

$10 = (2 + R - j)i_1 + ji_2$ -------------- (1)

$3 = -j(i_1 - i_2) \Rightarrow 3 = -ji_1 + ji_2$ ------------- (2)

From (1) & (2): $i_1 = \frac{7}{2+R}$; $i_2 = \frac{7}{2+R} - 3j$

From (2) $i_1 - i_2 = 3j$

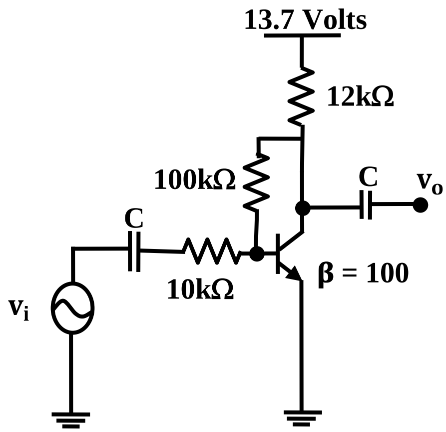
Power transfer from circuit A to circuit B

$P = i_1^2 R + (i_1 - i_2)^2(-j) + 3i_2$

$= \frac{49R}{(2+R)^2} + 9j + 3\left(\frac{7}{2+R} - 3j\right) = \frac{7(10R+6)}{(2+R)^2}$

$\frac{dP}{dR} = 0$ for max power $\Rightarrow R = 0.8\ \Omega$

33. The voltage gain A_v of the circuit shown below is



(A) $|A_v| \approx 200$
(B) $|A_v| \approx 100$
(C) $|A_v| \approx 20$
(D) $|A_v| \approx 10$

[Ans. D]

Approach – 1

This is voltage shunt feedback if we neglect base to emitter voltage. Feedback factor $P_f = \frac{1}{100 \times 103}$

$= \frac{1}{105}\Omega - 1$

Now with F/B,

$$A_2 = \frac{V_0}{I_i} = 12 \times 10^5 \Omega$$

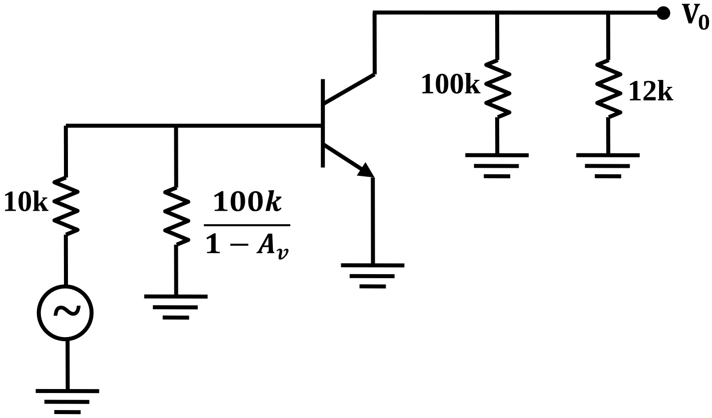
So with feedback

$$\frac{V_0}{I_f} = A_{2_f} = \frac{A_2}{1+\beta\ A_2} = \frac{12\times105}{\beta} \simeq 10^5$$

But $I_i = \frac{V_L}{10\times10^3} = \frac{V_L}{10^4}$

$$\frac{V_0}{V_L} \simeq \frac{10^5}{10^4} \simeq 10$$

Approach – 2



KVL in input loop, $13.7-(I_C+I_B)12k-100k\,(I_B)-0.7=0$

$\Rightarrow\ I_B-9.9\mu A;\ \mathrm{I_C}=\ \beta\,\mathrm{I_B}=0.99\mathrm{mA};\ \mathrm{I_E}=1\mathrm{mA}$

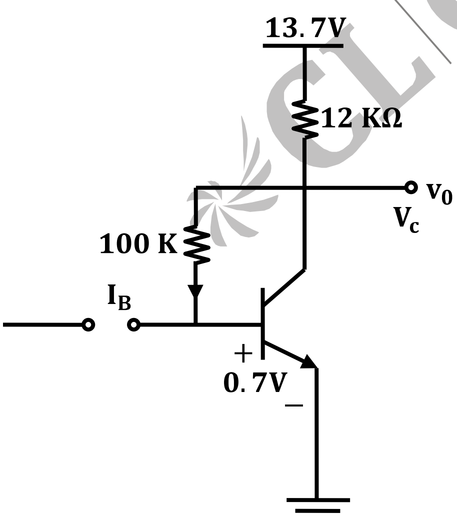
$\therefore\ r_e=\frac{26mA}{I_E}=26\,\Omega;\ \mathrm{z_i}=\ \beta\,\mathrm{r_e}=2.6\mathrm{k}\,\Omega;\ \therefore\ \mathrm{A_v}=\frac{(100\mathrm{k}||12\mathrm{k})}{26}=412$

$z_i'=z_i||\left(\frac{100k}{1+412}\right)=221\,\Omega;\ \mathrm{A_{vs}}=\mathrm{A_v}\frac{\mathrm{z_i}'}{z_i'+R_s}=(412)\left(\frac{221}{221+10k}\right)$

$|A_{vs}|\approx 10$

Approach – 3

This is a shunt – shunt feedback amplifier output voltage is sampled and current is feedback , , , DC circuit reduces to



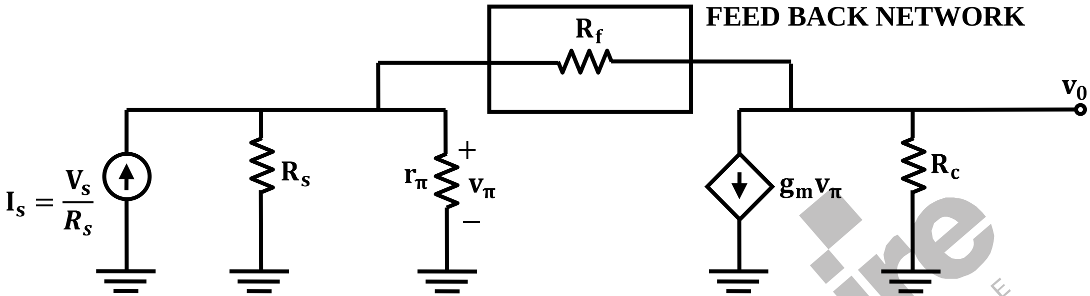
$\mathrm{V_C}=100\ \mathrm{I_B}+0.7$ ---------------- (1)

$\frac{13.7-\mathrm{V_C}}{12}=(\beta+1)\ I_B$ -------------- (2)

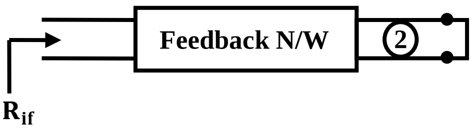
From (1) and (2)

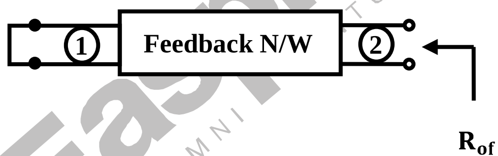
$$\frac{13.7 - 100\,I_B - 0.7}{12} \simeq \beta I_B = 100\,I_B$$

$\Rightarrow I_B = 0.01\,mA$ and $I_C = \beta I_B = 1\text{ mA}$

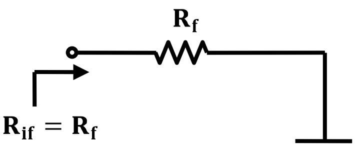


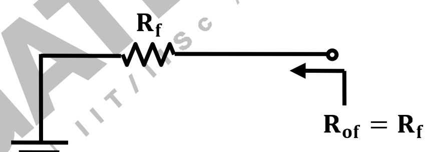
To divide R_f to input and output side



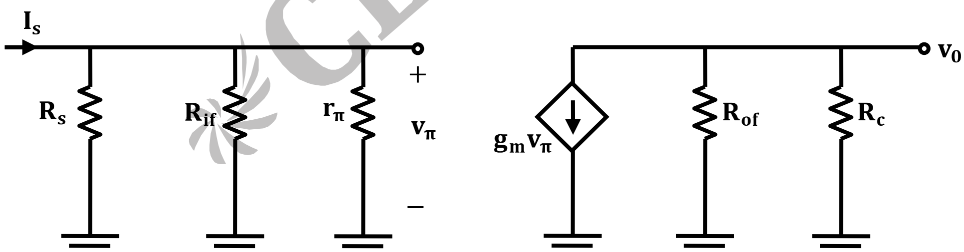


So





So, our circuit redeces to,



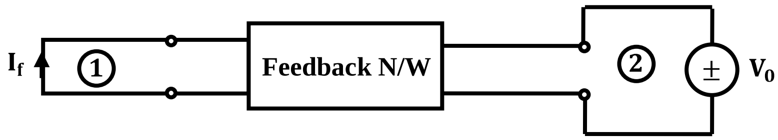
$$A = \frac{V_0}{I_S} = -\frac{g_m(R_f||R_c)v_\pi}{\dfrac{v_\pi}{(R_s||R_{if}||r_\pi)}} = -g_m\,(R_{of}||R_c)(R_s||R_{if}||r_\pi)$$

Now $R_{of} = R_{if} = 100k\Omega$, $R_c = 12k\Omega$, $r_\pi = \frac{V_T}{I_B} = 2.5k\Omega$, and $R_s = 10\text{ k}$

Also, $g_m = \frac{I_C}{V_T} = \frac{40\,mA}{V}$

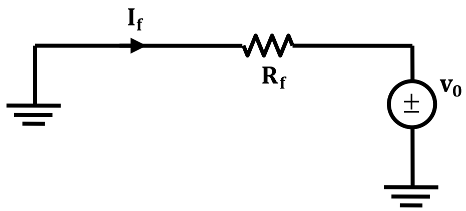
$A = -840.336\,k\Omega$ ---------------- (1)

Note: This is a trans-resistance amplifier. Now feedback factor, for, shunt – shunt configuration is



$\Downarrow$

$$\beta \equiv \left.\frac{I_f}{V_0}\right|_{V_I=0}$$



$$\frac{v_0}{I_f} = \frac{1}{\beta} \Rightarrow \beta = \frac{I_f}{v_0} = -\frac{1}{R_f}$$

So here $\beta = \frac{-1}{100k\Omega}$ ---------- (2)

So gain after feedback

$\frac{V_0}{I_S} = \frac{A}{1+A\beta} \Rightarrow \frac{v_0}{I_S} = \frac{-840.336k}{1+8.40336} \approx 10^5$ (Approx.)

So,

$$\frac{V_0}{V_I} = \frac{V_0}{I_S\, R_S} = \frac{1}{R_S} \times \frac{v_0}{I_S} = \frac{1}{10k} \times 10^5 = 10$$

So $|A_v| \approx 10$

34. The state variable description of an LTI system is given by

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{pmatrix} = \begin{pmatrix}0 & a_1 & 0\\ 0 & 0 & a_2\\ a_3 & 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} + \begin{pmatrix}0\\ 0\\ 1\end{pmatrix}u$$

$$y = (1 \quad 0 \quad 0)\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$$

where y is the output and u is the input. The system is controllable for

(A) $a_1 \neq 0, a_2 = 0, a_3 \neq 0$

(B) $a_1 = 0, a_2 \neq 0, a_3 \neq 0$

(C) $a_1 = 0, a_2 \neq 0, a_3 = 0$

(D) $a_1 \neq 0, a_2 \neq 0, a_3 = 0$

[Ans. D]

The controllability matrix

$= [B \quad AB \quad A^2B]$

$$A = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow \text{Controllability matrix} = \begin{bmatrix} 0 & 0 & a_1 a_2 \\ 0 & a_2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

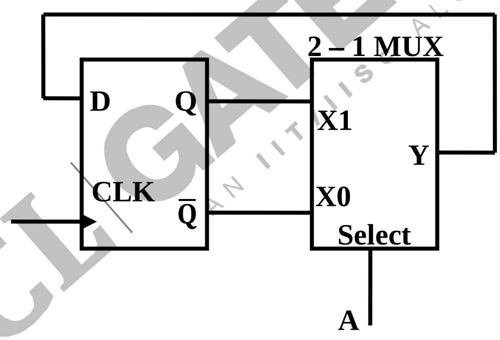
$\Rightarrow a_1 \neq 0$

$a_2 \neq 0$

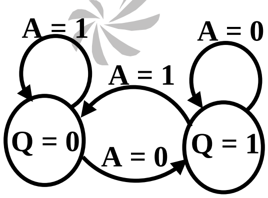
a_3 can be zero

For system to be controllable, determinant of control ability matrix should not be zero.

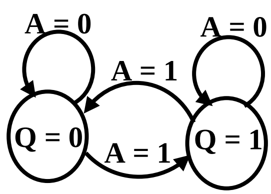
35. The state transition diagram for the logic circuit shown is



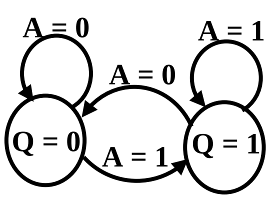
(A)



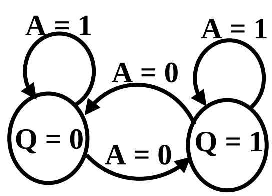
(B)



(C)

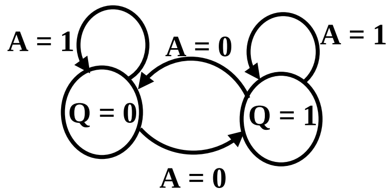


(D)



[Ans. D]

$$\left.\begin{matrix} A = 0, & Y = Q \\ A = 1, & Y = \overline{Q} \end{matrix}\right\} \text{whenever A = 1, output gets into same state}$$



Whenever A = 0, output gets toggled

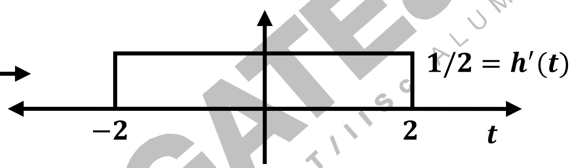
36. The Fourier transform of a signal h(t) is H(jω) = (2 cos ω) (sin 2ω)/ ω. The value of h(0) is

(A) 1/4
(B) 1/2
(C) 1
(D) 2

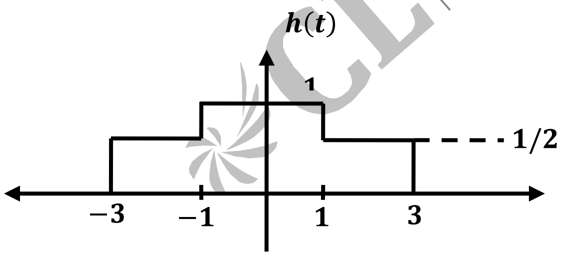
[Ans. C]

Approach – 1

$$\frac{\sin 2\omega}{\omega}$$



$$2\cos\omega\left(\frac{\sin 2\omega}{\omega}\right) = \left[e^{j\omega} + e^{-j\omega}\right]\left(\frac{\sin 2\omega}{\omega}\right) \leftrightarrow h(t) = h'(t-1) + h'(t+1)$$

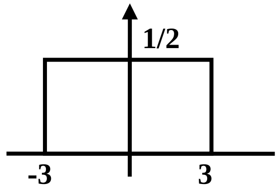


Approach – 2

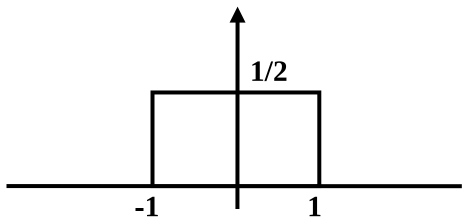
Given: $H(j\omega) = (2\cos\omega)\ (\sin 2\omega)/\omega$

Solution: $H(j\omega) = \frac{2\cos\omega\ \sin 2\omega}{\omega}$

$$H(j\omega) = \frac{\sin 3\omega}{\omega} + \frac{\sin\ \omega}{\omega}$$

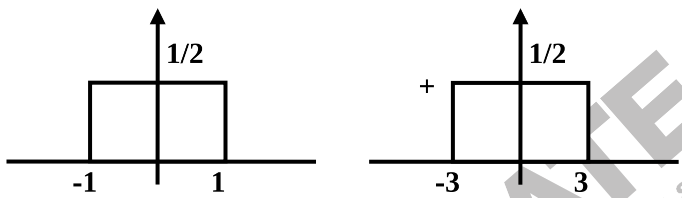


$\rightleftharpoons \quad \dfrac{\sin 3\omega}{\omega}$



$\rightleftharpoons \quad \dfrac{\sin \omega}{\omega}$

$h(t) =$



$$h(0) = \frac{1}{2} + \frac{1}{2}$$

$$h(0) = 1$$

37. Let y[n] denote the convolution of h[n] and g[n], where h[n] – $(1/2)^n$ u[n] and g[n] is a causal sequence. If y[0] = 1 and y[1] = 1/2, then g[1] equals

(A) 0 (C) 1

(B) 1/2 (D) 3/2

[Ans. A]

$$y[n] = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k g(n-k)$$

$$y[0] = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k g(-k) = 1$$

$$\Rightarrow \left(\frac{1}{2}\right)^O g(0) = 1$$

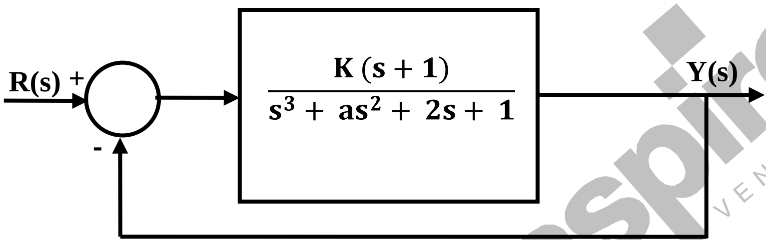
$\Rightarrow g(0) = 1$ since g(n) is Causal sequence $g(-1), g-2),\ldots\ldots\ldots\ldots = 0$

$$y[1] = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^{k} g[1-k]$$

$$\Rightarrow \left(\frac{1}{2}\right)^{0} g[1] + \left(\frac{1}{2}\right)^{1} g(0) = {}^{1}/_{2}$$

$$g[1] = 0$$

38. The feedback system shown below oscillates at 2 rad/s when



(A) K = 2 and a = 0.75
(B) K = 3 and a = 0.75
(C) K = 4 and a = 0.5
(D) K = 2 and a = 0.5

[Ans. A]

$$1 + G(S)H(S) = S^3 + as^2 + (2+k)s + 1 + k$$

s^3		$1\ (2+k)$
s^2		$a\ (2+k)$
s	$a(2+k) - (2+k)0$	
s^0		$(1+k)^a$

For system to oscillate,

$$a(2+k) - (1+k) = 0$$

$$a = \left(\frac{1+k}{2+k}\right)$$

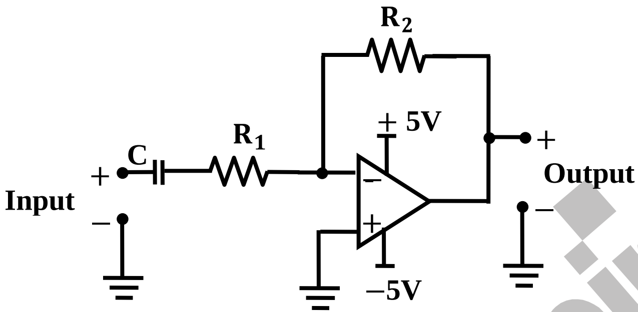
$$A.E \Rightarrow as^2 + (1+k) = 0$$

$$\Rightarrow s = \sqrt{\frac{1+k}{a}} = 2$$

$$\Rightarrow \left(\frac{1+k}{a}\right) = a$$

$\Rightarrow 2 + k = 4 \Rightarrow k = 2$
Thus a = 0.75

39. The circuit shown is a



(A) low pass filter with $f_{3dB} = \frac{1}{(R_1+R_2)C}$ rad/s

(B) high pass filter with $f_{3dB} = \frac{1}{R_1 C}$ rad/s

(C) low pass filter with $f_{3dB} = \frac{1}{R_1 C}$ rad/s

(D) high pass filter with $f_{3dB} = \frac{1}{(R_1+R_2)C}$ rad/s

[Ans. B]

The transfer function of the N/W is $\frac{V_0(s)}{V_i(s)} = \frac{-R_2}{R_1 + \frac{1}{CS}} = -\frac{R_2 CS}{R_1 CS + 1}$

This represents H.P filter with cutoff frequency at $\frac{1}{R_1 C}$

40. The input x(t) and output y(t) of a system are related as $y(t) = \int_{-\infty}^{t} x(\tau)\cos(3\tau)\ d\tau$. The system is

(A) time-invariant and stable
(B) stable and not time-invariant
(C) time-invariant and not stable
(D) not time-invariant and not stable

[Ans. D]

- Assume a bounded input x(t) = cos (3t)

$$y(t) = \int_{-\infty}^{t} cos^2\ (3\tau)\, d\tau$$

Thus, y(t) is unbounded, hence, system is not stable.

- Assume $x(t) = \delta(t)$

$$y(t) = \int_{-\infty}^{t} \delta(\tau)\cos(3\tau)\, d\tau$$

$= u(t)\cos(0) = u(t)$

Time shifted input $x\left(t - \frac{\pi}{6}\right) = \delta\left(t - \frac{\pi}{6}\right)$

$$y'(t) = \int_{-\infty}^{t} \delta\left(\tau - \frac{\pi}{6}\right)\cos(3\tau)\, d\tau$$

$= u(t)\cos\left(3 \times \frac{\pi}{6}\right) = 0$

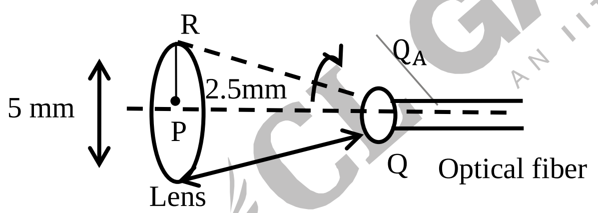
$y'(t) \neq y\left(t - \frac{\pi}{6}\right) \Longrightarrow$ System is not time – invariant

41. A double convex lens is used to couple a laser beam of diameter 5 mm into an optical fiber with a numerical aperture of 0.5. The minimum focal length of the lens that should be used in order to focus the entire beam into the fiber is

(A) 1.44 mm
(B) 2.50 mm
(C) 4.33 mm
(D) 5.00 mm

[Ans. Marks to All*] (*Ambiguous options)

One of the solution could be as given below:



N.A = 0. 5 PQ = focal length of lens (?)

NA is sine of acceptance angle

N.A = sin (θ_A)

$\sin^{-1}(0.5) = \theta_A$

$30^0 = \theta_n$

So, In right angled triangle

$\tan 30^0 = \frac{p}{b}$

$0.57 = \frac{2.5 \text{ mm}}{b}$

$b = \frac{2.5\text{mm}}{0.57}$

b = 4.38
4.3 mm

42. An analog voltmeter uses external multiplier settings. With a multiplier setting of 20 kΩ, it reads 440 V and with a multiplier setting of 80 kΩ, it reads 352 V. For a multiplier setting of 40 kΩ, the voltmeter reads

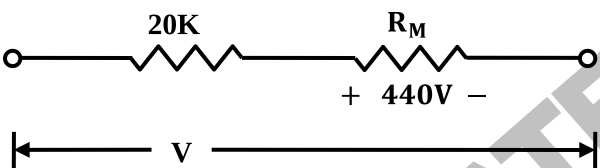
(A) 371 V
(B) 383 V
(C) 394 V
(D) 406 V

[Ans. Marks to All*] (*Ambiguous options)

One of the solution could be as given below:

Here the problem is solved by assuming the terminal voltage across the meter + the multiplier resistor remains same.

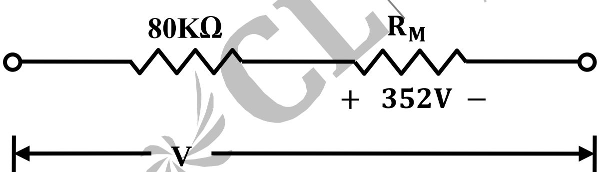
1) Given voltmeter reading as 440V when a multiplier resistance of 20 kΩ is used.



Let V be the terminal voltage and R_M be the meter resistance

$\therefore 440 = V\left[\frac{R_M}{R_M+20k}\right] \quad \rightarrow (1)$

2) Voltmeter reading was 352V for a multiplier resistance of 80 kΩ.



$\therefore 352 = V\left[\frac{R_M}{R_M+80k}\right] \quad \rightarrow (2)$

Solving (1) & (2), we get $R_M = 220k\Omega$

$\therefore$Terminal voltage $= 440 + 20k\left[\frac{440}{220k}\right]$

$= 480$ V

$\therefore$ For a multiplier resistance of 40 kΩ, the voltmeter reading is $480\left[\frac{220k}{220k+40k}\right] = 406.15V$

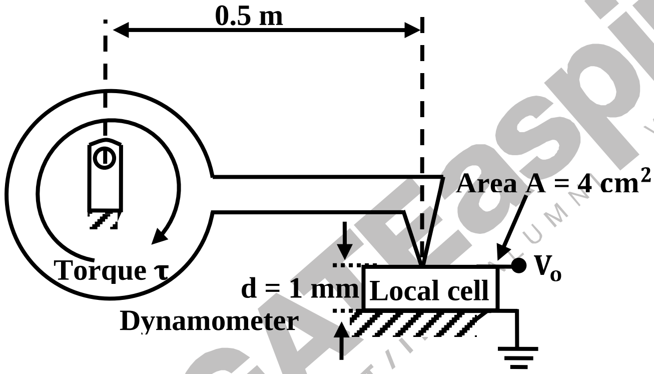
≈ 406 V

43. The open loop transfer function of a unity negative feedback control system is given by G(s) = $\frac{150}{s\,(s+9)(s+25)}$. The gain margin of system is

(A) 10.8 dB
(B) 22.3 dB
(C) 34.1 dB
(D) 45.6 dB

[Ans. C]

44. A dynamometer arm makes contact with the piezoelectric load cell as shown. The g-constant of the piezoelectric material is 50×10^{-3} Vm/N and the surface area of the load cell is 4 cm^2. If a torque r = 20 Nm is applied to the dynamometer, the output voltage V_o of the load cell is



(A) 4 V
(B) 5 V
(C) 10 V
(D) 16 V

[Ans. Marks to All*] (*Ambiguous options)

One of the solution could be as given below:

g = 50×10^{-3} $V_{m/N}$ Area 4 cm^2 = 4×10^{-4} m^2

τ = 20 N_m So, τ = F × Perp distance

20 Nm = f ×0.5 m

$\frac{20\text{N}}{0.5}$ = F

40 N = F

So, pressure = F/Area = $\frac{40\text{ N}}{4 \times 10^{-4}\text{ m}^2} = 10 \times 10^{+4}\text{ N/m}^2$.

V_o = g × t × p

$V_o = 50 \times 10^{-3}\,\frac{\text{Vm}}{\text{N}} \times 1\times 10^{-3}\text{ m} \times 10\times 10^4\,\frac{\text{N}}{\text{m}^2}$

$V_o = 50 \times 10^{-6} \times 10^5$

$V_o \Rightarrow 5$ V

45. Water (density: 1000 kgm^{-3}) stored in a cylindrical drum of diameter 1 m is emitted through a horizontal pipe of diameter 0.05 m. A pitot-static tube is placed inside the pipe facing the flow. At the time when the difference between the stagnation and static pressures measured by the pitot-static tube is 10 kPa, the rate of reduction in water level in the drum is,

(A) $\frac{1}{200\sqrt{5}} ms^{-1}$ (C) $\frac{1}{50\sqrt{10}} ms^{-1}$

(B) $\frac{1}{75\sqrt{10}} ms^{-1}$ (D) $\frac{1}{40\sqrt{5}} ms^{-1}$

[Ans. D]

ρ = 100 kg/ m^3 $(\rho - \rho_o)$ = 1000 KN/m^2

for a pitot tube:

v = $\sqrt{\frac{2(\rho-\rho_o)}{\rho}}$

v = $\sqrt{\frac{2 \times 10 \times 10^3}{1000}}$ v = $\sqrt{20}$ m/ sec

v = $2\sqrt{5}$ m/ sec

Including Velocity correction factor E = 20 from the given pipe diameter and tank diameter of 1ms 0.05m respectively we have

V = $\frac{1}{40\sqrt{5}}$ m/sec

46. A U-tube manometer of tube diameter D is filled with a liquid of zero viscosity. If the volume of the liquid filed is V_s the natural frequency of oscillations in the liquid level about its mean position, due to small perturbations, is

(A) $\frac{D}{2\sqrt{2\pi}} \sqrt{\frac{g}{V}}$ (C) $\frac{1}{2\sqrt{\pi}} \sqrt{\frac{gD}{V^{1/3}}}$

(B) $\frac{2\sqrt{2}}{\sqrt{\pi}} \frac{\sqrt{gV}}{D^2}$ (D) $\frac{1}{\sqrt{\pi}} \sqrt{\frac{g}{D}}$

[Ans. A]

47. The open loop transfer function of a unity gain negative feedback control system is given by G(s) = $\frac{s^2+4s+8}{s(s+2)(s+8)}$. The angle θ, at which the root locus approaches the zeros of the system, satisfies

(A) $|\theta| = \pi - \tan^{-1}\left(\frac{1}{4}\right)$ (C) $|\theta| = \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$

(B) $|\theta| = \frac{3\pi}{4} - \tan^{-1}\left(\frac{1}{3}\right)$ (D) $|\theta| = \frac{\pi}{4} - \tan^{-1}\left(\frac{1}{3}\right)$

[Ans. D]

Common Data Questions

Common Data for Question 48 and 49:

With 10 V dc connected at port A in the linear nonreciprocal two-port network shown below, the following were observed:

(i) 1 Ω connected at port B draws a current of 3 A

(ii) 2.5 Ω connected at port B draws a current of 2 A

48. With 10 V dc connected at port A, the current drawn by 7 Ω connected at port B is

(A) 3/7 A
(B) 5/7 A
(C) 1 A
(D) 9/7 A

[Ans. C]

The given network can be replaced by a Thevenin equivalent with V_{th} and R_{th} as Thevenin voltage and Thevenin Resistance.

Now we can write two equations for this

$V_{th} = 3(R_{th} + 1)$

$V_{th} = 2(R_{th} + 2.5)$

Solving these two equations we get $R_{th} = 2$ and $V_{th} = 9$.

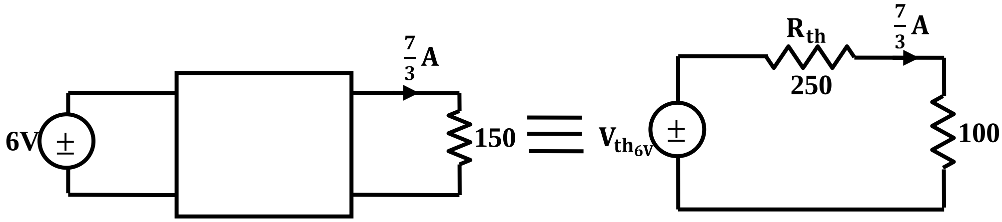
Now using the same equation with current unknown,

$9 = I \times 2 + 7 \times I \Rightarrow I = 1A$

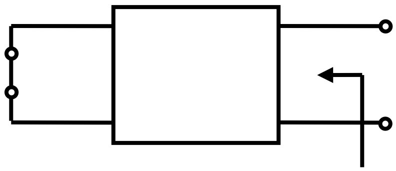
49. For the same network, with 6 V dc connected at port A, 1 Ω connected at port B draws 7/3 A. If 8 V dc is connected to port A, the open circuit voltage at port B is

(A) 6 V
(B) 7 V
(C) 8 V
(D) 9 V

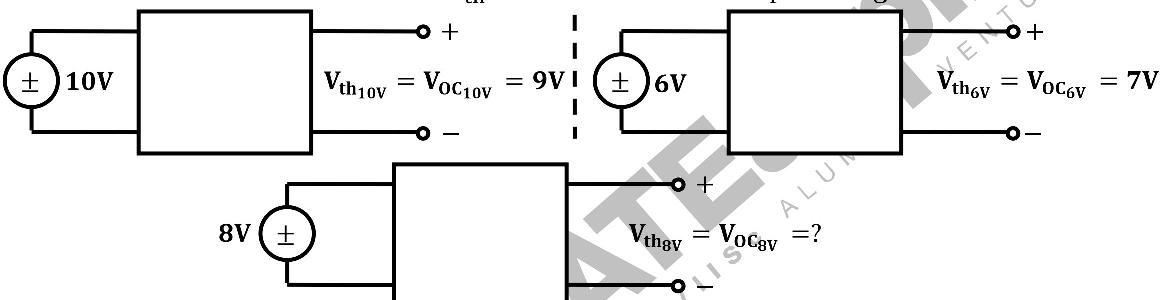
[Ans. C]



$\therefore V_{th_{6V}} = (R_{th} + 1)\frac{7}{3} = 7V$



R_{th} is same whatever the input voltage.



Since, two port network is linear non reciprocal, output (at port 2) can be expressed as a linear function of input.

i.e. $V_{OC} = aV_{in} + b$

9 = 10a + b -------------------(I)

7 = 6a + b --------------------(II)

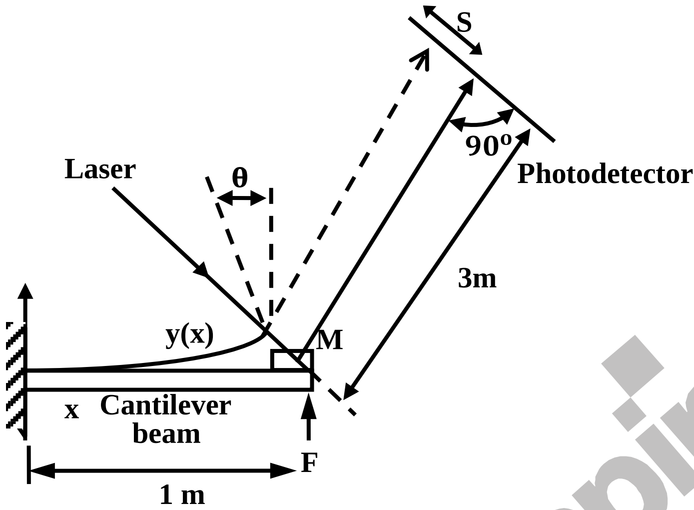
From (I) & (II),

a = 0.5, b = 4

$\therefore V_{OC_{8V}} = 0.5(8) + 4 = 8V$

Common Data for Questions 50 and 51:

The deflection profile y(x) of a cantilever beam due to application of a point force F (in Newton), as a function of distance x from its base, is given by $y(x) = 0.001\ F\ x^2\left(1 - \frac{x}{3}\right)$m. The angular deformation θ at the end of the cantilever is measured by reflecting a laser beam off a mirror M as shown in the figure.



50. The translation S of the spot of laser light on the photodetector when of F = 1 N is applied to the cantilever is

(A) 1 mm
(B) 3 mm
(C) 6 mm
(D) 12 mm

[Ans. C]

51. If linear variable differential transformers (LVDTs) are mounted at $x = \frac{1}{2}$ m and $x = \frac{1}{4}$ m on the cantilever to measure the effect of time varying forces, the ratio of their output is

(A) 12/7
(B) 40/11
(C) 176/23
(D) 112/15

[Ans. B]

Output of LVDT is proportional to the displacement of the core.
Therefore LVDT1 / LVDT2 = Disp 1/ Disp 2
Disp 1 (@ x = 0.5m) = 5/24 (using the relation given in problem).
Disp2 (@ x = 0.25m) = 11/192.
So Disp1/Disp2 = 40 /11

Linked Answer Questions

Statement for Linked Answer Question 52 and 53:

Transfer function of a compensator is given as

$$G_c(s) = \frac{s+a}{s+b}$$

52. $G_c(s)$ is a lead compensator if

(A) $a = 1, b = 2$
(B) $a = 3, b = 2$
(C) $a = -3, b = -1$
(D) $a = 3, b = 1$

[Ans. A]

$\phi = tan^{-1}\frac{\omega}{a} - tan^{-1}\frac{\omega}{\beta}$

For phase lead ϕ should be positive

$\Rightarrow tan^{-1}\frac{\omega}{a} > tan^{-1}\frac{\omega}{\beta}$

$\Rightarrow a < b$

Both option (A) and (C) satisfies,

It may be observed that option (C) will have poles and zero in RHS of s – plane, thus not possible (not a practical system).

Therefore, it can be concluded that Option (A) is right.

53. The phase of the above lead compensator is maximum at

(A) $\sqrt{2}$ rad/s
(B) $\sqrt{3}$ rad/s
(C) $\sqrt{6}$ rad/s
(D) $1/\sqrt{3}$ rad/s

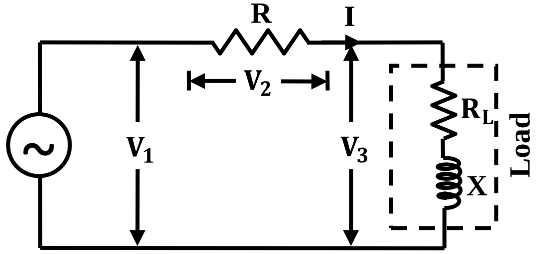
[Ans. A]

ω = geometric mass of two carrier frequencies

$= \sqrt{2 \times 1} = \sqrt{2}\, rad/sec$

Statement for Linked Answer Questions 54 and 55:

In the circuit shown, the three voltmeter readings are $V_1 = 220$ V, $V_2 = 122$ V, $V_3 = 136$ V.



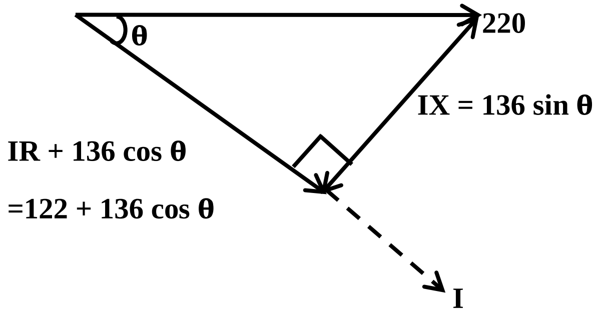
54. The power factor of the load is

(A) 0.45 (C) 0.55

(B) 0.50 (D) 0.60

[Ans. A]

Voltage dropped across R and R_L will in same i.e, in phase of current and voltage dropped across inductor will be 90 to current phasor.



$$\cos\theta = \frac{V_1^2 - V_2^2 - V_3^2}{2V_1V_2} = \frac{220^2 - 122^2 - 136^2}{2\times122\times136} = 0.45$$

55. If $R_L = 5\Omega$, the approximate power consumption in the load is

(A) 700 W (C) 800 W

(B) 750 W (D) 850 W

[Ans. B]

Approach1

Now power is dissipated only in resistance

So $P = V_R\, I_R \cos\emptyset$.

$$P = 136\cos\emptyset \times \frac{136\cos\emptyset}{5} \times 1 \quad \left[\begin{array}{c}\text{Consider}\\ \text{only resistance}\\ \text{voltage and}\\ \text{resistance current}\end{array}\right.$$

Here $\cos\emptyset = 1$ because current in resistance and voltage across the resistance are in same phase

(Voltage across R_L is $136\cos\emptyset$)

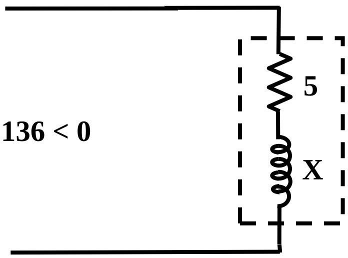
$$P = 136 \times 0.45 \times \frac{136\times0.45}{5} \times 1$$

$$= 749.2 \cong 750\text{ W}$$

Approach2

Consider only load part (along X)

Power in resistance = **136 < 0**



P = V I cos Ø

$= 136 \times \frac{136}{\sqrt{5^2+X^2}} \times \cos Ø$ ----------(i)

We have calculated power factor as 0.45

$\therefore 0.45 = \frac{5}{\sqrt{5^2+X^2}}$

$\therefore \frac{1}{\sqrt{5^2+X^2}} = \frac{0.45}{5}$ --------- (ii)

Put value of $\frac{1}{\sqrt{5^2+X^2}}$ in (i)

$P = 136 \times \frac{136}{5} \times 0.45 \times 0.45$

$= 749.2 \cong 750$ W.

General Aptitude (GA) Questions (Compulsory)

Q. 56 – Q. 60 carry one mark each.

56. Choose the most appropriate alternative from the options given below to complete the following sentence:

If the tired soldier wanted to lie down, he _______ the mattress out on the balcony.

(A) should take
(B) shall take
(C) should have taken
(D) will have taken

[Ans. A]

57. If $(1.001)^{1259} – 3.52$ and $(1.001)^{2062} = 7.85$, then $(1.001)^{3321} =$

(A) 2.23
(B) 4.33
(C) 11.37
(D) 27.64

[Ans. D]

$(1.001)^{1259} = 3.52$ and $(1.001)^{2062} = 7.85$, then $(1.001)^{1259} \times (1.001)^{2062} = 3.52 \times 7.85$ or $(1.001)^{(1259+2062)} = 3.52 \times 7.85$ or $(1.001)^{3321} = 27.64$

58. One of the parts (A, B, C, D) in the sentence given below contains an ERROR. Which one of the following is INCORRECT?

I requested that he should be given the driving test today instead of tomorrow.

(A) requested that
(C) the driving test
(B) should be given
(D) instead of tomorrow

[Ans. B]

The correct statement should be -" i requested that he be given the driving test today instead of tomorrow."

59. Which one of the following options is the closest in meaning to the word given below?

Latitude

(A) Eligibility
(C) Coercion
(B) Freedom
(D) Meticulousness

[Ans. B]

Latitude means freedom to choose what you do or the way that you do it.

60. Choose the most appropriate word from the options given below to complete the following sentence:

Given the seriousness of the situation that he had to face, his _____ was impressive

(A) beggary
(C) jealousy
(B) nomenclature
(D) nonchalance

[Ans. D]

Nonchalance means behaving in a calm and relaxed way; giving the impression that you are not feeling any anxiety.

Q. 61 – Q. 65 carry two marks each.

61. Raju has 14 currency notes in his pocket consisting of only Rs. 20 notes and Rs. 10 notes. The total money value of the notes is Rs. 230. The number of Rs. 10 notes that Raju has is

(A) 5
(C) 9
(B) 6
(D) 10

[Ans. A]

Let the number of Rs. 10 notes = x

And the number of Rs. 20 notes = y

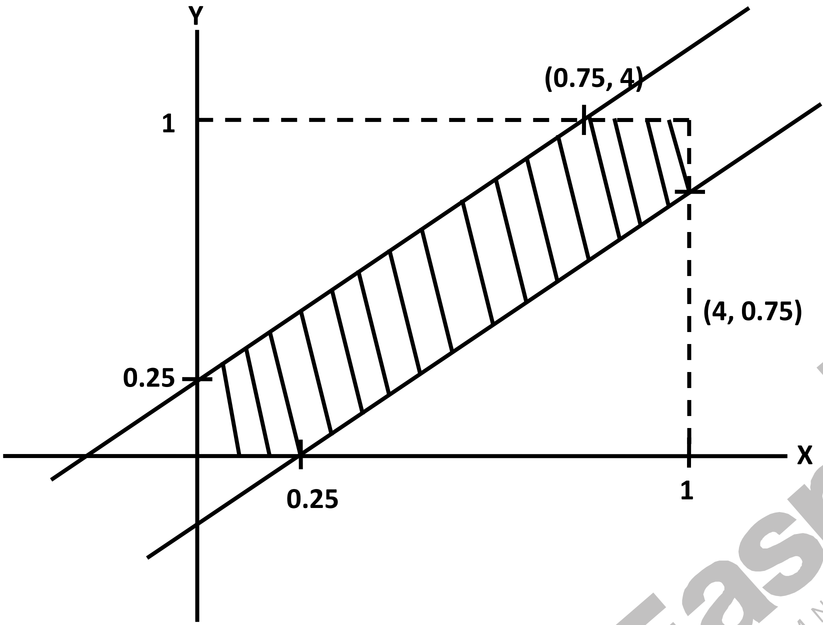
Now,
$x + y = 14$ and $10x + 20y = 230$.
Solving them, we get $x = 5$ and $y = 9$.

62. **One of the legacies of the Roman legions was discipline. In the legions, military law prevailed and discipline was brutal. Discipline on the battlefield kept units obedient, intact and fighting, even when the odds and conditions were against them.**
Which one of the following statements best sums up the meaning of the above passage?
(A) Through regimentation was the main reason for the efficiency of the Roman legions even in adverse circumstances.
(B) The legions were treated inhumanly as if the men were animals.
(C) Discipline was the armies' inheritance from their seniors.
(D) The harsh discipline to which the legions were subjected to led to the odds and conditions being against them.

[Ans. A]
The passage states that the strict discipline kept the armies intact even when condition were against them.

63. A and B are friends. They decide to meet between 1 PM and 2 PM on a given day. There is a condition that whoever arrives first will not wait for the other for more than 15 minutes. The probability that they will meet on that day is
(A) 1/4
(B) 1/16
(C) 7/16
(D) 9/16

[Ans. C]
We can solve this question graphically.
Let x axis represents the time when A reaches the meeting place and y axis represents the time when B reaches the same place.



Given conditions are

Total area is represented by

$0 \le x \le 1$ and $0 \le y \le 1$.

Total area = 1 × 1 = 1 unit

Desired area is represented by

$0 \le x \le 1, 0 \le y \le 1, x - y \le 1/4$ and $y - x \le 1/4$

$$\text{Desired area} = 1 - \left(\frac{1}{2} \times \frac{3}{4} \times \frac{3}{4}\right) - \left(\frac{1}{2} \times \frac{3}{4} \times \frac{3}{4}\right) = \frac{7}{16}$$

$$\text{Required probability} = \frac{\textit{desired area}}{\textit{total area}} = \frac{7}{16 \times 1} = \frac{7}{16}$$

64. The data given in the following table summarizes the monthly budget of an average household.

Category	**Amount (Rs.)**
Food	4000
Clothing	1200
Rent	2000
Savings	1500
Other expenses	1800

The approximate percentage of the monthly budget NOT spent on savings is

(A) 10%
(B) 14%
(C) 81%
(D) 86%

[Ans. D]
Total Income = 10500
Savings =1500

Percentage of budget spent on savings = $\frac{1500}{10500} \times 100 = 14.28\%$

Percentage of budget not spent on savings = 86%

65. There are eight bags of rice looking alike, seven of which have equal weight and one is slightly heavier. The weighing balance is of unlimited capacity. Using the balance, the minimum number of weighing required to identify the heavier bag is
(A) 2
(B) 3
(C) 4
(D) 8

[Ans. A]