SAMPLE OF THE STUDY MATERIAL

PART OF CHAPTER 3

Symmetrical Components & Faults Calculations

3.0 Introduction

* Fortescue's work proves that an unbalanced system of 'n' related phasors can be resolved into 'n' systems of balanced phasors called the symmetrical components of the original phasors.
* The method of symmetrical components is a general one, applicable to any polyphase system.
* Usually, we apply symmetrical component method to symmetrical and unbalanced 3-ϕ circuits. The term 'symmetrical' in power systems is used for a poly phase N/W if and only if all the phases are having same impedance (i.e., in magnitude as well as phase angle). A N/W is said to be balanced if and only if all the phases containing voltages / currents in same magnitude but should be displaced by same phase angle.
* According to Fortescue's theorem, three unbalanced phasors of a three phase system can be resolved into three balanced systems of phasors. The balanced sets of components are :

1. Positive sequence components consisting of three phasors equal in magnitude, displaced from each other by 120° in phase, and having the same phase sequence as the original phasors.
2. Negative sequence components consisting of three phasors equal in magnitude, displaced from each other by 120° in phase, and having the phase sequence opposite to that of the original phasors
3. Zero sequence components consisting of three phasors equal in magnitude and with zero phase displacement from each other.

* Three sets of balanced phasors which are the symmetrical components of three unbalanced phasors. Shown in below figures.

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Positive Sequence Components Negative Sequence Components Zero Sequence Components

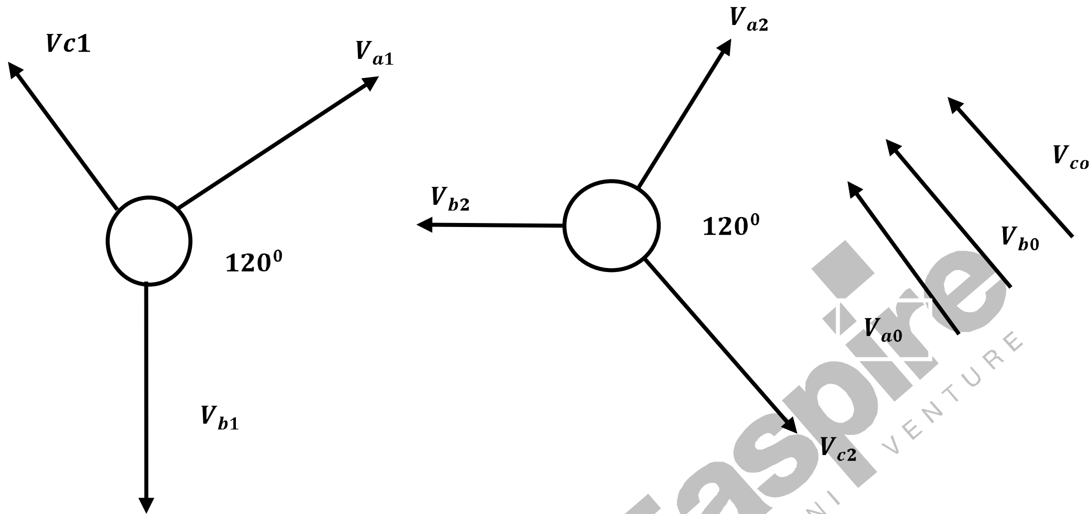


Fig. 3.1

3.1 Components Zero Sequence Components

Considering the positive sequence components, the vector V_{b1} lags behind V_{a1} by 120° electrical and the vector V_{c1}leads the vector V_{a1} by 120° electrical since these three are also equal in magnitude

$$|V_{a1}| = |V_{b1}| = |V_{c1}|$$

$$V_{b1} = V_{a1}\angle -120^0$$

$$V_{c1} = V_{a1}\angle 120^0$$

* For the positive sequence components, Stator and rotor field directions are same. Considering the negative sequence components, the vector V_{b2} leads V_{a2} by 120° electrical, and the vector V_{c2} lags behind V_{a2} by 120° electrical.

* Since these three are also equal in magnitude

$|V_{a2}| = V_{b2}| = |V_{c2}|$

$V_{b2} = V_{a2}\angle 120^0$

$V_{c2} = V_{a2}\angle -120^0$

* For the negative sequence components, Stator and rotor field directions are reverse. For zero sequence components, we have $V_{a0} = V_{b0} = V_{c0}$

Since the three un balanced vectors V_a, V_b, and V_c can be resolved into three sets of balanced vectors, the vector $V_a$is equal to the sum of the positive sequence component $V_{a1}$of phase a, the negative sequence component of phase a, and zero sequence component V_{a2} of phase a.and zero sequence component $V_{a0}$of phase a.

$$i.e., V_a = V_{a1} + V_{a2} + V_{a0}$$

* Since all power systems considered to be linear, so superposition principle holds good. Similarly

$$V_b = V_{b2} + V_{b2} + V_{b0}$$

$$V_c = V_{c1} + V_{c2} + V_{c0}$$

3 unbalanced vectors (V_a, V_b, V_c)resolved into a total of 9 vectors, i.e, $V_{a1}, V_{a2}, V_{a0}$and V_{b1}, V_{b2}, V_{b0} and V_{c1} , V_{c2}, V_{c0}. Out of these, only 3 are unknown. So, only three components are linearly independent. (i.e. from the knowledge of the three vectors V_{a0}, V_{a1}, V_{a2} we can find out all the remaining vectors)

OPERATORS: An operator 'a' is introduced, which when operates upon a phasor rotates it by +120° without changing the magnitude of the phasor upon which it operates. It is represented as

$$a = 1\angle 120^0 = e^{J120} = -0.5 + J0.866$$

$$a^2 = 1\angle -120^0 = e^{-J120} = -0.5 - J0.886$$

$$a^3 = 1\angle 360^0 = 1\angle 0^0 = e^{J360} = 1$$

$$\therefore 1 + a + a^2 = 0$$

$$a^4 = a; \quad a^5; = a^2; \quad a^6 = 1;$$

$$1 - a = \sqrt{3}\angle -30^0; \qquad a - 1 = \sqrt{3}\angle 150^0$$

$$a^2 - 1 = \sqrt{}\angle 210^0 \qquad 1 - a^2 =$$

$\sqrt{3}\angle 30^0$

$$a - a^2 = J\sqrt{3}; \qquad a^2 - a = -J\sqrt{3}$$

$$a^2 + a = -1$$

* Assuming phases ‘a’ as the reference, the relationship between the symmetrical components of phases 'b' and 'c' in terms of symmetrical component of phase 'a' can be written.

(phase sequence is abc)

$$V_a = V_{a1} + V_{a2} + V_{a0}$$

$$V_b = a^2 V_{a1} + aV_{a2} + V_{a0}$$

$$V_c = aV_{a1} + a^2 V_{a2} + V_{a0}$$

or in matrix form

$$\begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{pmatrix}$$

$$\underline{V}^{abc} = [T] \quad \underline{V}^{012}$$

Where, T = symmetrical transformation matrix

$V_{a0} = (1/3)\,[V_a + V_b + V_c]$

$V_{a1} = (1/3)\,[V_a + a\,V_b + a^2\,V_c]$

$V_{a2} = (1/3)\,[V_a + a^2\,V_b + a\,V_c]$

or in matrix form

$$\begin{pmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{pmatrix} = (1/3) \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix} \begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix}$$

$$\underline{V}^{012} = [T^{-1}] \quad \underline{V}^{abc}$$

where, T^{-1} = inverse symmetrical transformation matrix

Current instead of Voltage:

$I_a = I_{a1} + I_{a2} + I_{a0}$

$I_b = a^2\,I_{a1} + a\,I_{a2} + I_{a0}$

$I_c = a\,I_{a1} + a^2\,I_{a2} + I_{a0}$

or in matrix form

$$\begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} = (1/3) \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{pmatrix}$$

$$\underline{I}^{abc} = [T] \quad \underline{I}^{012}$$

Where, T symmetrical transformation matrix

$I_{a0} = (1/3)\,[\,I_a + I_b + I_c]$

$I_{a1} = (1/3)\,[\,I_a + a\,I_b + a^2\,I_c]$

$I_{a2} = (1/3)\,[\,I_a + a^2\,I_b + a\,I_c]$

or in matrix form

$$\begin{pmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{pmatrix} = (1/3) \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix} \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix}$$

$$\underline{I}^{012} = [T^{-1}] \; \underline{I}^{abc}$$

where, T^{-1} = inverse symmetrical transformation matrix

Average 3 – phase power in Terms of Symmetrical Components:

$$P = 3\,[|V_{a0}|\,|I_{a0}|\cos\theta_0 + |V_{a1}|\,|I_{a1}|\cos\theta_1 + |V_{a2}|\,|I_{a2}|\cos\theta_2]$$

Impedance:

$$\underline{V}^{abc} = [Z^{abc}]\,\underline{I}^{abc}$$

$$[T]\,\underline{V}^{012} = [Z^{abc}]\,[T]\,\underline{I}^{012}$$

$$\underline{V}^{012} = [T^{-1}]\,[Z^{abc}]\,[T]\,\underline{I}^{012}$$

$$\underline{Z}^{012} = [T^{-1}]\,[Z^{abc}]\,[T] \quad (\text{since } Z^{012} = \underline{V}^{012} / \underline{I}^{012})$$

$$[Z^{abc}] = \begin{pmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{pmatrix}$$

Z_s & Z_m are self & mutual impedances

$$[Z^{012}] = \begin{pmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{pmatrix}$$

Sequence Impedance and Networks:

The impedance of a circuit when positive sequence currents alone are flowing is called the impedance to positive sequence current. Similarly, when only negative sequence currents are present, the impedance is called the impedance to negative sequence current. When only zero sequence currents are present, the impedance is called the impedance to zero sequence current.

Let Z_a, Z_b, Z_c, & Z_n are three phase impedances and neutral impedance

$Z_{a0} = 1/3\,(Z_a + Z_b + Z_c)$ zero sequence

$Z_{a1} = 1/3\,(Z_a + aZ_b + a^2 Z_c)$ positive sequence

$Z_{a2} = 1/3\,(Z_a + a^2 Z_b + aZ_c)$ negative sequence

3.2 Sequence N/Ws of a synchronous Generator:

3.2.1 Positive sequence Network:

The positive sequence network of a synchronous generator is a balanced and symmetrical three phase network with positive sequence generated voltages (E_{a1}, E_{b1}, E_{c1}) in the three phase, symmetrical three phase star impedance Z, in each phase and positive sequence currents (I_{a1}, I_{b1}, I_{c1}) flowing in the three phase of shown in the below figure.

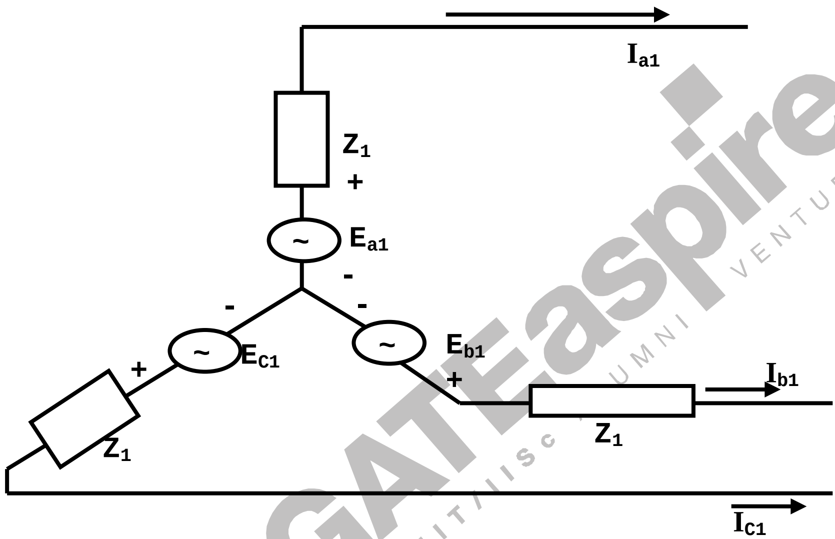
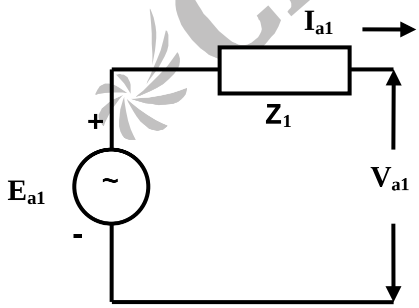


Fig. 3.2

The three phase system can be replaced by a single phase network as shown in the below:



'E' for the Generated voltage.

'V' for the Terminal voltage.

Fig. 3.3

The equation of the positive sequence network is $V_{a1} = E_{a1} - I_{a1} . Z_1$

3.2.2 Negative sequence network:

In any general synchronous generator which is designed to generate, balanced voltages the negative sequence generated voltages are always zero.

The negative network containing negative sequence impedance Z_2 in each phase and negative sequence currents flowing through these carrying negative sequence voltage drops as shown below figure.

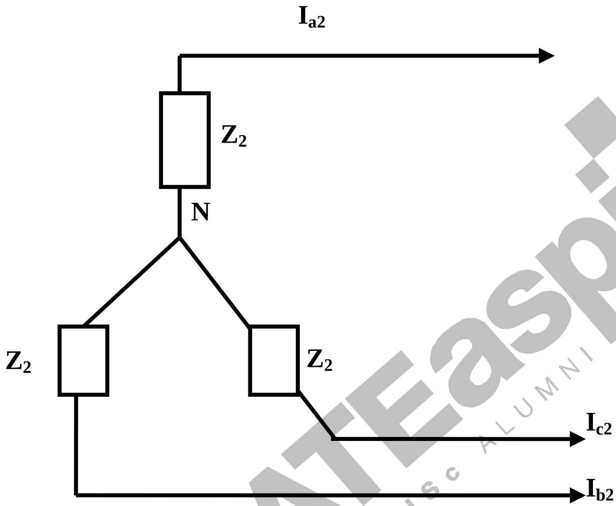


Fig. 3.4

The three phase network can be replaced by a single – phase network as shown in the below figure.

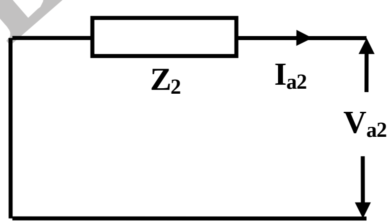


Fig. 3.5

The equation for the negative sequence network is $V_{a2} = - I_{a2}.Z_2$

3.2.3 Zero sequence Network:

The equation for the zero sequence network is $V_{a0} = - I_{a0} Z_0$

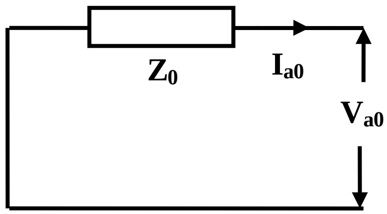


Fig. 3.6

The equivalent circuit for the zero sequence network is different from the positive and negative sequence networks the impedance offered to zero sequence currents depends on grounding of the star point.

If the star point is not grounded, then the zero sequence impedance is infinity.

If the star point is grounded through some impedance Zn, then the equivalent zero sequence impedance is equal to the sum of the zero sequence phase impedance and is equal to $3Z_n$.

- ∗ Balanced three phase system consists of positive sequence components only; the negative and zero sequence components being zero.
- ∗ The presence of negative or zero sequence currents in a three phase system introduces asymmetry and is indication of an abnormal condition of the circuit in which these components are found.
  - The vector sum of the positive and negative sequence currents of an unbalanced three phase system is zero. The resultant solely consists of three zero sequence currents i.e.,
  - Vector sum of all sequence currents in three phase unbalanced system = $\vec{I}_{R0} + \vec{I}_{y0} + \vec{I}_{B0}$
- ∗ In a three phase, 4 – wire unbalanced system, the magnitude of zero sequence component is one – third of the current in the neutral wire.
- ∗ In a three – phase unbalanced system, the magnitude of negative sequence component cannot exceed that of the positive sequence component. If the negative sequence component are greater, the phase sequence of the resultant system would be reversed.
- ∗ The current of a single phase load drawn from a three phase system comprises equal positive, negative and zero sequence component

Example:

The line to ground voltage on the high voltage side of a step up Transformer are 100 kV, 33 kV, 38 kV on phases a, b, & c respectively. The voltage of phase a leads That of phase b by 100° & lags That of phase c by 176.5°. Determine analytically The symmetrical component of voltage

$V_a = 100\angle 0$, $V_b = 33\angle -100°$, $V_c = 38\angle 176.5°$

Solution:

$V_{a1} = \frac{1}{3}(V_a + \lambda V_b + \lambda^2 V_c)$

$\Rightarrow \frac{1}{3}\ [100\ \angle 0 + 33\angle - 100°, \angle 120 + 38\angle 176.5°\angle - 120°]$

$\Rightarrow 50.56 + J\ 14.32 = 52.548\ \angle 15.81$

$V_{a2} \Rightarrow \frac{1}{3}\ [V_a + \lambda^2 V_c + \lambda\ V_c]$

$\Rightarrow \frac{1}{3}[100 + 33\ \angle - 100\ \angle - 120 + 38\ \angle 176.5\ \angle 120]$

$V_{a2} = 30.55 - J\ 4.26 \Rightarrow 30.85\ \angle - 7.94$

$V_{a0} = \frac{1}{3}\ (V_a + V_b + V_c)$

$\Rightarrow 18.79 - J\ 10.06\ \Rightarrow 21.31\ \angle - 28.16.$

3.3 FAULT CALCULATIONS:

Faults can be classified as two types:

1. Series faults 2. Shunt faults

* Shunt faults are characterized by increase in current and decrease in voltage, frequency and power factor.
* Series faults are characterized by decrease in current and increase in voltage, frequency and power factor.

The series faults are classified as

1. One open conductor fault 2. Two open conductors fault

The shunt type of fault are classified as:

1. Single line to ground fault
2. Line to Line fault
3. Double line to Ground fault
4. Three phase fault
   * The first three faults are the unsymmetrical faults.
   * The three phase fault is symmetrical faults.
   * Severity & occurrence of Faults:

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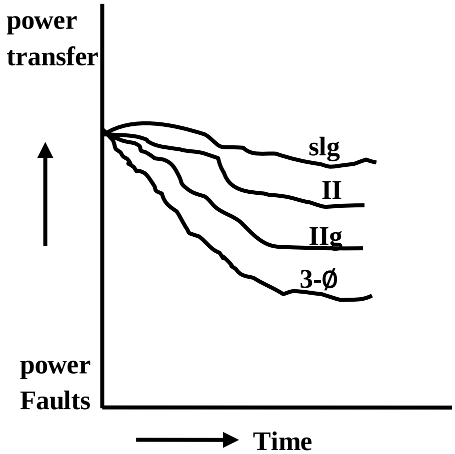

Fig. 3.7

	Fault	Severity	Occurrence
1)	3-Ø (LLL,LLLG)	Severe	5%
2)	Phase to phase ground (LLG)	Severe	10%
3)	Phase to phase fault (LL)	Less Severe	15%
4)	Single line to ground Faults (LG)	Very less	70%

* 1 cause of short circuit → insulation failure.

- Overvoltage caused by lightning or switching surge.
- Insulation contamination → salt spray, pollution.
- Mechanical causes → overheating, abrasion.

3.3.1 Faults on T.L.

- Most common – lines are exposed to elements of nature 60 – 70% lightning stroke → over voltage causes insulation to flash over
- Line to ground short circuit or line to line s.c.
- High winds → topple tower, tree falls on line.
- Winds an ice loading → mechanical failure of insulation.
- For, salt spray, dirty insulation → conduction path → insulation failure.

S. C. other elements.

Cabels (10 – 15%), C.B. (10 – 12%) generator, motor, X-mer (10 – 15%)
→ much less common → over loading for extended periods → deterioration of insulation

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3.3.2 Voltage of the Neutral:

The potential of the neutral when it is grounded through some impedance or is isolated, will be at ground potential unbalanced conditions such as unsymmetrical faults. The potential of the neutral is given as $V_n = I_n Z_n$ where Z_n is the neutral grounding impedance and I_n is neutral current. Here negative sign is used as the current flow from the ground to the neutral of the system and potential of the neutral is lower than the ground.

For a 3- phase system,

$$I_n = I_a + I_b + I_c$$

$$= (I_{a1} + Ia_2 + Ia_0) + (a^2 Ia_1 + aIa_2\ Ia_0) + (a\ I_{a1} + a^2\ Ia_2 + Ia_0)$$

$$= I_{a1}\ (1+a+a^2) + Ia_2\ (1+a+a^2) + 3Ia_0$$

$$= 3Ia_0$$

$$\therefore V_n = -3I_{a0}\ Z_n$$

Since the positive sequence and negative sequence components of currents through the neutral are absent, the drops due to these currents are also zero.

3.3.3 Single Line to Ground Fault:

1. Most frequently occurring fault.
2. Usually assumed the fault on phase – a for analysis purpose, phase – b and phase – are healthy.

The boundary conditions are

a) $V_a = 0$ ……….(1)
b) $I_b = 0$ ……….(2)
c) $I_c = 0$ ……….(3)

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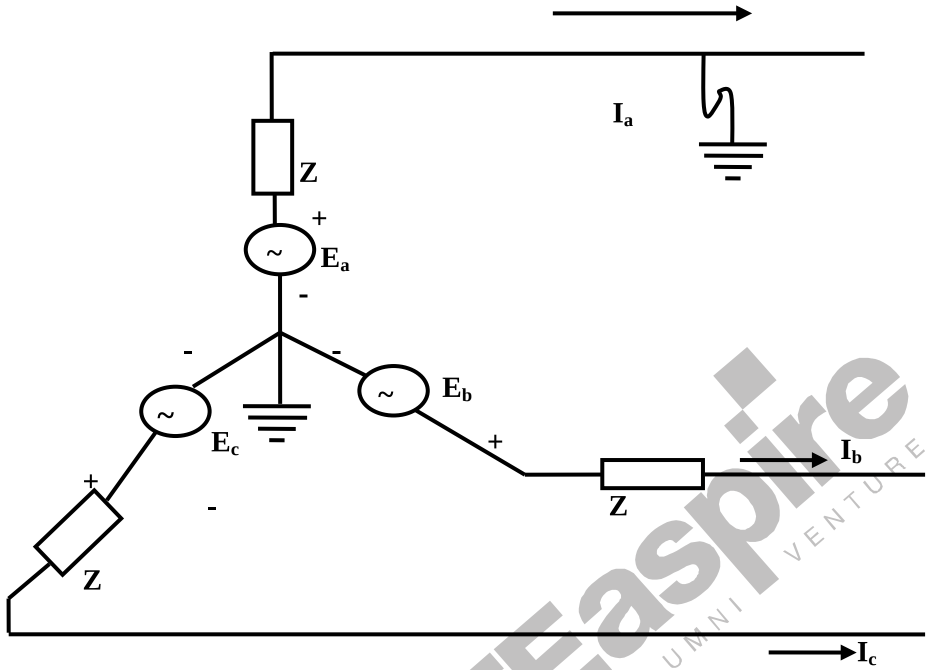

Fig. 3.8

The sequence network equations are

$V_{a0} = - I_{a0} Z_0$ …………..(4)

$V_{a1} = E_a - I_{a1} Z_1$ ……………(5)

$V_{a2} = I_{a2} Z_2$ ……………(6)

The solution of these six equations will give six unknowns V_{a0}, V_{a1}, V_{a2} and I_{a0}, I_{a1}, and I_{a2}

Since $I_{a1} = (1/3)\ [I_a + a\ I_b + a^2\ I_c]$

$I_{a2} = (1/3)\ [I_a + a^2\ I_b + a\ I_c]$

$I_{a0} = (1/3)\ [I_a + I_b + I_c]$

Substituting the values of I_b and I_c from equations (2), (3)in above three equations

$I_{a1} = I_{a2} = I_{a0} = I_a/3$ ………………….(7)

Equation (1) can be written in terms of symmetrical components

$V_a = 0 = V_{a1} + V_{a2} + V_{a0}$

Now substituting the values of V_{a0}, V_{a1} and V_{a2} from the sequence network equation,

$E_a - I_{a1} Z_1 - I_{a2} Z_2 - I_{a0} Z_0 = 0$ ……………… (8)

Since $I_{a1} = I_{a2} = I_{a0}$

Equation (8) becomes

$$E_a - I_{a1} Z_1 - I_{a1} Z_2 - I_{a1} Z_0 = 0$$

$$I_{a1} = \frac{Ea}{Z_1 + Z_2 + Z_0}$$

$I_f = I_a = I_{a1} + I_{a2} + I_{a0}$; $I_{a1} = I_{a2} = I_{a0}$ and +ve, −ve and zero sequence networks are connected in series.

$$I_f = \frac{3\ Ea}{Z_1 + Z_2 + Z_0}$$

single Line of Ground fault with Z_f :

$$I_{a1} = \frac{Ea}{Z_1 + Z_2 + (Z_{g0} + 3Z_n) + 3\ Z_f}$$

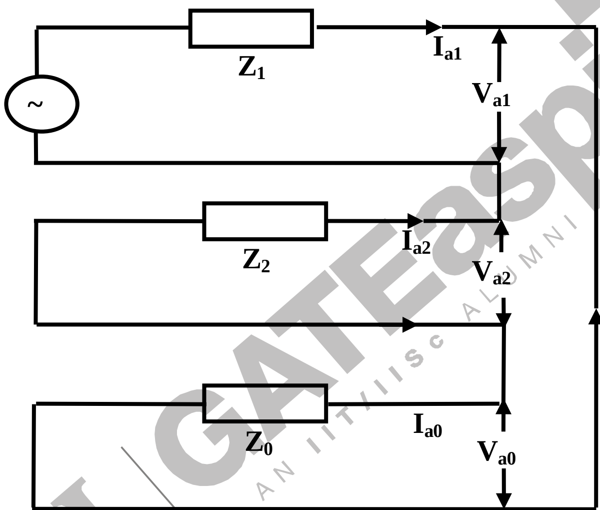

Fig. 3.9

Conclusion:

1. The three sequence networks are connected in series.
2. If the neutral of the generator is not grounded, the Zero network is open circuited.

Line to Line Fault

The line to line fault takes place on phases ‘b’ and ‘c’

The boundary conditions are

$I_a = 0$ …………. (1)

$I_b + I_c = 0$ …………. (2)

$V_b = V_c$ ....(3)

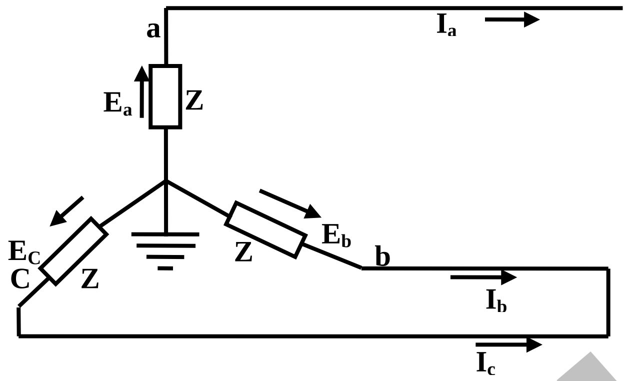

Fig. 3.10

The sequence network equations are

$V_{a0} = -I_{a0} Z_0$ ……………..(4)

$V_{a1} = E_a = I_{a1} Z_1$ ………………(5)

$V_{a2} = - I_{a2} Z_2$ ………………(6)

The solution of these six equations will give six unknowns V_{a0}, V_{a1}, V_{a2}, and I_{ao}, I_{a1} and I_{a2}

Since $I_{a1} = (1/3) [I_a + a I_b + a^2 I_c]$

$I_a = (1/3) [I_a + a^2 I_b + a I_c]$

$I_{a1} = (1/3) [I_a + I_b + I_c]$

Substituting the values of I_a, I_b and I_c from equations in above three equations

$I_{a1} = (1/3) [0 + a I_b - a^2 I_b]$

$= (1/3) [a - a^2] I_b$

$I_{a2} = (1/3) [0 + a^2 I_b - a I_b]$

$= (1/3) [a^2 - a] I_b$

$I_{a0} = (1/3) [0 + I_b - I_b]$

$I_{a0} = 0$

Which means for a line to line fault the zero sequence component of current is absent and positive sequence component of current is equal in magnitude but opposite in phase to negative sequence component of current, i.e.,

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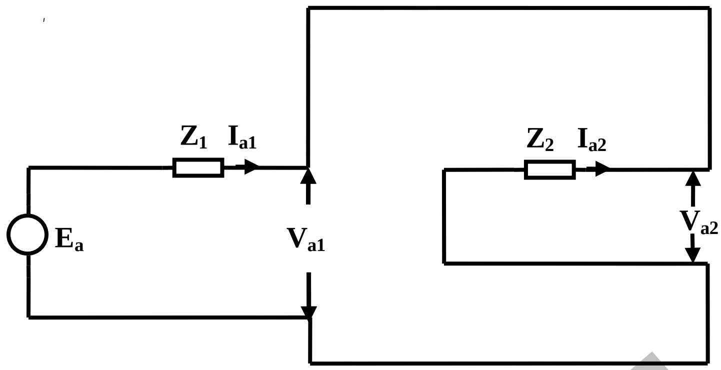

Fig. 3.11

$I_{a1} = -I_{a2}$ …………… (7)

Since $V_b = a^2V_{a1} + aV_{a2} + V_{a0}$ ………….. (8)

$V_C = aV_{a1} + a^2V_{a2} + V_{a0}$ …………..(9)

Substituting the equations (8) and (9) equations (3)

$a^2V_{a1} + aV_{a2} + V_{a0} = aV_{a1} + a^2V_{a2}V_{a0}$

∴ $V_{a1} = V_{a2}$ …………(10)

i.e., positive sequence component of voltage equals the negative sequence component of voltage. This also means that the two sequence networks are connected in opposition. Now making use of the sequence network equation and the equation (10) ∴ $V_{a1} = V_{a2.}$

$E_a - I_{a1} Z_1 = -I_{a2} Z_2 = I_{a1} Z_2$

$I_{a1} = \frac{Ea}{Z_1 + Z_2}$

$I_f = I_b = - I_c = a^2I_{a1} + aI_{a2} + I_{a0}$ ($I_{a2} = - I_{a1}$, $I_{a0} = 0$)

$= (a^2 - a)Ia_1 = -J\sqrt{3}\ I_{a1}$

$= \frac{-J\sqrt{3}\ Ea}{Z_1 + Z_2}$

Conclusion:

1. The connection of sequence currents are connected in parallel.
2. The phase difference between I_{a1} and I_{a2} for line – to – line fault should be 180° ($I_{a1} = I_{a2}$). Line to Line fault with Z_f

$I_{a1} = \frac{Ea}{Z_1 + (Z_2 + Z_f)}$

Double Line to Ground Fault: Here sequence networks are connected in parallel.

Double line to ground fault takes place on phases 'b' and 'c' The boundary conditions are

$I_a = 0$ ……………….(1)

$V_b = 0$ ……………….(2)

$V_c = 0$ ……………….(3)

The sequence network equations are

$V_{a0} = -I_{a0} Z_0$ ……….(4)

$V_{a1} = E_a - I_{a1} Z_1$ ……….(5)

$V_{a2} = -I_{a2} Z_2$ ....(6)

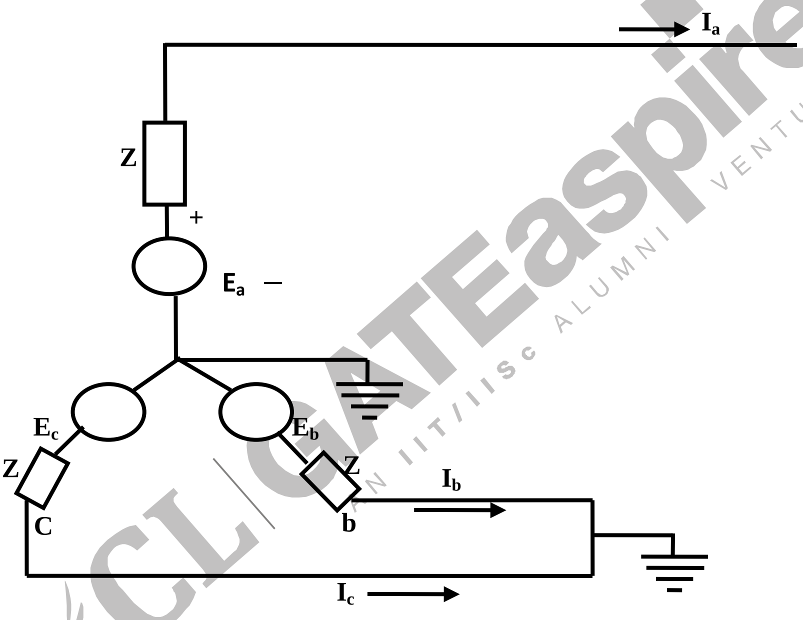

Fig. 3.12

The solution of these six equations will give six unknowns V_{a0}, V_{a1}, V_{a2} and I_{a0}, I_{a1} and I_{a2}.

Since $V_{a0} = (1/3)\,[V_a + V_b + V_c]$

$V_{a1} = (1/3)\,[V_a + aV_b + a^2 V_c]$

$V_{a2} = (1/3)\,[V_a + a^2V_b + a V_c]$

Using above three equations and substituting for V_a, V_b and V_c from the equation (2) and (3)

$V_{a0} = (1/3)\,[V_a + 0 + 0\,]$

$= V_a/3$

$V_{a1} = (1/3)\,[V_a + a.\,0 + a^2.\,0\,]$

$= V_a/3$

$V_{a2} = (1/3)\ [V_a\ + a^2.\ 0 + a.0\ ]$

$= V_a/3$

$V_{a0} = V_{a1} = V_{a2}$ …………….(7)

Using this relation of voltages and substituting in the sequence network equations

$V_{a0} = V_{a1}$

$-I_{a0}Z_0 = E_a - I_{a1}\ Z_1$

$I_{a0} = \frac{-[E_a - I_{a1}\ Z_1]}{Z_0}$………..(8)

Similarly

$V_{a2} = V_{a1}$

$-I_{a2}Z_2 = E_a - I_{a1}\ Z_1$

$I_{a2} = \frac{-[E_a - I_{a1}\ Z_1]}{Z_2}$ ……….(9)

Now from equation (1)

$I_a = I_{a1} + I_{a2} + I_{a0} = 0$

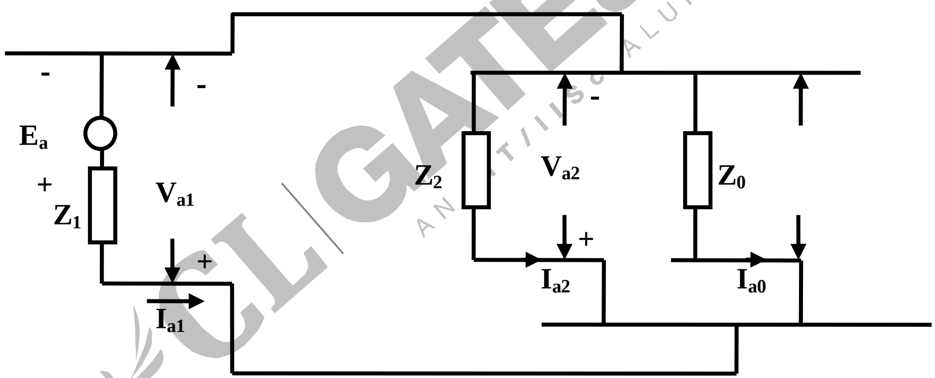

Fig. 3.13

Substituting values

of I_{a2} and I_{a0} from the equation(9) and (8)

$I_{a1} = \frac{[E_a - I_{a1}\ Z_1]}{Z_0} - \frac{[E_a - I_{a1}\ Z_1]}{Z_2} = 0$

Therefore $I_{a1} = \frac{E_a}{Z_1 + \frac{Z_0\ Z_2}{Z_0 + Z_2}}$

$I_f = I_b + I_c = 3\ I_{a0}$

$I_{a0} = -I_{a1}\ \frac{Z_2}{Z_0 + Z_2}$

$I_f = -3\ I_{a1}\ \frac{Z_2}{Z_0 + Z_2}$

Conclusion:

- Zero sequence and negative sequence networks are parallel and this is in series to the positive sequence.

Double line to ground fault with Z_f

$$I_{a1} = \frac{E_a}{Z_1 + \frac{Z_2(Z_0 + 3Z_n + 3Z_f)}{Z_2 + Z_0 + 3Z_n + 3Z_f}}$$

3.4 Three Phase Fault

The boundary conditions are

$I_a + I_b + I_c = 0$

$V_a = V_b = V_c$

I_a is taken as reference $I_b = a^2 I_a$ and $I_c = aI_a$

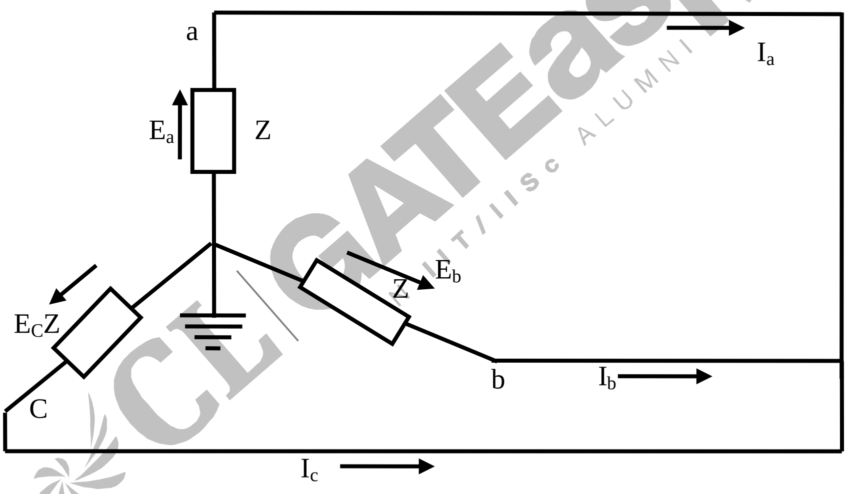

Fig. 3.14

$I_{a1} = 1/3\ (I_a + a\ I_b + a^2 . I_c)$

and substituting the values of I_b and I_c

$I_{a1} = 1/3\ (I_a + a.\ a^2\ I_a + a^2\ a\ I_a)$

$Ia_1 = Ia$

$I_{a2} = 1/3\ (I_a + a^2\ I_b + a\ I_c)$

$= 1/3\ (I_a + a^4\ I_b + a^2\ I_c)$

$= 1/3\ (I_a + a\ I_a + a^2\ I_c)$

$= \frac{I_a}{3} (1 + a + a^2)$

$= 0$

$I_{a0} = 1/3 (I_a + I_b + I_c)$

$= 0$

$V_{a1} = 1/3 (V_a + a V_b + a^2 V_c)$

$= 1/3 (V_a + a V_a + a^2 V_a)$

$= V_a (1 + a + a^2) = 0$

$V_{a1} = 1/3 (V_a + a^2 V_b + a.V_c)$

$= 0$

Since $V_{a1} = 0 = E_a - I_{a1} Z_1$,

$\therefore I_{a1} = E_a / Z_1$

* The frequent fault in transformer line is single line to ground fault.
* The most severe fault is 3-phase fault.
* The most severe fault nearer to generator is single line to ground fault.

From the analysis of the various faults, the following observations are made:

1. Positive sequence currents are present in all types of faults.
2. Negative sequence currents are present in all unsymmetrical faults.
3. Zero sequence currents are present when the neutral of the system is grounded and the fault also involves the ground, and magnitude of the neutral current is equal to $3 I_{a0.}$

* When currents are entering a delta connected winding, positive sequence components of line currents lead the corresponding positive sequence components of phase currents (winding current) in delta winding by 90°.
* Positive sequence components of line currents on the star side of a star delta transformer, lead the corresponding positive sequence line currents on the delta side by 90°.
* Positive sequence components of line and phase voltages on the star side of a star delta transformer lead the corresponding positive sequence line and equivalent phase voltages respectively on the delta side by 90°.

Proof:

Balanced three phase system consists of positive sequence components only; the negative and zero sequence components being zero.

Balanced system, Voltages must be in the form of

$V_b = a^2 V_a \qquad V_c = aV_a$

From Inverse symmetrical transformation,

$V^{012} = [T^{-1}]\ V^{abc}$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$= (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ a^2 V_a \\ a\, V_a \end{bmatrix}$$

$$= (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} V_a$$

$$= 1/3 \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} V_a \qquad \therefore 1 + a + a^2 = 0$$

$$a^3 = 1,\ a^4 = a^3 \,.\, a = a$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ V_a \\ 0 \end{bmatrix}$$

So for balanced 3 – ϕ system, $V_{a0} = V_{a2} = 0;\ V_{a1} = V_a$

Load Flow

* Load cannot be same for all time in the system. The power flow idea is to find out the voltage at different bus bar, sub - station, node point & the flow of power on these lines, with given constraints and specifications.

Types of Buses:

1. **Load bus:**
   In this type of bus, P_i and Q_i are known.
   The unknowns are $|V_i|$ and δ_i.

2. **Slack Bus/Reference Bus/Swing Bus:-**
   This bus is a special type of bus. Here real and reactive powers are not specified only $|V_i|$ and δ_i are known.

3. **Generator/Voltage control/PV Bus:-**
   In this type of bus P_i and $|V_i|$ are known. Q_i and δ_i are not known.

Power System Network

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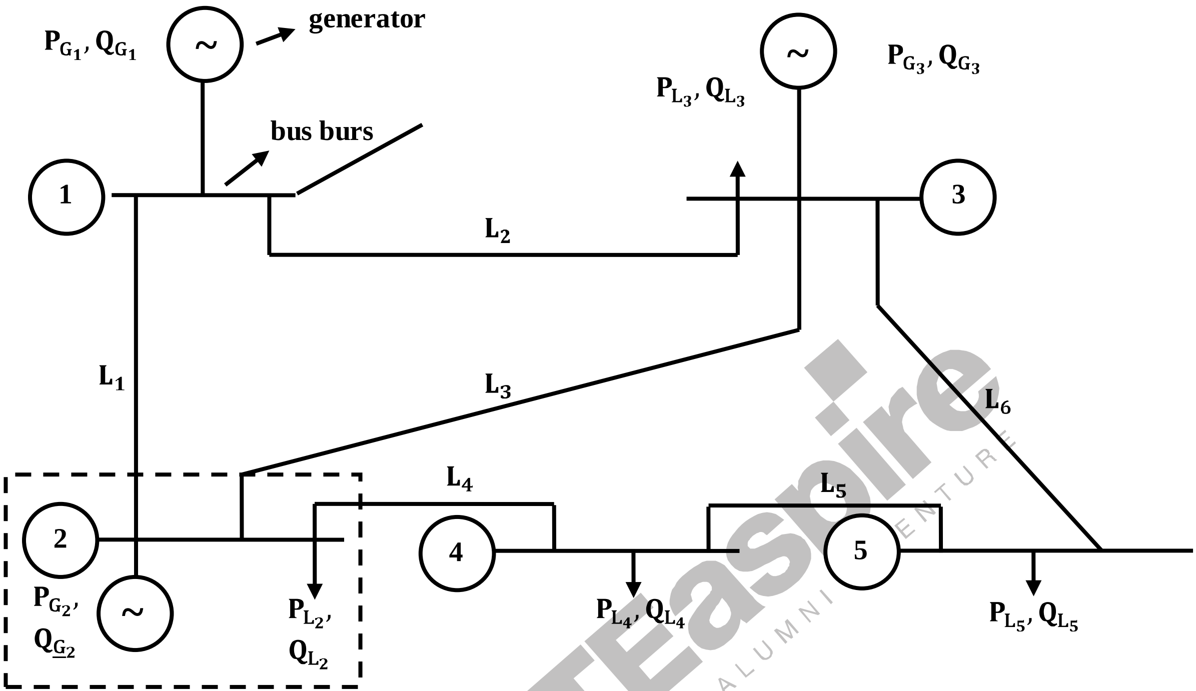


Fig. 3.15

L → Load

G → Generator

[Those bus who has self generator are called generated bus bar.]

[Who's don't have generator called load bus bar.]

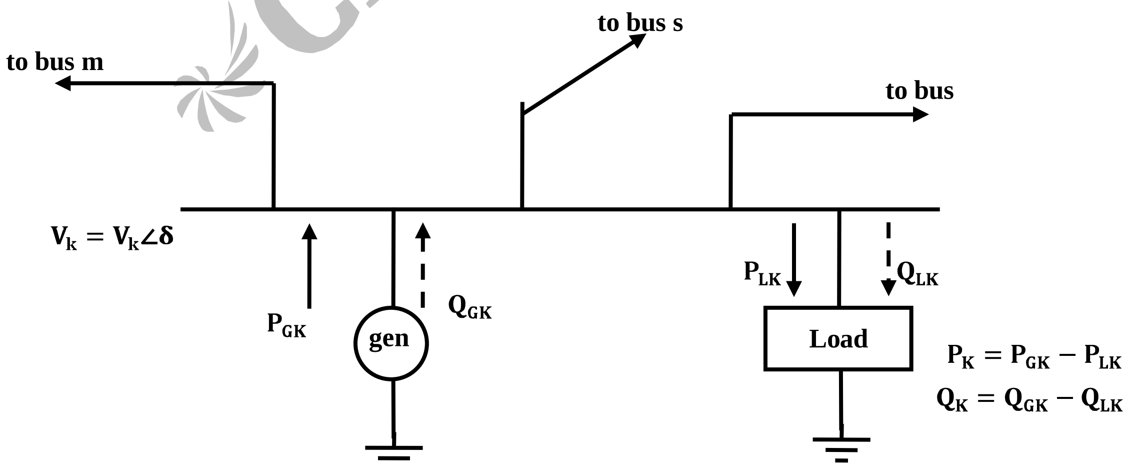


Fig. 3.16

Where $P_K \rightarrow$ injected power (real) in to bus.

& $Q_K \rightarrow$ injected (reactive) power into bus.

For load bus $P_{GK} = Q_{GK} = 0$

So we can write

$P_K = -P_{LK}$ (i)

$Q_K = -Q_{LK}$ (ii)

On the behalf of equation (i) and (ii) load bus are drawing the power.

$\Downarrow$

Example:

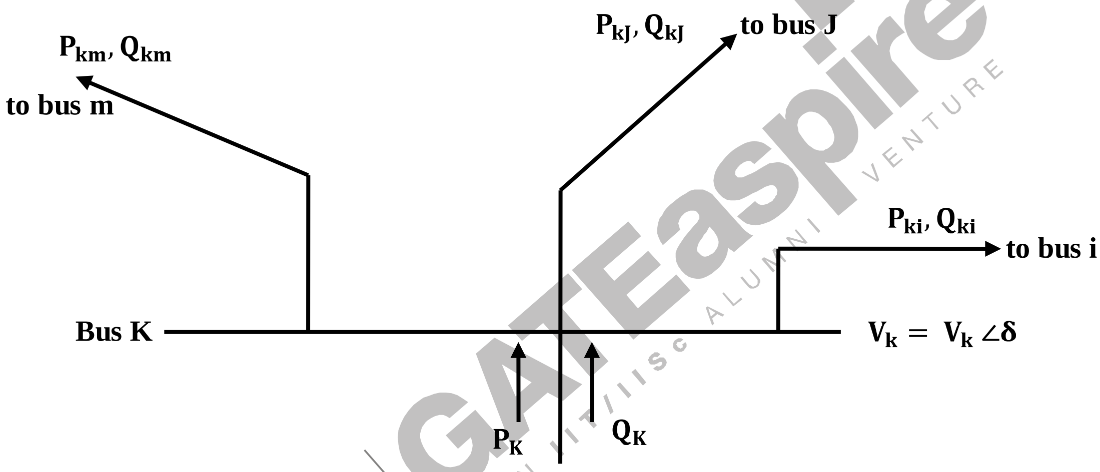


Fig. 3.17

$P_K = P_{GK} - P_{LK}$ $Q_K = Q_{GK} - Q_{LK}$

$P_K = P_{ki} + P_{kj} + P_{km}$ $Q_K = Q_{ki} + Q_{kj} + Q_{km}$

Static analysis of Power Network.

Mathematical model of the Network T.L. – nominal π model.

Bus power injection –

$S_K = V_K i_K^* = P_K + JQ_K$.

$P_K = P_{GK} - Q_{LK}, Q_K = Q_{GK} - Q_{LK}$.

But Admittance matrix ⇒

Taking small example

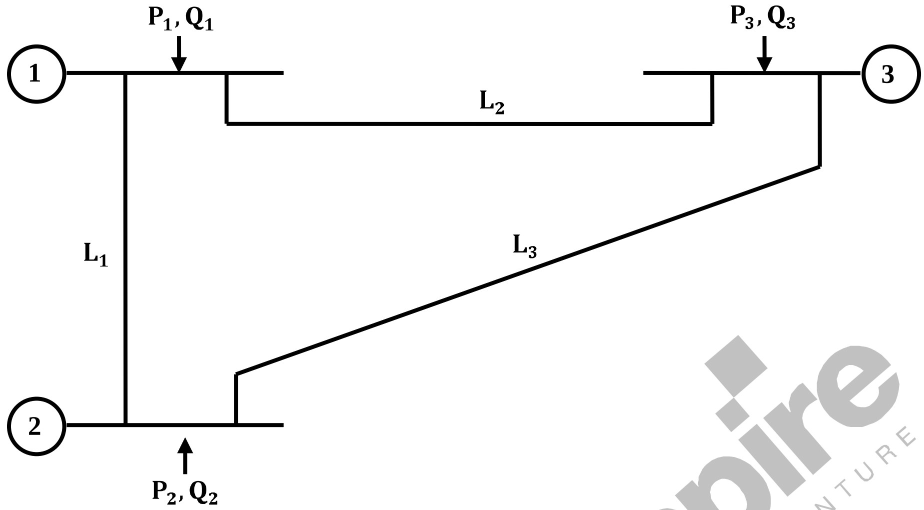


Fig. 3.18

equivalent π model.

⇓

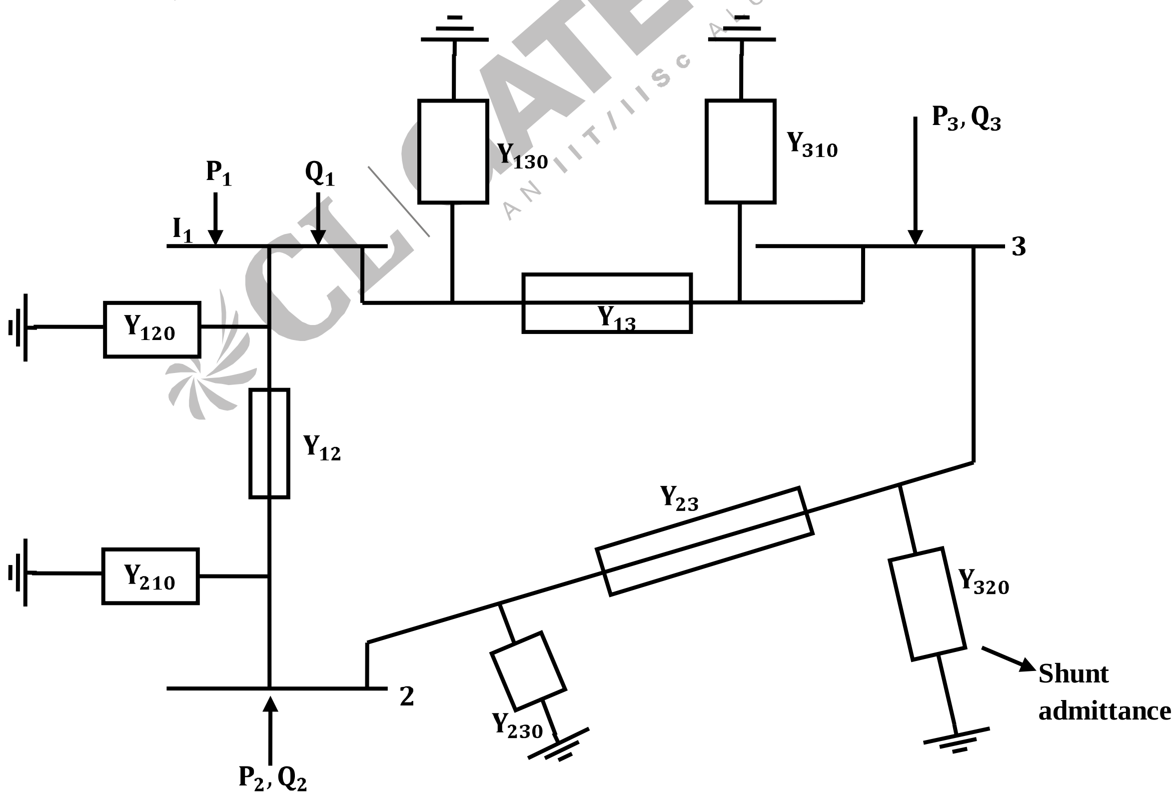


Fig. 3.19

Now power equation converting in current equation. So we can write equation.

$I_1 = Y_{120} V_1 + Y_{12} (V_1 - V_2) + Y_{130} V_1 + Y_{13} (V_1 - V_3)$

$I_2 = Y_{210} V_2 + Y_{12} (V_2 - V_1) + Y_{230} V_2 + Y_{23} (V_2 - V_3)$

$I_3 = Y_{310} V_3 + Y_{13} (V_3 - V_1) + Y_{320} V_3 + Y_{23} (V_3 - V_2)$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{120} + Y_{12} + Y_{130} + Y_{13} & -Y_{12} & -Y_{13} \\ -Y_{21} & \frac{(Y_{210} + Y_{12} + Y_{230} + Y_{23})}{Y_{22}} & -Y_{23} \\ -Y_{31} & -Y_{32} & (Y_{310} + Y_{13} + Y_{320} + Y_{23}) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$\rightarrow Y_{33}$

$\Downarrow$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$Y_{11} = Y_{22} = Y_{33} \Rightarrow$ self admittance. (Driving point admittance)

$Y_{12} = Y_{21} = ------- X_{31} \rightarrow$ transfer admittance or mutual adce.

$I_{BUS} = Y_{BUS} \quad V_{BUS}$

$V_{BUS} = Z_{BUS} I_{BUS}$ $\qquad Z_{BUS} = [Y_{BUS}]^{-1}$

1. Y_{BUS} formatting is so easy & & Z matrix comp.
2. Y_{BUS} is symmetric matrix

   $Y_{12} = Y_{21} = -Y_{12}$

   $Y_{13} = Y_{31} = -Y_{13}$

   $Y_{23} = Y_{32} = -Y_{23}$

   Y_{bus} matrix is sparse matrix. (Sparse means most of the elements of Y_{BUS} matrix are zero) Because there are 90% elements are zero.
3. Dimension of Y_{BUS} is $(N \times N) \rightarrow N =$ No. of bus

Power Flow Equation

$I_{BUS} = Y_{BUS} \; V_{BUS}$

$$I_K = \sum_{n=1}^{N} Y_{Kn} \; V_n$$

$S_K = P_K + JQ_K = V_K I_K^*$

$$P_K + JQ_K = V_K \left[\sum_{n=1}^{N} Y_{Kn} \; V_n\right]^*, K = 1, 2, -----N.$$

$V_n = V_n \cdot e^{J\delta n}$.

$Y_{Kn} = Y_{Kn} \, e^{j\theta_{kn}}$, k, n = 1, 2, - - - - - N.

$$P_K + JQ_K = V_K \sum_{n=1}^{N} Y_{Kn}\ V_n\ e^{J(\delta_K - \delta_n - \theta_{Kn})}$$

$$P_K = V_K \sum_{n=1}^{N} V_n\ Y_{Kn} \cos\ (\delta_K - \delta_n - \theta_{Kn})$$

$$Q_K = V_K \sum_{n=1}^{N} V_n\ Y_{Kn} \sin(\delta_K - \delta_n - \theta_{Kn})$$

Characteristics of power flow equation

Power flow equation are algebraic – static system

______________________nonlinear – Iterative solution

Relate P, Q in terms of V, δ & Y_{BUS} elements

$-P, Q \rightarrow f\ (V, \delta)$

There are two methods to solve these non –linear equations:

1. Newton Raphson method
2. Gauss – Seidal method

Newton Raphson (N.R.) method has got quadratic convergence and is fast as compared to Gauss – Seidal and always converge. But N.R. method requires more time per iteration. Gauss – Seidal has got linear convergence, convergence is affected by choice of slack bus and the presence of sense capacitor.

Example:

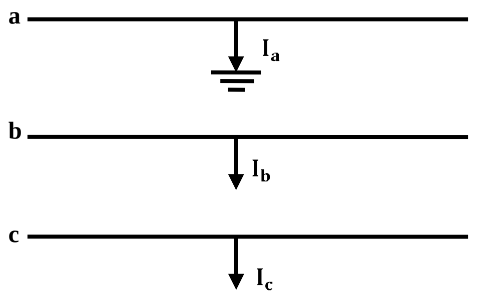
Assume a three – phase system with a sustained supply voltage of 2,300 volts from line to neutral and with line impedance $Z_1 = j\ 10\Omega, Z_2 = j10\Omega$ and $Z_0 = (10 + j\ 10)\Omega$. Compute the magnitude of fault current for a single line to ground fault at the end of the line.

Solution:

Supply voltage = 2300 V line/neutral

$Z_1 = j\ 10\ \Omega\ Z_2 = j\ 10\ \Omega\ Z_0 = (10 + j\ 10)\Omega$

For a single line to ground fault at "a" in Figure below.



$I_b = 0, I_c = 0, V_a = 0$

then $I_{a_1} = I_{a_2} = I_{a_0}$

$$I_{a_1} = \frac{V_f}{Z_0 + Z_1 + Z_2}$$

where V_f is prefault voltage

$$= \frac{2300}{j10 + j10 + 10 + j10} = \frac{2300}{10 + j30}$$

Also $I_a = I_{a_1} + I_{a_2} + I_{a_0} = 3I_{a_1}$

$$= \frac{3 \times 2300}{10 + j30} = 218.2\ \angle\underline{-71.57° A}$$

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