IIT-JEE – 2010 (Code-5)

Answer Keys

(Paper–II)

Chemistry

1. C
2. A
3. D
4. B
5. C
6. D
7. 2
8. 6
9. 2
10. 3
11. 7
12. B
13. C
14. B
15. B
16. A
17. D
18. A - p,s
    B - p,q,r,t
    C - p,q
    D - p,q
19. A - r,s
    B - t
    C - p,q
    D - r

Mathematics

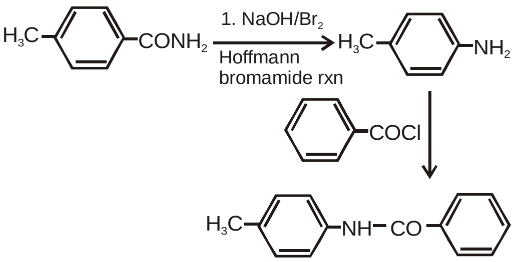
20. B
21. C
22. D
23. D
24. A
25. B
26. 1
27. 3
28. 4
29. 3
30. 0
31. D
32. C
33. A
34. C
35. A
36. B
37. A - t
    B - p,r
    C - q,s
    D - r
38. A - q
    B - p
    C - p,s,t
    D - q,r,s,t

Physics

39. B
40. A
41. B
42. D
43. C
44. D
45. 3
46. 4
47. 8
48. 2
49. 6
50. D
51. B
52. C
53. A
54. C
55. C
56. A - r,s,t
    B - q,r,s,t
    C - p,q
    D - q,r,s,t
57. A - p,r
    B - q,s,t
    C - p,r,t
    D - q,s

Chemistry

1. C



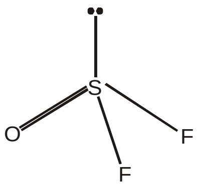
2. A If Hund's rule violated then electrnic configuration is

$\sigma 1s^2, \sigma * 1s^2, \sigma 2s^2, \sigma * 2s^2, \pi 2p_x^{\ 2}$

$B.O. = \frac{16-41}{2} = 1$

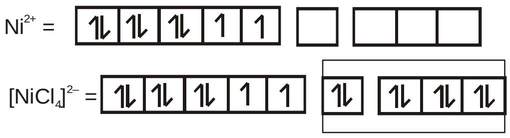
Bond order is 1 and diamagnetic.

3. D



(Pyramidal)

4. B



$[NiCl_4]^{2-}$ has two unpaired e^- so

$\mu = \sqrt{n(n+2)} = \sqrt{2(2+2)} = \sqrt{8} = 2.82$ B.M.

5. C In P, —OH group is activating and o-, p-directing but —COOH is deactivating.
In Q, —OCH_3 is more activating and o-, p- directing than —CH_3.
In S, —CO— is deactivating and —O— is activating.

6. D $\text{Packing Efficiency} = \frac{\text{Area occupied by atoms}}{\text{Total area of unit cell}}$

From unit cell,

$4r = L\sqrt{2} \Rightarrow r = \frac{L}{2\sqrt{2}}$; where r is the radius of atom.

$PE = \frac{2\pi r^2}{L^2} = \frac{2\pi L^2}{8L^2} = \frac{\pi}{4}$ $\quad \%PE = \frac{\pi}{4} \times 100 = 78.54$

7. 2 F always show –1 oxidation state. Na always show +1 oxidation state.

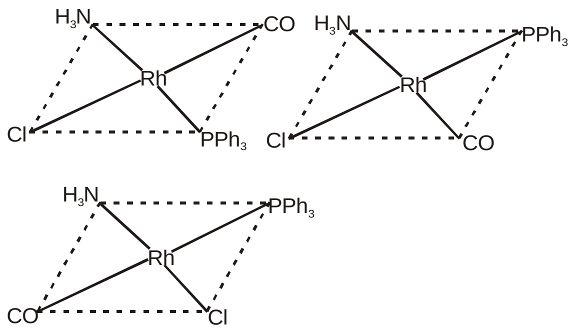
8. 6 $H_2SO_4, H_3PO_3, H_2CO_3, H_2S_2O_7, H_2CrO_4, H_2CO_3$ are biprotic acid.

9. 2 $w_S = (4 \times 1.5) + (1 \times 1) + (0.75 \times 0.25) = 8.875$

$w_D = \int_{0.5}^{5.5} \frac{2}{V} dV \approx 4.6$

$\frac{w_D}{w_S} \approx 2$

10. 3



11. 7 $\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{108 \times 3}{6.023 \times 10^{23} \times 4\pi r^3}$

$r^3 = \frac{108 \times 3}{6.023 \times 10^{23} \times 4 \times 3.14 \times 10.5} = 4.08 \times 10^{-24}\ cm^3$

$r = 1.4 \times 10^{-10}$ m

Area of one atom $= 6.15 \times 10^{-20}$

Number of atoms in 10^{-12} m^2 area $= \frac{10^{-12}}{6.15 \times 10^{-20}}$

$= 1.6 \times 10^7$

12. B S_1 has one radial node and spherically symmetrical 1s l=0.
Radial node = (n – l –1)
or n –0 – 1 = 1 or n = 2
Therefore, S_1 = 2s

13. C $E_n = -13.6 \times \frac{Z^2}{n^2} = -13.6 \times \frac{9}{4}$

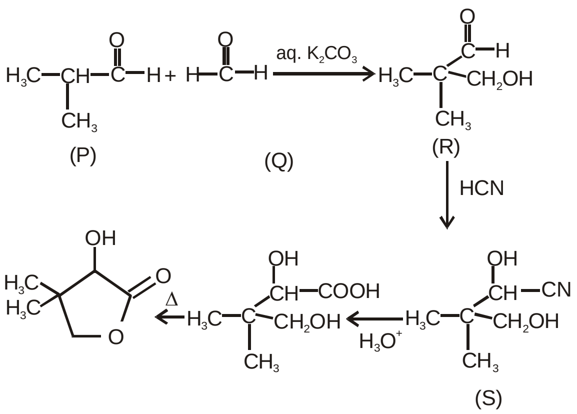
= 2.25 times to the energy of H atom in ground state.

14. B S_2 has one radial node and energy is equal to ground state energy of H atom.

So $-13.6 = -13.6 \times \dfrac{Z^2}{n^2} \Rightarrow n = 3$

Now radial nodes $(n - l - 1) = 1$
or $l = 1$

Solution of Qs 15, 16, 17



15. B

16. A

17. D

18. $(A \to p,s)$; $(B \to p,q,r,t)$; $(C \to p,q)$; $(D \to p,q)$

19. $(A \to r,s)$; $(B \to t)$; $(C \to p,q)$; $(D \to r)$

Mathematics

20. B $x = 0,\ f(0) = 2$

$e^{-x}f'(x) - e^{-x}f(x) = \sqrt{1+x^4}$

$f'(0) - 2 = 1\ \ f'(0) = 3$

$f^{-1}\left(f(x)\right) = x$

$f^{-1}\left(f(x)\right)f'(x) = 1$

$f^{-1}(2) = \dfrac{1}{3}$

21. C OG refers to signal being green originally.
GR green received
RR red received

$P\left(\dfrac{OG}{GR}\right) = \dfrac{P(OG \cap GR)}{P(GR)}$

$P(GR) = \dfrac{4}{5}\times\dfrac{3}{4}\times\dfrac{3}{4} + \dfrac{4}{5}\times\dfrac{1}{4}\times\dfrac{1}{4} + \dfrac{1}{5}\times\dfrac{3}{4}\times\dfrac{1}{4} + \dfrac{1}{5}\times\dfrac{1}{4}\times\dfrac{3}{4}$

$P(GR) = \dfrac{46}{80}$

$P(OG \cap GR) = \dfrac{4}{5}\times\dfrac{3}{4}\times\dfrac{3}{4} + \dfrac{4}{5}\times\dfrac{1}{4}\times\dfrac{1}{4}$

$P(OG \cap GR) = \dfrac{40}{80}$

$\Rightarrow\ P\left(\dfrac{OG}{GR}\right) = \dfrac{40}{46}$

22. D No. of required subsets will be 41

23. D $A_r = {}^{10}C_r,\ B_r = {}^{20}C_r,\ C_r = {}^{30}C_r$

$\sum_{r=1}^{10} {}^{10}C_r\left({}^{20}C_{10}\,{}^{20}C_r - {}^{30}C_{10}\,{}^{10}C_r\right)$

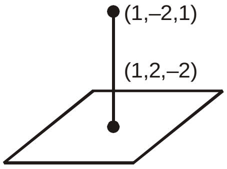
$\Rightarrow\ {}^{20}C_{10}\sum_{r=1}^{10} {}^{10}C_r\,{}^{20}C_r - {}^{30}C_{10}\sum_{r=1}^{10} {}^{10}C_r\,{}^{10}C_r$

$\Rightarrow\ {}^{20}C_{10}({}^{30}C_{10} - 1) \quad - {}^{30}C_{10}({}^{20}C_{10} - 1)$

$= {}^{30}C_{10} - {}^{20}C_{10}$

$= C_{10} - B_{10}$

24. A $\frac{x-1}{1}=\frac{y+2}{2}=\frac{y-1}{-2}=\lambda$



$(\lambda+1, 2\lambda-2, 1-2\lambda)$

$x+2y-2z-\alpha=0$

distance $5=\frac{|1-4-2-\alpha|}{\sqrt{1+4+4}}$

$|\alpha+5|=15$

$\alpha=10$

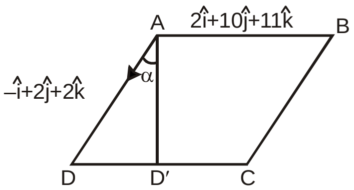
$(\alpha+1)+2(2\alpha-2)-2(1-2\lambda)-10=0$

$9\lambda-15=0 \quad \lambda=\frac{15}{9},\frac{5}{3}$

$\left(\frac{5}{3}+1, 2\frac{5}{3}-2, 1-\frac{5}{3}\right)$

$\left(\frac{8}{3},\frac{4}{3},-\frac{7}{3}\right)$

25. B



$\cos(90+\alpha)=\frac{2(-1)+(10)(2)+11(2)}{\sqrt{2^2+10^2+11^2}\sqrt{(-1)^2+(2)^2+(2)^2}}$

$=\frac{40}{153}$

$-\sin\alpha=\frac{8}{9}$

$\cos\alpha=\frac{\sqrt{17}}{9}$

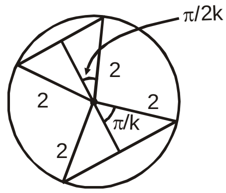
26. **1**

$f'(x)=2010(x-2009)(x-2010)^2(x-200)$

$f'(x)=\frac{1}{g(x)}.g'(x) \qquad g(x)=e^{f(x)}$

$g'(x)=g(x)\,f'(x)=e^{f(x)}f'(x) \quad g'(x)>0 \text{ if } f'(x)>0$

27. **3**



$2\cos\frac{\pi}{2k}+2\cos\frac{\pi}{k}=\sqrt{3}+1$

$\Rightarrow \cos\frac{\pi}{k}+\cos\frac{\pi}{2k}=\frac{\sqrt{3}}{2}+\frac{1}{2}$

$\Rightarrow \cos\frac{\pi}{k}=\frac{1}{2}$ or $\cos\frac{\pi}{2k}=\frac{\sqrt{3}}{2}\Rightarrow\frac{\pi}{k}=\frac{\pi}{3}$

$k=3$

28. **4**

$(\text{adj}A)=I|A|$

$\text{adj}=|A|^{n-1}=|A|^2$

$|A|^2+|B|^2=10^6$

$|A|=\begin{vmatrix}2k-1 & 2\sqrt{x} & 2\sqrt{k}\\ 2\sqrt{k} & 1 & -2k\\ -2\sqrt{k} & 2x & -1\end{vmatrix}, \quad |B|=0$

$|A|=(2k-1)(-1+4k^2)-2\sqrt{k}\left(-2\sqrt{k}-4k\sqrt{k}\right)$

$+2\sqrt{k}\left(2\sqrt{k}(2k)+2\sqrt{k}\right)$

$|A|=\left(8k^3+12k^2+6k+1\right)=(2k+1)^3$

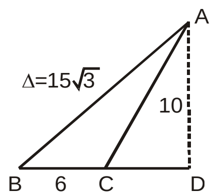
$(2k+1)^3=10^6 \qquad (2k+1)=10$

$2k=10$

$k=\frac{9}{2}$

$[k]=4$

29. **3**



$\Delta=15\sqrt{3}$

Area of triangle $=\frac{1}{2}\times AD\times BC$

$\Rightarrow 15\sqrt{3}=\frac{1}{2}\times AD\times 6$

$\Rightarrow AD=5\sqrt{3}$

Further $AC^2 = CD^2 + AD^2 \Rightarrow CD = 5$

$\therefore\ AB^2 = \left(5\sqrt{3}\right)^2 + (11)^2 = 196 \Rightarrow AB = 40$

$\therefore\ s = 15 \text{ and } \Delta = 15\sqrt{3} \Rightarrow r = \sqrt{3},\ r^2 = 3$

30. 0

$a_1 = 15$

further $2a_{k-1} = a_k + a_{k-2} \Rightarrow a_{k-2}, a_{k-1}, a_k$ are in A.P.

$\Rightarrow$ a_1, a_2, a_3...,a_{11} are in A.P.

as $\dfrac{a_1^2 + a_2^2 + \cdots + a_{11}^2}{11} = 90$

Assume $a_6 = 15 - 5d$, where d is the common difference of A.P.

$$\Rightarrow (a_6 - 5d)^2 + (a_6 - 4d)^2 + \cdots + (a_6)^2 + \cdots + (a_6 + 4d)^2 + (a_6 + 5d)^2 = 990$$

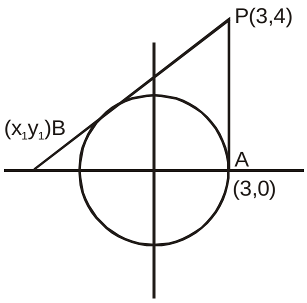
$\Rightarrow a_6^2 + 10d^2 = 90$

$\Rightarrow (15 + 5d)^2 + (10d)^2 = 90$

$\Rightarrow d = -3,\ -\dfrac{9}{7}$, only d = –3 is applicable as $a_2 < \dfrac{27}{2}$

Hence $a_6 = 0$ and $\dfrac{a_1 + a_2 + \cdots + a_{11}}{11} = 0$

31. D



Equation of tangent from (3, 4) $\dfrac{x}{3} + \dfrac{y}{4} = 1$

Solving with the curve $\dfrac{x^2}{9} + \dfrac{1}{4}\left[\left(\dfrac{3-x}{3}\right)4\right]^2 = 1$

solving we get $x = 3, -\dfrac{9}{5}$, $y = 0, \dfrac{8}{5}$

$A(3,0), B\left(-\dfrac{9}{5}, \dfrac{8}{5}\right)$

32. C Equation of line through B and perpendicular to PA is

$y = \dfrac{8}{5}$ as AP is parallel to y-axis

As orthocentre (G) lies on the above line, let us assume G as $\left(x_1, \dfrac{8}{5}\right)$

$m_{PG}.m_{AB} = -1$

$$\left(\frac{\frac{8}{5} - 4}{x_1 - 3}\right)\left(\frac{\frac{8}{5} - 0}{-\frac{9}{5} - 3}\right) = -1 \qquad x_1 = \frac{11}{5}$$

orthocentre $M\left(\dfrac{11}{5}, \dfrac{8}{5}\right)$

33. A P(3, 4), A(3, 0), $B\left(-\dfrac{9}{5}, \dfrac{8}{5}\right)$ let point be C(h, k)

$PC = \sqrt{(h-3)^2 + (k-4)^2}$

line $AB = 8x + 24y - 2y = 0$

Distance of C from $AB = \left|\dfrac{8x + 24y - 24}{\sqrt{640}}\right| = \left|\dfrac{x + 3y - 3}{\sqrt{10}}\right|$

$(x + 3y - 3)^2 = 10\left[(x-3)^2 + (y-4)^2\right]$

$x^2 + 9(y-1)^2 + 6x(y-1) = 10x^2 - 60x + 90 + 10y^2 - 160y + 160$

$x^2 + 9y^2 - 18y + 9 + 6xy - 6x = 10x^2 + 10y^2 - 60x - 160y + 160$

$9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

34. C $f(x) = 4 \times 3 + 3x^2 + 2x + 1$

$f'(x) = 12x^2 + 6x + 2 > 0\ \forall x \in R$

$f(+\infty) \to \infty$

$f(-\infty) \to -\infty$

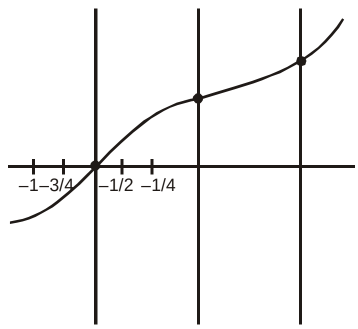
$f(0) > 0$

$f\left(-\dfrac{1}{4}\right) > 0$

$f\left(-\dfrac{1}{2}\right) > 0$

$f\left(-\dfrac{3}{4}\right) < 0$

$f(-1) < 0$



real root lies in $\left(-\dfrac{3}{4}, \dfrac{-1}{2}\right)$

35. A $I_1 = \int_0^{1/2} f(x)\,dx,\ I_2 = \int_0^{3/4} f(x)\,dx$

$= \left| x + x^2 + x^3 + x^4 \right|_0^{1/2}$

$I_2 = \left| x + x^2 + x^3 + x^4 \right|_0^{3/4}$

$I_1 = \frac{15}{16},\ I_2 = \frac{525}{256}$

36. B

37. $(A \to t),\ (B \to p,r),\ (C \to q,s),\ (D \to r)$

(A) Equation of plane passing through origin will be
A(x – 0) + B(x – 0) + C(z – 0) = 0

If this plane contain line1 $\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+1}{1}$,
the following conditions must be true

A ×1 + B × –2 + C × 1 = 0 …(i)
(direction ratio of normal to the plane are A,B,C)
and
A(2) + B(1) + C(–1) = 0 …(ii)
(point (2, 1, –1) lies on the plane)
Solving (i) and (ii) we get
A = 3B and C = + 5B
Hence equation of the plane passing through origin and containing line 1 will be
x + 3y + 5z = 0
Similarly we can find equation of plane passing through origin and containing line 2 as
3x + y – 5z = 0
The equation of line which intersect line 1 and line 2 will be
x + 3y + 5z = 0 = 3x + y – 5z
Direction ratios of this line will be in ratio of 5, –5, 2
Hence the equation of the desired line will be

$\frac{x}{5} = \frac{y}{-5} = \frac{z}{2}$ ..... line 3

To find out P, we will solve line 1 and line 3.

let us assume $\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+1}{1} = r$ and

$\frac{x}{5} = \frac{y}{-5} = \frac{z}{2} = s$

r+2 = 5s ; 1-2r = -5s ; r-1 = 2s
we get , r = 3 .
Hence , P = (5, -5 , 2)
Similarily by solving line 3 and line 2 , we get
Q = (10/3 , -10/3 , 4/3)
$PQ^2 = 6$

(B) $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}\frac{3}{5}$

$\tan^{-1}\frac{x+3-x+3}{1+x^2-9} = \sin^{-1}\frac{3}{5}$

$\tan^{-1}\frac{6}{x^2-8} = \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4}$

$\Rightarrow \frac{6}{x^2-8} = \frac{3}{4}$

$x^2 - 8 = 8$

$x^2 = 16 \quad x = \pm 4.$

(C) $\vec{a}.\vec{b} = 0 \Rightarrow \vec{a} \perp \vec{b}$

$(\vec{b}-\vec{a}).(\vec{b}+\vec{c}) = 0$

$\Rightarrow |\vec{b}|^2 + \vec{b}.\vec{c} = \vec{a}.\vec{c}$ …(ii)

$2|\vec{b}+\vec{c}| = |\vec{b}-\vec{a}|$

$3|\vec{b}|^2 + 4|\vec{c}|^2 + 8\vec{b}.\vec{c} = |\vec{a}|^2$ …(iii)

$\because \vec{a} = \mu\vec{b} + 4\vec{c}$

$\therefore |\vec{a}|^2 = \mu^2|\vec{b}|^2 + 16|\vec{c}|^2 + 8\mu\vec{b}.\vec{c}$

$\therefore (3-\mu^2)b^2 = 12c^2 + 8(\mu-1)\vec{b}.\vec{c}$

Further, $\mu b^2 = -4\vec{b}.\vec{c}$ and $4c^2 = \vec{a}.\vec{c} - \mu\vec{b}.\vec{c}$

$\therefore 4c^2 = b^2 + \vec{b}.\vec{c} - \mu\vec{b}.\vec{c}$

$\therefore (3-\mu^2)b^2 = 3b^2 + 3\vec{b}.\vec{c} - 3\mu\vec{b}.\vec{c} + 8(\mu-1)\vec{b}.\vec{c}$

$\therefore (-\mu^2)b^2 = 5(\mu-1)\vec{b}.\vec{c}$

$\therefore (-\mu^2)b^2 = 5(\mu-1)\left(-\frac{\mu}{4}\right)b^2$

$\Rightarrow 4\mu^2 = 5\mu^2 - 5\mu \quad \Rightarrow \mu = 0, 5$

(D) $f(x) = \dfrac{\sin\frac{9x}{2}}{\sin\frac{x}{2}}$ it is an even function

$$\therefore \frac{2}{\pi}\int_{-\pi}^{\pi} F(x)dx = \frac{4}{\pi}\int_0^{\pi}\frac{\sin\frac{9x}{2}}{\sin\frac{x}{2}}dx = \frac{4}{\pi}\int_0^{\pi}\frac{2\sin\frac{9x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}dx$$

$$= \frac{4}{\pi}\int_0^{\pi}\frac{\sin 5x}{\sin x} + \frac{\sin 4x}{\sin x}dx \quad \because \int_0^{\pi}\frac{\sin 4x}{\sin x}dx = 0$$

$$= \frac{4x^2}{\pi}\int_0^{\pi/2}\frac{\sin 5x}{\sin x}dx$$

$$= \frac{8}{\pi}\int_0^{\pi/2}\frac{\sin 3x\cos x + \cos 3x\sin 2x}{\sin x}dx$$

$$\frac{8}{\pi}\int_0^{\pi/2}(3 - 4\sin^2 x)\cos 2x + 2\cos 3x\cos x\,dx$$

$$\frac{8}{\pi}\int_0^{\pi/2}(3 - 2(1-\cos 2x))\cos 2x + \cos 4x + \cos 2x\ dx$$

$$\frac{8}{\pi}\int_0^{\pi/2}\cos 2x + 1 + \cos 4x + \cos 4x + \cos 2x\ dx$$

$$= \frac{8}{\pi}\left[\frac{2\sin 4x}{4} + \frac{2\sin 2x}{2} + x\right]_0^{\pi/2}$$

$$= \frac{8}{\pi}\left[0 + 0 + \frac{\pi}{2} - 0\right] = 4$$

38. $(A \to q),\ (B \to p),\ (C \to p,s,t),\ (D \to q,r,s,t)$

(A) z = x + iy

$$\left|x + iy - i\,|\,(x+iy)\,|\,\right| = \left|x + iy + i\sqrt{x^2+y^2}\right|$$

$$\left|x + i\left(y - \sqrt{x^2+y^2}\right)\right| = \left|x + i\left(y + \sqrt{x^2+y^2}\right)\right|$$

$$x^2 + y^2 + x^2 + y^2 - 2y\sqrt{x^2+y^2}$$
$$= x^2 + y^2 + x^2 + y^2 + 2y\sqrt{x^2+y^2}$$
$$4y\sqrt{x^2+y^2} = 0$$

$\Rightarrow$ y = 0

Im(z) = 0

(B) $|z+4| + |z-4| = 10$

This condition for ellipse

ae = 4 2a = 10

a = 5 $e = \frac{4}{5}$

Let $z = x + iy$

$$\left|(x+4)^2 + iy\right| - \left|(x-4) + iy\right| = 10$$

$$\sqrt{(x+4)^2 + y^2} + \sqrt{(x-4)^2 + y^2} = 10$$

$$x^2 + 16 + 8x + y^2 + 100 - 20\sqrt{(x+4)^2 + y^2}$$
$$= x^2 + 16 - 8x + y^2$$

$$16x + 100 = 20\sqrt{(x+4)^2 + y^2}$$

$$4x + 25 = 5\sqrt{(x+4)^2 + y^2}$$

$$16x^2 + 625 + 200x = 25x = 25x^2 + 400 + 200x + 25y^2$$

$$9x^2 + 25y^2 = 225$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

(C) Let $w = 2(\cos\theta + i\sin\theta)$

$$z = w - \frac{1}{w} = 2(\cos\theta + i\sin\theta) - \frac{1}{2}(\cos\theta + i\sin\theta)^{-1}$$

$$= 2(\cos\theta + i\sin\theta) - \frac{1}{2}(\cos\theta - i\sin\theta)$$

$$= \frac{3}{2}\cos\theta + \frac{5}{2}i\sin\theta$$

If z = x + iy

$$x = \frac{3}{2}\cos\theta \quad y = \frac{5}{2}\sin\theta$$

$$\left(\frac{2x}{3}\right)^2 + \left(\frac{2}{5}y\right)^2 = \cos^2\theta + \sin^2\theta$$

$$\frac{x^2}{\frac{9}{4}} + \frac{y^2}{\frac{25}{4}} = 1$$

$$\frac{9}{4} = \frac{25}{4}(1 - e^2)$$

$$e^2 = 1 - \frac{9}{25} \Rightarrow e = \frac{4}{5}$$

(D) $|w| = 1$

Let $w = \cos\theta + i\sin\theta \Rightarrow \frac{1}{w} = \cos\theta - i\sin\theta$

$$z = w + \frac{1}{w} = 2\cos\theta$$

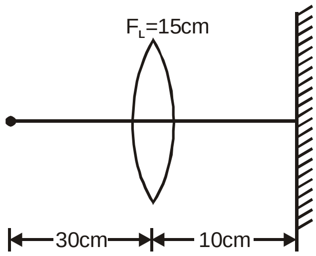
$$|\,z\,| = 2\cos\theta$$

If z = x + iy, $x = 2\cos\theta,\ y = 0$

$\text{Re}(z) \le 2\ \ \text{Im}(z) = 0$

Physics

39. B



For the lens, as the object is at 2F the 1st image will be at 2F.

This image acts as an object (virtual) for the mirror and hence it forms the 2nd image 20 cm in front of the mirror, which, acts as a virtual object for the lens place at 10 cm from the lens to its left in the direction of incident light.

$\therefore$ For final image $\frac{1}{v}-\frac{1}{10}=\frac{1}{15}$

v = 6 cm to the left of the lens.

$\therefore$ the image is 16 cm to the left of the mirror and is real.

40. A Pressure at a point on a charged spherical shell due to its own charge is $P=\sigma\left(\frac{\sigma}{2\varepsilon_0}\right)=\frac{\sigma^2}{2\varepsilon_0}$

Thus, force on each hemi-spherical shell will be

$$F_E = P.A. = \frac{\sigma^2}{2\varepsilon_0}\left(\pi R^2\right)$$

Hence, external force required to hold the shell is proportional to

$$F_E \propto \frac{\sigma^2 R^2}{\varepsilon_0}$$

41. B Length of pipe = 0.8 m

Length of string = 0.5 m

Tension = 50 N

velocity of sound = 320 m/s

$V_{sound} = V_{string}$

$4L_{pipe} = L_{string}$

$$\frac{320}{4\times 0.8}=\frac{1}{0.5}\sqrt{\frac{50}{\mu}}$$

$$\therefore\ \mu=\frac{1}{50}\Rightarrow\frac{m}{0.5}=\frac{1}{50}$$

$$\therefore\ m=\frac{1}{100}\text{kg}=10\,\text{gm}$$

42. D $1\text{ MSD} = 1\text{mm},\ 1\text{VSD}=\frac{16}{20}=0.8\,\text{mm}$

$\therefore$ LC = 1 MSD – VSD = 0.2 mm

43. C Area under the F-t graph represents the change in momentum

$$\text{Area}=\frac{1}{2}\left[4\times 3-2\times 1.5\right]=4.5$$

$$\therefore\ \Delta P = 4.5\,\text{kg}-\text{m/s}=P_f-P_i=P_f\ \text{ as }\ P_i=0$$

$$KE_f=\frac{P_f^2}{2m}=\frac{(4.5)^2}{2\times 2}=5.06$$

44. D $\frac{4}{3}\pi r^3\rho g = qE = 6\pi\eta r v$

$$\therefore\ r=\frac{qE}{6\pi\eta v}$$

$$q=\left[\frac{\left(6^3\pi^2\eta^3v^3\right)3}{4E^2\rho g}\right]^{1/2}$$

45. 3

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$v_1=\frac{25}{3}\text{m, so } u_1=-50\text{m}$$

$$v_2=\frac{50}{7}\text{m, so } u_2=-25\text{m}$$

Displacement of object = 25 m

Time = 30 s

$$\therefore\ \text{speed} = \frac{25}{30}\times\frac{18}{5}=3\,\text{km/hr}$$

46. 4

$$T_1V_1^{\gamma-1}=T_2V_2^{\gamma-1}\qquad T_1V_1^{\gamma-1}=\alpha T_1\left(\frac{V_1}{32}\right)^{2/5}$$

$$\therefore\ \alpha=(32)^{2/5}=4$$

47. 8

$$\left|\frac{dN}{dt}\right|=\lambda N_0e^{-\lambda t}\qquad \ln\left|\frac{dN}{dt}\right|=\ln(\lambda N_0)-\lambda t$$

$$Y=C+mx$$

$$m=-\frac{1}{2}\qquad \lambda=\frac{1}{2}$$

$$\frac{N}{N_0}=\frac{1}{P}=e^{-\lambda t}=e^{-2.08}$$

ln P = 2

or P = e^2

$P\approx 8$

48. **2**

Time constant of the circuit = $R_{eq}L_{eq}$

$\tau = 4s$

Instantaneous voltage

$V = V_0\left(1 - e^{-t/\tau}\right)$

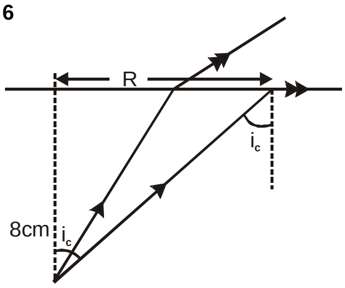
$4 = 10\left(1 - e^{-t/4}\right)$

$e^{-t/\tau} = 0.6$

$\frac{t}{\tau} = \log\left(\frac{5}{3}\right) = \log 5 - \log 3$

Hence t = 2s

49. **6**



$\sin i_c = \frac{3}{5}$

$\therefore\ \tan i_c = \frac{3}{4} = \frac{R}{8}$ $\therefore$ R = 6 cm

50. D $I\omega = \frac{nh}{2\pi}$

$\Rightarrow\ \omega = \frac{nh}{2\pi I}$

Therefore K.E. $= \frac{1}{2}I\omega^2$

$= \frac{1}{2}\times I\left(\frac{nh}{2\pi I}\right)^2$

$= \frac{n^2h^2}{8\pi^2 I}$

51. B Change in rotational energy = $h\nu$

$\frac{h^2}{8\pi^2 I}\left(2^2 - 1^2\right) = h\nu$

$\Rightarrow\ I = \frac{3h}{8\pi^2\nu} = 1.87\times10^{-46}\,\text{kgm}^2$

52. C $I = \mu r^2$ [where μ is the reduced mass r is the distance between C and O]

$1.87\times10^{-46} = \left(\frac{m_1m_2}{m_1+m_2}\right)r^2$

on solving r = 1.3 × 10^{-10} m

53. A Since force due to surface tension

F = T.l [l is the length line in contact)

$= T.2\pi r$ (given r << R)

Therefore answer is (A)

54. C When drop detaches; total upward force = Total downward force

$2\pi rT = mg$

$\Rightarrow\ 2\pi rT = \frac{4}{3}\pi R^3\rho g$

$R^3 = \frac{3rT}{2\rho g}$

$R = 2\times10^{-3}\,m$

55. C Surface energy $= 4\pi R^2 T$

$= 4\pi\times4\times10^{-6}\times0.11$

$= 5.4\times10^{-6}\,J$

56. $(A\rightarrow r,s,t)$, $(B\rightarrow q,r,s,t)$, $(C\rightarrow p,q)$, $(D\rightarrow q,r,s,t)$

For circuit p

I = 0, V_1 = 0, V_2 = V

Since it is an L-C circuit connected to a D.C. source, capacitor will cease the flow of current in steady state.

For circuit q

It is an R-L circuit connected to a D.C. source, thus V_1 across the inductor will be zero and V_2 is equal to V, $I \neq 0$ and hence $V_2 > 0$.

For circuit r

It is an R-L circuit connected to an A.C. source where X_L = LW = 6 × 10^{-3} × $2\pi\times50$ = 1.88 ohm. But, R is 2 ohm. Therefore X_L < R, hence $I \neq 0$, $V_1 < V_2$ and V_2 is proportional to I.

For circuit s

It is an L-C circuit connected to an A.C. source, where $X_C > X_L$. Hence $I \neq 0$, $V_2 > V_1$ and V_2 is proportional to I.

For circuit t

it is an R-C circuit connected to an A.C. source where X_C < R. Hence $I \neq 0$.

V_2 is proportional to I and $V_2 < V_1$

57. $(A\rightarrow p,r)$, $(B\rightarrow q,s,t)$, $(C\rightarrow p,r,t)$, $(D\rightarrow q,s)$

Ray bonds towards the normal on entering denses medium from rarer and away from the normal when going from denser to rares.